

Range-angle performances of coherent MIMO long-range radars

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Abstract

Active antennas and modular generation and reception opens the way to radar systems where different waveforms are simultaneously transmitted and received through multiple channels (MIMO radars). Such an increase in the degrees of freedom is shown to allow improvements in angular resolution compared to standard wide beam. It can also be used to optimize the trade-offs between angular and range resolution and sidelobes. In this paper, the basic properties are analyzed on a new class of space-time circulating codes adapted to 2D antennas, and the benefits are demonstrated through fair comparisons of transmit/receive ambiguity functions.

1. Introduction

The advent of active antenna systems, combined with digital waveform generation and coding, opens the way to flexible and high performance radar systems, where multiple simultaneous transmissions will provide higher resolutions in Doppler and angle, and better capabilities for target analysis in complex environments [1-4].

Some modern radars use a widened beam on transmit, allowing for long illumination time and thus better extraction of targets, and combining multiple receiving antennas for digital beamforming of focused pencilbeams in parallel. Such digital beamforming generally does not essentially change the power budget, compared to standard pencil beam exploration, since the lower gain on transmit (due to wider illumination) is traded against a longer integration time (made possible by the simultaneous observation of different directions). In fact, the main benefit provided by wide beam illumination and associated digital beamforming is an improved velocity resolution, especially useful for identification purposes, or for detection of slow targets in clutter.

However, this velocity resolution comes at a cost: the non-directive beam on transmit, which induces a poorer rejection of competing echoes coming from adjacent directions, and a poor clutter rejection, since only half the dBs are obtained, compared to focused beam illumination. In order to recover this angular separation on transmit (which was basic to standard pencil beam techniques), it is necessary to code the transmitted signals (space-time coding, aka “coherent colocated MIMO”), such that the signals transmitted in the different directions be separable on receive.

Space-time coding is thus of interest for detection of transitory targets (helicopter pop-up, periscope), or slow small moving targets in competition with clutter and/or strong targets.

In this paper, the main performances of such systems will be analyzed for surface radars, taking into account the sidelobes of the ambiguity function, which define the final rejection performance for such systems. It will be shown that, compared to wide beam standard digital beamforming on receive, space-time coding allows an improvement in angular resolution and accuracy by a factor 2 ($\sqrt{2}$ in each dimension of a flat antenna array). The resulting ambiguity function will also be shown to provide acceptable sidelobe level with matched filtering – to be further improved with mismatched filtering or adaptive processing, as usual.

In order to clarify the issue, comparisons will be made on the example of circulating codes, which exhibit very characteristic ambiguity functions. For this purpose, original results on the adaptation of circulating signals to 2D scanning situations will be provided. This will allow a clean and fair comparison between focused beam, wide beam, and wide beam space-time coded systems, when matched filtering is assumed. For operational systems, of course, mismatched or adaptive processing will generally be required, as for every existing radar.

2. Objectives and tools

The baseline comparison will be made on a typical pulse-Doppler radar, assuming that there is no significant Doppler effect during the duration of one pulse (Doppler effect will then be processed, as usual, from pulse to pulse).

2.1 Definitions

Let us first consider a simple coherent system where multiple signals $s_T^1(t), s_T^2(t), \dots, s_T^N(t)$ are transmitted through a linear array of N identical antennas or sub-arrays, at positions $\vec{x}(1), \vec{x}(2), \dots, \vec{x}(N)$. The signal transmitted in a given direction θ_0 is the sum of all transmitted signals, with appropriate phase shifts corresponding to this direction defined by the wavevector $\vec{k}(\theta_0)$.

This signal, after reflection by a target at range $c\tau/2$ is then processed with a filter matched to the time delay τ and direction θ , and digital beamforming on receive, providing the output described in [5] as:

$$\chi_{\theta_0}(\tau, \theta) = \left[\sum_{n=1}^N e^{j(\vec{k}(\theta_0) - \vec{k}(\theta)) \cdot \vec{x}(n)} \right] \cdot \sum_{m=1}^N e^{j(\vec{k}(\theta_0) \cdot \vec{x}(n) - \vec{k}(\theta) \cdot \vec{x}(m))} \int s_T^n(t) (s_T^m(t + \tau))^* dt$$

The ambiguity function is thus a 3-dimensional function, giving for each aiming direction θ_0 the delay-angle ambiguity.

These definitions are easily extended to bi-dimensional angular exploration, with 2 angular variables in elevation and azimuth, leading in this case to a 5-dimensional ambiguity function (2 angles, for aiming and for replica, and time delay).

More details and illustrations about the use of ambiguity functions for space-time coding systems are provided in [5-7].

2.2 Trade-off analysis on circulating codes

For this comparison, we will focus here on circulating codes [8-10], which exhibit very basic properties of the ambiguity function. Circulating codes are generated by the same waveform $s^n(t) = s(t - (n-1)\Delta t)$ circulating with a relative time shift Δt through N MIMO transmitter channels. The relative time shift Δt between adjacent circulating signals is equal to 1-time sample, $\Delta t = 1/\Delta F$, where ΔF is the signal bandwidth.

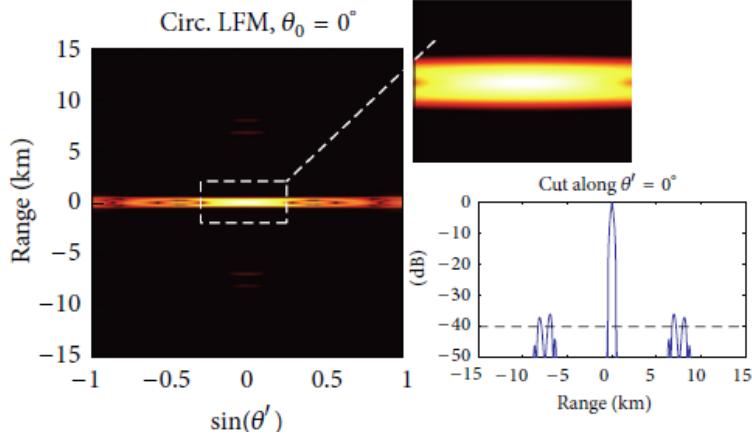


Figure 1: Circulating chirp: Cuts of the 3-D ambiguity function $|\chi_{\theta_0}(\tau, \theta)|^2$ at $\tau=0$ and $\theta_0=0^\circ$

The ambiguity function for a circulating signal $s_T(t)$, with Fourier transform $\tilde{s}_T(f)$, is then written:

$$\chi_{\theta_0}(\tau, \theta) = \left[\sum_{n=1}^N e^{j(\vec{k}(\theta_0) - \vec{k}(\theta)) \cdot \vec{x}(n)} \right] \cdot \sum_{m=1}^N e^{j(\vec{k}(\theta_0) \cdot \vec{x}(n) - \vec{k}(\theta) \cdot \vec{x}(m))} e^{2\pi j(m-n)\frac{f}{\Delta f}} \int |\tilde{s}_T(f)|^2 e^{-2\pi j f \tau} df$$

The cuts of this ambiguity function are shown on Figure 1, for a circulating chirp (linear frequency modulation); the linear antenna array is made of 8 elementary antennas, spaced $\lambda/2$ from each other, the carrier frequency is 10 GHz,

and the BT product is 256, with $100\mu\text{s}$ pulse length. Since the properties of transmission are of interest, the receiving antenna is assumed to be isotropic, with only one receiving element.

The main property of this “circulating chirp” coding is that the ambiguity function is very clean, with sidelobes almost everywhere lower than -50dB, but the range resolution is degraded by a factor 8, as is obvious on the range response. This is due to the “circulating code”: the correlation peaks from the different chirps are adjacent to each other, thus combining to build a peak which is N times larger, N being the number of transmitted codes.

In comparison with a standard wide beam situation, for which there is no angular separation on transmit, but the full range resolution is obtained, it then appears that range resolution has been traded against angular resolution: space-time coding allows to obtain, in the same wide angular coverage, the full angular resolution and rejection of the transmit antenna, at the price of a degradation of the range resolution by the same amount, equal to the number of antenna elements (ie the number of space-time codes).

3. A fair comparison: wide beam vs space-time coding

In order to design a fair comparison, it is essential that the coverage of the compared systems be identical, and the bandwidth (and time bandwidth product) also be the same. For a more realistic analysis, the antenna is a 2-D array made of $4 \times 4 = 16$ equal shape and equal size rectangular sub-arrays, each sub-array’s size is close to 4×4 wavelengths. Each of the 16 sub-arrays transmits a chirp with 2 MHz instantaneous bandwidth, and a BT product = 256; For Scenario A: a single identical chirp is transmitted through only one sub-array; For Scenario B: a circulating chirp is transmitted through the 16 sub-arrays.

It is important to note that, for this bi-dimensional situation (azimuth/elevation), the circulating codes can not “circulate” properly (or that would lead to undersampling in one dimension). Therefore, the chirps are made to circulate randomly from one sub-array to another. More precisely, 16 circulating chirps are generated, and these chirps are then distributed randomly to the different sub-arrays, producing a kind of shuffling of the circulating codes. Actually, that randomness induces a similar effect to the one produced in so-called Delft codes [11] with an initial phase shift from code to code: such circulating codes allow to recover the range resolution of the whole bandwidth, at the cost of a plateau area over N range cells, N being the total number of codes (or sub-arrays).

The receiving antenna has the same size as the whole transmit antenna – close to 16×16 wavelengths – , and uses full Digital Beam Forming (without sub-arrays), with -35dB Taylor weighting. Transmit and receive antennas are co-located.

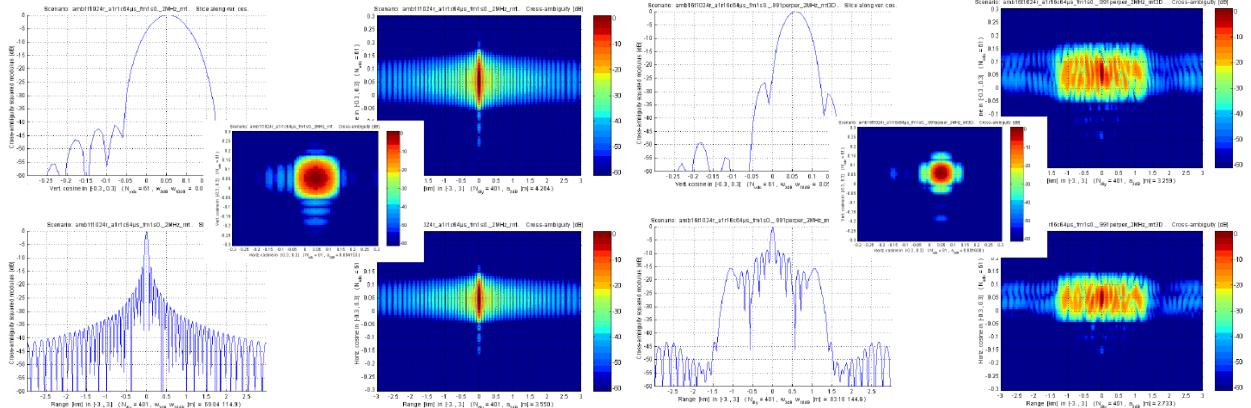


Figure 2: Comparison between standard wide beam (left) and space-time coded wide beam (right), chirp waveform

The results are presented on Figure 2, left (standard) and right (space-time coding). On each side is shown:

- Top left: Angular diagram (transmit-receive) in elevation;
- Top right: Range Elevation cut of the ambiguity function;
- Bottom right: Range Azimut cut of the ambiguity function;

- Bottom left: range profile, in the direction of the target;
- Middle: 2D-footprint of the beam, in the range cell of the target;

From these results, the following comments can be made:

1. The 2D angular resolution (measured at 3dB level) is improved by a factor 2.2.
2. The sidelobe distribution is a serious limitation in both cases, and mismatched filtering (or adaptive or compressive sensing) should be implemented, as is always the case for operational radars.
3. Outside the plateau area, the range-angle sidelobes of circulating chirp are very low – typically lower than 45 dB
4. Anyway, the designer should optimize the trade-off between sidelobe level, and range and angle resolutions, through selection of space-time codes – circulating codes being good candidates for such trade-offs through adaptation of their parameters.
5. Anyway, some kind of sidelobe suppression has to be implemented, through mismatched filtering, or adaptive / compressive sensing.

4. Conclusion

The increased degrees of freedom provided by space-time coding on transmit opens the way to adaptive systems where range and angle resolution can be traded, depending on the application. Compared with modern wide beam DBF Systems, it provides an improvement in both accuracy and resolution larger than 2, for 2-dimensional antennas.

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