An Energy-Efficient Power Control Scheme in Small Cell Networks

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Abstract

This paper studies the power control scheme in small cell networks. The object is to improve the system energy efficiency. Based on the characteristic of small cells, a distributed power control scheme which is based on supermodel game theory is proposed. The uniqueness of the Nash equilibrium of the proposed power control game is proved. By conducting the game in an iterative manner, the stable state is shown to be finally reached.

1. Introduction

The global wireless traffic is increasing explosively. A 1000-fold data traffic increase in a decade is expected [1]. To satisfy the requirement of wireless traffic in the future, the small cell is deployed embedded in the macro cell system. The small cell is a kind of low power node which includes picocell and femtocell specified by the 3rd Generation Partnership Project (3GPP) [2]. Though a single small cell consumes tiny power, in the dense deployment scenario where the cell quantity is large, the power consumption can be huge. High energy efficiency power control scheme is necessary to reduce the waste of energy [3][4]. To some extent, certain small cells are deployed randomly rather than planned by the telecommunication provider. This randomness prefers a distributed power control manner. This paper considers the energy-efficient power control issue. The problem is formulated as a supermodular game [5]. The uniqueness of the Nash equilibrium of the game is proved. Simulation results show that the proposed scheme can iteratively improve the system performance and the stable state can be finally reached.

2. System Model and Basic Assumptions

Fig.1 shows the system model of small cell networks. There are $M$ small cells in a limited area and each small cell only provides services to one small cell users. The set of small cells is $\mathcal{S} = \{1, 2, 3, \ldots, M\}$, and the user set is denoted as $\mathcal{U} = \{1, 2, 3, \ldots, M\}$. To simplify the notions, we assume that the $i$th small cell provides service to the $i$th small cell user. The served users can feedback the interference condition of other $M - 1$ small cells to its serving small cell. Each small cell collects the feedback information and makes the decision on power control. It is assumed that the small cell can get the interference level of its user precisely.

![Small Cell and its UE](image)

Fig. 1 System model of small cell networks

Besides, it is supposed that the small cells are uniformly distributed in the area and its user equipment (UE) locates in the vicinity. The channel is assumed to be flat and changing in a slower pace than the power control pace.

3. Power Control Scheme Description

Based on the assumptions, the throughput of the $i$th small cell is:
\[
Th_i = BW \times \log\left(1 + \frac{p_i \times g_{ii}}{\sigma^2 + \sum_{j=1, j \neq i}^{M} p_j \times g_{ji}}\right), \quad (1)
\]

where \(BW, \sigma^2, p_j, g_{ji}\) denote the bandwidth, the noise power, the transmit power of the \(j\)th small cell and the channel gain between the UE in the \(i\)th small cell and the base station in the \(j\)th small cell. For simplicity, the interference of the \(i\)th UE can be expressed as:

\[
I_i = \sum_{j=1, j \neq i}^{M} p_j \times g_{ji}, \quad (2)
\]

From (1) it indicates that the throughput is monotonically increasing with \(p_i\), given the interference. The energy efficiency of user in the \(i\)th small cell is defined as [6]:

\[
EE_i = \frac{Th_i}{(BW \times p_i)} = \frac{\log\left(1 + \frac{p_i \times g_{ii}}{\sigma^2 + \sum_{j=1, j \neq i}^{M} p_j \times g_{ji}}\right)}{p_i}. \quad (3)
\]

Let \(x\) denote \(p_i\) and \(y\) denotes \(\frac{\sigma^2 + \sum_{j=1, j \neq i}^{M} p_j \times g_{ji}}{g_{ii}}\). Based on these assumptions we can get that:

\[
\frac{\partial EE_i}{\partial x} = \frac{\frac{x}{x+y} - \frac{\log(1 + \frac{x}{y})}{x^2}}{x^2} = g(x), \quad (4)
\]

For the function

\[
g(x) = \frac{x}{x+y} - \frac{\log(1 + \frac{x}{y})}{x^2}; \quad x \geq 0, \quad y \geq 0,\quad (5)
\]

it is easy to get that:

\[
g(0) = 0, \quad (6)
\]

and

\[
\frac{\partial g}{\partial x} = \frac{-x}{(x+y)^2} \leq 0. \quad (7)
\]

So we have:

\[
g(x) \leq g(0) = 0. \quad (8)
\]

Based on equation (8) it is concluded that \(EE_i\) is monotonically decreasing with \(p_i\) given the interference. In order to efficiently use the power resource, the game below is formulated to optimize the transmit power of each small cell to achieve a better performance. The game is represents as \(G = \{M; P_1, P_2, \ldots, P_M; \pi_1, \pi_2, \ldots, \pi_M\}\), where \(M\) represents the number of players in the game, \(P_i\) represents the strategy space of the \(i\)th small cell. In the game, \(P_i \in [p_{\text{min}}, p_{\text{max}}]\) is all the possible transmit power value of the \(i\)th small cell. \(\pi_i(p_1, p_2, \ldots, p_i, \ldots, p_M) = \pi_i(p_i, p_{-i})\) represents the payoff of the \(i\)th small cell:

\[
\pi_i(p_i, p_{-i}) = \frac{\log\left(1 + \frac{p_i \times g_{ii}}{\sigma^2 + I_i}\right)}{p_i} + \beta \times p_i; \quad \beta > 0, \quad (9)
\]

where \(\beta\) is a positive const. The last polynomial of (9) is a reward factor. Since (3) is monotonically decreasing, if there is no reward factor, each small cell will transmit with the lowest power, which may be not the best strategy from the system performance aspect. The first order and second order partial derivatives of equation (9) is shown respectively as:

\[
\frac{\partial \pi_i}{\partial p_i} = \frac{1}{p_i^2} \times \left[\frac{1}{1 + \frac{\sigma^2 + I_i}{g_{ii} \times p_i}} - \log\left(1 + \frac{p_i \times g_{ii}}{\sigma^2 + I_i}\right)\right] + \beta, \quad (10)
\]

and

\[
\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = \frac{g_{ki} \times g_{ij}^2}{(\sigma^2 + I_i)(g_{ii} \times p_i + \sigma^2 + I_i)^2} > 0. \quad (11)
\]

The game can be solved by an iterative method. Let \(P^0 = (p_1^0, p_2^0, \ldots, p_M^0) = (p_{\text{max}}, p_{\text{max}}, \ldots, p_{\text{max}})\) and \(P^0 = P\). Let \(p_i^1 = \frac{BR_i(p_{-i}^0)}{p_i^0}\) denote the best response of the \(i\)th small cell user given the strategy \(p_{-i}^0\). Let
\( P_i^1 = \{ p_i \in P_i^0 : p_i \leq p_i^1 \} \) and \( P_i^1 \) is the dominant strategy space for the first iteration. This conclusion is proved as below. From equation (11) we have that for any \( p_i > p_i^1 \),

\[
\pi_i(p_i, p_{-i}) - \pi_i(p_i^1, p_{-i}) \leq \pi_i(p_i, p_i^0) - \pi_i(p_i^1, p_i^0).
\]  

(12)

Because \( p_i^1 \) maximize the payoff given the strategy \( p_i^0 \), it can be concluded that for any \( p_i > p_i^1 \),

\[
\pi_i(p_i, p_i^0) - \pi_i(p_i^1, p_i^0) \leq 0.
\]  

(13)

So for the \( i \)th small cell, the strategy \( p_i > p_i^1 \) is the dominated strategy. Define:

\[
p_i^{k+1} = \overline{BR}(p_{-i}^k),
\]

and

\[
P_i^{k+1} = \{ p_i \in P_i^k : p_i \leq p_i^{k+1} \}.
\]

(14)

(15)

By the mathematical induction it can be obtained that if \( p_i^k \leq p_i^{k-1} \),

\[
p_i^{k+1} = \overline{BR}(p_{-i}^k) \leq \overline{BR}(p_{-i}^{k-1}) = p_i^k.
\]  

(16)

From the induction above, we can see that \( p_i^k \) is a decreasing sequence for each small cell. Define \( \overline{p}_i = \lim_{k \to \infty} p_i^k \) and only the strategy \( p_i \leq \overline{p}_i \) is dominant. Then for all \( i, p_i \),

\[
\pi_i(p_i^{k+1}, p_{-i}) \geq \pi_i(p_i, p_{-i}).
\]  

(17)

Taking limits as \( k \to \infty \),

\[
\pi_i(\overline{p}_i, p_{-i}) \geq \pi_i(p_i, p_{-i}).
\]  

(18)

This is the Nash equilibrium of the game. In fact, each small cell controls its transmit power based on the solution of the supermodular game. The system will come to the stable state in an iterative manner.

### 4. Simulation Results

The simulation parameters are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of small cells</td>
<td>20</td>
</tr>
<tr>
<td>Number of UEs per small cell</td>
<td>1</td>
</tr>
<tr>
<td>Noise power density (dBmW/Hz)</td>
<td>-174</td>
</tr>
<tr>
<td>Maximum transmit power (mW)</td>
<td>20</td>
</tr>
<tr>
<td>Minimum transmit power (mW)</td>
<td>2</td>
</tr>
<tr>
<td>Simulation square (m^2)</td>
<td>150 x 150</td>
</tr>
<tr>
<td>Small cell distribution</td>
<td>Uniform</td>
</tr>
<tr>
<td>UE distribution</td>
<td>Uniformly in the circle with R=35m. The center of the circle is the serving small cell</td>
</tr>
<tr>
<td>Path loss (dB), ( r ) in km</td>
<td>-(128 + 37.6*\log_{10}(r))</td>
</tr>
<tr>
<td>System bandwidth (KHz)</td>
<td>200</td>
</tr>
</tbody>
</table>

Fig.2 shows the variation of system throughput during the conduction of the game. It is shown that generally the system throughput increases iteratively during the conduction and finally reaches to a stable state. Fig.3 illustrates the total sum payoff of all the small cells in the system. In a distributed system, the stability is important. From Fig.3, it is seen that the energy efficiency of the system converges to a stable state after about six times iterations, which shows the robustness of the convergence of the proposed scheme.

### 5. Conclusion

In this paper, the energy-efficient power control issue is formulated as a supermodular game. The existence and uniqueness of the Nash equilibrium of the formulated game is proved. Based on the solution, the optimal power control scheme is obtained in an iterative manner.
6. Acknowledgments

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7. References


