

# Electromagnetic Wave Lenses and Reflectors Designed with Transformation Electromagnetics

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## Abstract

Lenses and reflectors are common devices in manipulating electromagnetic (EM) wave propagation and are largely utilized in radiation and receiving systems of electromagnetic engineering. The recently proposed significant concept of transformation electromagnetics provided an effective tool to design devices that could control the EM wave with desired ray traces. In this presentation, we will demonstrate several examples of applying the transformation electromagnetics in the design of a novel EM wave lens that could focus EM wave with better resolution than the Abbe-Rayleigh diffraction limit and a planar retrodirective reflector. The design procedures, device performances as well as the consideration of practical realizations will be analyzed.

## 1. Introduction

Recently, based on the form-invariant of Maxwell's equations under different coordinate transformations, the transformation electromagnetics (TE) proposed firstly by J. Pendry and U. Leonhardt in [1, 2] has triggered great interest of applying it to various electromagnetic (EM) device designs due to its potential ability to arbitrarily manipulate the EM wave propagation with the pre-defined distribution of material constitutive parameters implemented through artificial metamaterials. More intensive theoretical and experimental explorations have brought in many interesting design ideas and practical approaches. Besides the invisibility cloak [1-3], EM wave concentrators, rotators, shifter and other interesting EM devices have also been proposed by utilizing the TE method [4-7]. TE has also been utilized to design lens, antenna and related devices. For example, planar antenna and lens have been designed through coordinate transformation, which show similar performance with conventional counterparts, but with lower profile [8-9].

Lenses and reflectors are important devices in manipulating EM wave propagation and are often applied in radiation and receiving systems of electromagnetic engineering. In this presentation, we will utilize the TE method in the design of novel EM lens and reflector. Two examples will be demonstrated to show how the TE concept could improve the performance of the EM lens and reflector. The first example is a super gradient index lens that is capable of focusing EM waves with resolution better than the Abbe-Rayleigh diffraction limit. The second example is a retrodirective reflector designed on a planar conducting sheet. Besides the design procedure through TE concept, we will also discuss on how these devices could be simplified and practically realized.

## 2. Super Graded Index Lens

Lens made of material with gradient refractive index (GRIN), could ideally focusing a parallel entrance beam onto a point on an optical axis, as known in the gradient-index optics [10-11]. These lenses (one example is the well-known Luneberg lens) in the framework of ray optics transform a parallel entrance beam (the object is at infinity) into a convergent output homocentric beam (as shown schematically in Fig. 1(a)). However, like any optical instruments, ideal focus spot is always prevented in the GRIN lens by Abbe-Rayleigh diffraction limit which proscribes the visualization of features smaller than half wavelength of light.

Here, we try to circumvent this focus limit in the GRIN lens by re-designing the refractive index distribution with TE method. TE has been successfully applied to modulate EM beams [5, 6]. In our previous work we have also proposed different ways to modulate EM beam to establish either beam expander or compressor with linear coordinate transformation [12]. By designing the material constitutive tensors through different kind of coordinate transformations, the beam width of an incident Gaussian EM wave could be modulated by a metamaterial slab. The finite-embedded coordinate transformation method enables these transformed structures to let the modulated beam unchanged while leaving the metamaterial region. We apply similar scheme to a planar GRIN lens to compress the EM beam that propagate inside the lens to obtain fine focus better than ordinary GRIN lens, as schematically demonstrated in Fig. 1(b).

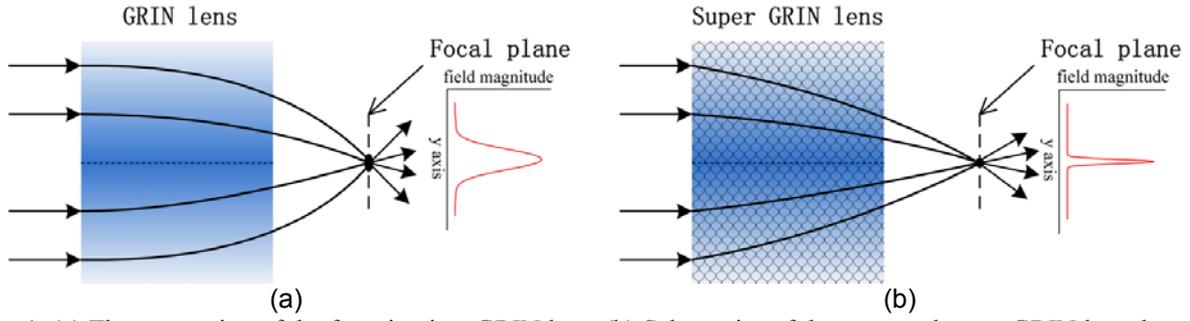


Figure 1. (a) The ray tracing of the focusing in a GRIN lens, (b) Schematics of the proposed super GRIN lens that could focus with resolution beyond the Abbe-Rayleigh diffraction limit.

To make further EM beam compress to form the so-called super GRIN lens, we applied coordinate transformation on the region of a normal slab of ordinary GRIN lens (assuming two-dimensional (2D) case for simplicity). The ordinary GRIN lens has a refractive index profile described as  $n(r) = n(0)(1 - 1/2a^2r^2)$ , where  $r$  is the distance to the center of the lens,  $n(0)$  is the refractive index at the center of the lens, and  $a$  is a constant [13]. Both a linear and a parabolic coordinate transformation were utilized on the ordinary GRIN lens. Through the standard TE procedure, two planar super GRIN lens of the same size were obtained with certain material parameters distribution determined by the coordinate transformation and described by both a permittivity tensor and a permeability tensor. To explore the focus performance of the super GRIN lens, we studied the propagation of a Gaussian beam incident normally onto the planar lenses. Full-wave numerical simulation based on the finite element method was carried out to calculate the EM field distribution. Assuming the beam width at the input and the focal plane are denoted as  $w_1$  and  $w_2$ , respectively, the focusing is restricted by  $w_1w_2 \geq 2\lambda_0 f / \pi^2$  under the Abbe-Rayleigh diffraction limit, where  $\lambda_0$  and  $f$  are the wavelength and the foci, respectively. As indicated in Fig. 2, the two super GRIN lenses could focus the incident Gaussian beam to a better spot than the ordinary GRIN lens. The focus performances through the ordinary GRIN and the two super GRIN lenses are compared and summarized in Tab. 1. Both the linear and parabolic super GRIN lens could achieve focused beam width better than that imposed by the Abbe-Rayleigh diffraction limit, while the beam focused by the ordinary GRIN lens is limited by the Abbe-Rayleigh diffraction limit. The super GRIN lens design through nonlinear parabolic beam compression has a slightly better focus than the one through linear compression. We will also discuss the material requirement for the super GRIN lens and consider about the practical realization.

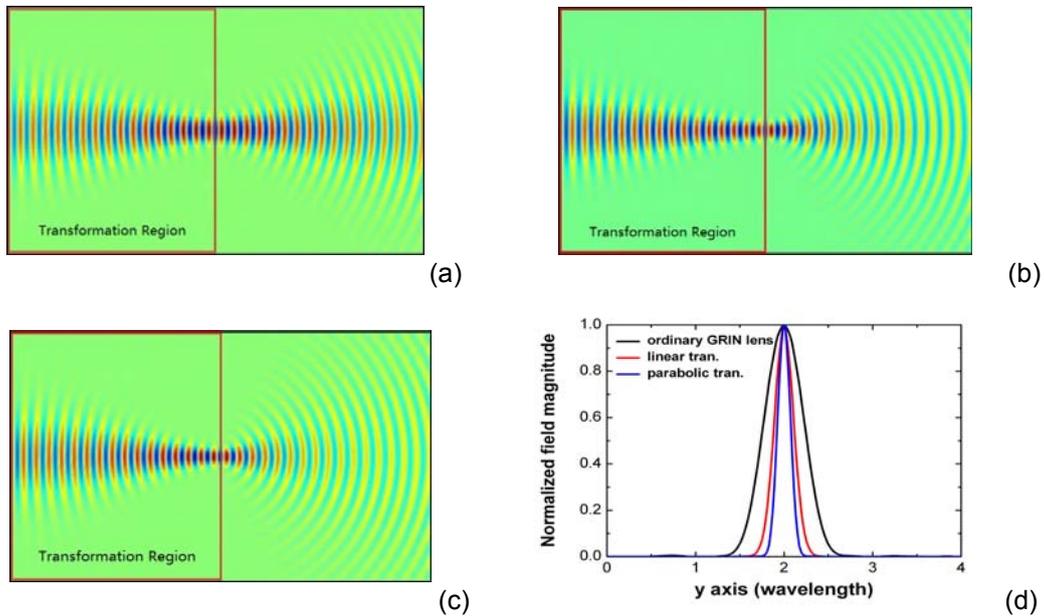


Figure 2. (a) The field distribution for a Gaussian beam propagating through an ordinary GRIN lens (a), a super GRIN lens with linear transformation (b), or a super GRIN lens with a parabolic transformation (c). (d) Comparison of the intensity profiles along the focal planes of the three kinds of GRIN lenses.

Table 1. Comparison of the focused beam width with the theoretical limit.

Lens Type	$w_1$ (m)	$w_2$ (m)	$w_1w_2$	$2\lambda_0f/\pi^2$
original GRIN lens	1	0.3099	0.3099	0.304
linear super GRIN lens	1	0.2190	0.2190	0.304
parabolic super GRIN lens	1	0.1758	0.1758	0.304

### 3. Planar Retrodirective Reflector

Retrodirective reflector that can scatter EM waves in the direction anti-parallel to that of the incoming EM beam is quite useful in microwave engineering such as to enhance radar cross sections of objects, or in satellite communications, as well as in identification application and military application. Conventional retrodirective reflector is often realized by the so-called corner reflector structure which consists of two or three conducting metal sheets at  $90^\circ$  angles to each other (shown in Fig. 3(a)). However, it is quite desirable to construct retrodirective reflectors with planar or low-profile features in many applications.

We employ the TE method to transform a corner reflector to a planar PEC with a dielectric cover structure to form a planar retrodirective reflector. For simplicity, we restrict the problem to a 2D case. We utilize a specifically chosen linear coordinate transformation to squeeze the  $90^\circ$  degree air filled PEC corner into a planar PEC with a triangle dielectric cover as schematically shown in Fig. 3(b). The material parameters for the triangle cover can be easily calculated through standard TE procedure. The resulted dielectric cover becomes a homogeneous birefringent material under such linear transformation; therefore, we could further reduce the material parameters to a non-magnetic form, and then realize the cover with multilayer structure of alternating isotropic dielectrics based on the effective medium theory similar to that used in our previous design of a dielectric ground plane invisibility cloak [14].

Fig. 3(c) shows the performance of a proof-of-concept example of the retrodirective reflector working at microwave regime. Assuming the size of the reflector to be 12.5 cm long and 8.3 cm thick, we can finally obtain the resulted multilayer dielectric cover structure with alternating dielectric of  $\epsilon_1 = 1$  and  $\epsilon_2 = 33$ , and aligned at  $\theta = 31.7^\circ$  or  $\theta = 148.3^\circ$  in the left or the right part, respectively. The retrodirective reflecting is clearly demonstrated from the near field magnetic field distributions and the far field scattering patterns calculated at 8 GHz in Fig. 3(c) and (d), despite of slight specular reflecting at the incident boundary and slight scattering at the other boundary of the cover, which are attributed to the impedance mismatch at the boundaries. It is also found that the retrodirective reflector could have a broadband performance due to the non-dispersive and low loss feature of the multilayer structure of normal dielectric.

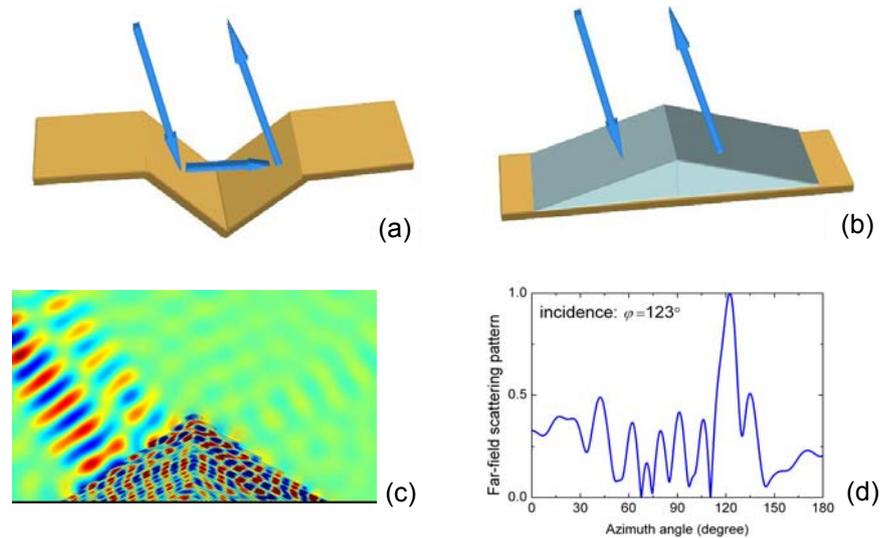


Figure 3. Schematic of a traditional PEC corner reflector (a), and the proposed retrodirective reflector realized with dielectric cover on a planar PEC sheet (b). The near-field magnetic field distribution (c), and normalized far-field magnetic field scattering pattern (d) of a multilayer retrodirective reflector with alternating air layer and dielectric layer with relative permittivity of 33 for incident wave at azimuth angle of  $123^\circ$ .

## 4. Conclusion

In this presentation we reported examples on how to apply the TE method to design novel EM wave device such as focusing lens and planar reflector. We show that a super GRIN lens design with TE procedure could have focus ability with resolution not restricted by the Abbe-Rayleigh diffraction limit. We also show that a retrodirective reflector could be realized on a planar PEC sheet with a dielectric cover designed by TE method. These examples demonstrated the significant ability that TE method could provide in the EM wave manipulation.

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