Nonparaxial propagation of a (1+1)-dimensional Pearcey beam

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We introduce a virtual source that generates a family of a Pearcey wave. We derive a closed-form expression for the (1+1)-dimensional Pearcey wave that in the appropriate limit yields the paraxial accelerating and non-diffracting Pearcey beam (PB). From the perturbative series representation of a complex-source-point spherical wave, we derive an infinite series nonparaxial correction expression for PB. The infinite series expression of a PB can provide accuracy up to any order of diffraction angle. From the integral representation of the Pearcey wave, the first three orders of nonparaxial corrections to the paraxial Pearcey beam are derived.

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I. INTRODUCTION

As is well known, self-accelerating beams have paid much attention since the concept of Airy wave packets was introduced from quantum mechanics [1] into optics in 2007 [2, 3]. Airy beams propagate along parabolic trajectories and have nondiffracting and self-healing properties [2–4]. In the last few years, such accelerating beams have been applied in manipulating microparticles [5], inducing curved plasma filaments [6], synthesizing versatile bullets of light [7], as well as carrying out autofocusing and supercontinuum experiments [8]. Many kinds of finite-energy accelerating beams were introduced. In 2010, Barwick [9] presented the accelerating regular polygon beams. In 2012, Ido Kaminer et al [10] introduced nonparaxial spatially accelerating shape-preserving beams. Zhang et al [11] experimentally demonstrated the linear and nonlinear nonparaxial accelerating beams (NABs) propagating along a circular trajectory. Recently, Zhang et al [12] explored the nonparaxial Mathieu and Weber accelerating Beams; Parinaz Aleahmad et al [13] theoretically shown the dynamics of self-similar accelerating 3D vectorial spherical wave functions. Ring et al [14] studied the auto-focusing and self-healing of Pearcey beams. However, the nonparaxial propagation of Pearcey beam (PB) has remained unexplored.

In this conference, by using the virtual source method [15–25] introduced by Deschamps [15] and systematically extended by Felsen and his collaborators [16, 17], we introduce the virtual source that generates the Pearcey wave, and we obtain a closed-form expression for the Pearcey wave, and the paraxial approximation and the first three order series nonparaxial correction are derived from this expression.

II. NONPARAXIAL PROPAGATION OF A (1+1) DIMENSIONAL PEARCEY BEAM

The (1+1) dimensional electric field distribution of the initial PB at the $z = 0$ plane can be expressed as [26, 27]

$$E(x,0) = \mathrm{Pe}(\frac{x}{w_0}, c_0), \tag{1}$$

where $\mathrm{Pe}(\cdot)$ is the Pearcey function, w_0 denotes arbitrary transverse scale, c_0 is a complex constant.

To establish a scalar Pearcey wave function $E(x, z)$ which travels along the positive z axis and satisfies the homogeneous Helmholtz equation for $z > 0$, we require that in the paraxial approximation, the input field distribution of the beam described by $E(x, z)$ reduces to Eq. (1). In the virtual source method, the beam is supposed to be generated by a source of strength S_{cs} situated at $x = 0$ and $z = z_{\mathrm{cs}}$ exterior to the physical space $z > 0$. The wave function $E(x, z)$ satisfies the inhomogeneous Helmholtz equation

$$\begin{aligned}\Big(\frac{\partial^2}{\partial x^2} \;+\; \frac{\partial^2}{\partial z^2} + k^2\Big)E(x,z) = -S_{cs} \\ \times\;\exp[-D(x)]\delta(x)\delta(z - z_{\mathrm{cs}}),\end{aligned} \tag{2}$$

where $D(x) = w_0^4\partial^4/\partial x^4$, k is the wave number.

Applying the Fourier transform pairs,

$$E(x,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{E}(f_x,z)\exp(if_x x)df_x, \tag{3}$$

$$\tilde{E}(f_x,z) = \int_{-\infty}^{+\infty} E(x,z)\exp(-if_x x)dx, \tag{4}$$

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where f_x is the spatial frequency in the x direction. From Eq. (2), $\tilde{E}(f_x, z)$ is obtained and substituted into Eq. (3), one can find that,

$$\begin{aligned} E(x,z) &= \frac{iS_{cs}}{4\pi}\int_{-\infty}^{+\infty} df_x \exp(if_x x)\exp(iw_0^4 f_x^4) \\ &\times \ 1/\zeta \exp[i\zeta(z - z_{cs})], \qquad (5) \end{aligned}$$

where $\mathrm{Re}(z - z_{cs}) > 0$, $\zeta = (k^2 - f_x^2)^{1/2}$. Expanding ζ into series and keeping the first and second terms, we obtain $\zeta = k(1 - f_x^2/(2k^2))$. Under paraxial approximation, i.e., $k^2 \gg f_x^2$, we replace ζ of the exponential part in Eq. (5) with the approximation and the other terms with k, Eq. (5) simplifies to,

$$\begin{aligned} E(x,z) &= \exp[ik(z - z_{cs})]\frac{iS_{cs}}{4\pi k} \\ &\times \int_{-\infty}^{+\infty} df_x \exp(if_x x)\exp(iw_1^4 f_x^4) \\ &\times \ \exp[-if_x^2(z - z_{cs})/(2k)], \qquad (6) \end{aligned}$$

Applying the Pearcey integral,

$$\mathrm{Pe}(s,t) = \int_{-\infty}^{+\infty} dq \exp(iq^4 + iq^2 t + iqs), \qquad (7)$$

the integral in Eq. (6) can be evaluated as,

$$\begin{aligned} E(x,z) &= \frac{iS_{cs}}{4\pi k w_0}\exp[ik(z - z_{cs})] \\ &\times \ \mathrm{Pe}(x/w_0, -i(z - z_{cs})/(2kw_0^2)). \qquad (8) \end{aligned}$$

In the input plane i.e., $z = 0$, Eq. (8) simplifies to

$$\begin{aligned} E(x,0) &= \frac{iS_{cs}}{4\pi k}\exp(-ikz_{cs}) \\ &\times \ \mathrm{Pe}(x/w_0, iz_{cs}/(2kw_0^2)). \qquad (9) \end{aligned}$$

Comparison Eqs. (9) and (1), one can obtain the parameters z_{cs} and S_{cs},

$$z_{cs} = -2ic_0 k w_0^2 = -4ic_0 z_R, \qquad (10)$$

$$S_{cs} = -4i\pi k w_0 \exp(4kc_0 z_R), \qquad (11)$$

where $z_R = 1/2kw_0^2$. With the expressions for z_{cs} and S_{cs}, and Eq. (5), the exact integral expression for E can be obtained,

$$\begin{aligned} E(x,z) &= kw_0 \exp(4kc_0 z_R)\int_{-\infty}^{+\infty} df_x \exp(if_x x) \\ &\times \ \exp(iw_0^4 f_x^4) 1/\zeta \exp[i\zeta(z + 4ic_0 z_R)], \quad (12) \end{aligned}$$

From Eq. (10), it is not difficult to find that the source in Eq. (2) lies external to $z > 0$, and Eq. (12) is the exact solution of the homogeneous equation corresponding to Eq. (2). Eq. (12) simplifies to the correct paraxial approximation in the appropriate condition if all the nonparaxial contributions and the evanescent waves are ignored.

By performing the power expansion $1/\zeta$ and $\exp[i\zeta(z - z_{cs})]$ in Eq. (12), in the product of both series terms up to order $(kw_0)^{-2j}$ are retained, we can obtain the jth-order corrections. The nonparaxial solution approaches the exact solution Eq. (12) when the parameter j increases. Particularly, when j becomes infinite, the nonparaxial solution becomes the exact solution. Here, we will give the first three nonparaxial corrections of the Pearcey beam for $j = 3$, Eq. (12) can be written in the form,

$$\begin{aligned} E(x,z) &= w_0 \exp(4kc_0 z_R)\int_{-\infty}^{+\infty} df_x \exp(if_x x) \\ &\times \ \exp[-if_x^2 z'/(2k) + iW_0 f_x^4] \\ &\times \ [1 + f_x^2/(2k^2) + 3f_x^4/(8k^4) \\ &+ \ (5/k + iz')f_x^6/(16k^5)], \qquad (13) \end{aligned}$$

where $W_0 = w_0^4 - z'/(8k^3)$. The integral is evaluated with the following result:

$$\begin{aligned} E(x,z) &= w_0/W_0^{1/4}\exp(4kc_0 z_R) \\ &\times \ [1 - 1/(2k^2)\partial^2/\partial x^2 + 3/(8k^4)\partial^4/\partial x^4 \\ &- \ (5/k + iz')/(16k^5)\partial^6/\partial x^6] \\ &\times \ \mathrm{Pe}(x/W_0^{1/4}, -z'/(2k\sqrt{W_0})). \qquad (14) \end{aligned}$$

Eq. (14) is the nonparaxial expression of the Pearcey beam with the first three nonparaxial corrections.

III. CONCLUSIONS

In conclusion, we have demonstrated a virtual source that yielded the (1+1) dimensional Pearcey beam. The Pearcey wave is expressed in the form of a spectral integral and the differential representation. Under the paraxial approximation, the integral simplifies to the paraxial Pearcey beam. By comparing the virtual source for generating the the complex source point spherical wave and by acting an exponential operator on it, we obtain the source for the (1+1) dimensional Pearcey wave. From the integral representation of the (1+1) dimensional Pearcey wave, we obtain the first three orders of nonparaxial corrections for the corresponding paraxial Pearcey wave.

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