A Noise-Robust Radar Target Classification Method Based on Complex Probabilistic Principal Component Analysis

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Abstract

We develop a noise-robust radar target classification method to discriminate the moving vehicle and walking human. The traditional real-valued Probabilistic Principal Component Analysis (PPCA) model is extended to the complex-value domain for modeling the low resolution radar echoes from the ground moving targets. The denoising preprocessing is accomplished by signal reconstruction with the proposed Complex Probabilistic Principal Component Analysis (CPPCA) model, where we utilize the Bayesian Inference Criterion (BIC) to adaptively select the principal components. After denoising, a 3-dimensional timefrequency feature vector is extracted from the denoised micro-Doppler signatures of the two kinds of ground targets, and the classification is performed via Support Vector Machine (SVM) classifier. In the experiments based on the measured data, the proposed classification scheme shows the good classification and denoising performance under the relatively low SNR condition. In the real application, the advantage in SNR can effectively extend the classification distance between the target and radar.

1. INTRODUCTION

When radar illuminates a moving target, the carrier frequency of the returned signal will be shifted. This phenomenon is known as Doppler effect. If the target or any structural component of the target has oscillatory or rotation motion, which is referred to as micro-motion, such micro-motion may induce the additional Doppler modulation that generates sideband frequencies around the target's Doppler frequency, which is called as micro-Doppler effect. Since Chen *et al.* introduced the notion of micro-Doppler to radar area, lots of researches on the applications of micro-Doppler effects have been presented in radar area. Papers [1], [2] show the micro-Doppler information contained in the time-frequency spectrum of different human gait motions can be utilized for the human activity classification. A hierarchical micro-Doppler feature extraction method for discriminating the moving wheel and tracked vehicles is discussed in [3]. Paper [4] studies on the classification problem between pedestrians and vehicles.

For radar target classification problem, the training data are usually collected via simulations or via some cooperative measurement experiments, where the signal-to-noise ratio (SNR) is usually high; while the test samples are usually got in the non-cooperative circumstance (e.g., at the battle time), where the high SNR cannot be guaranteed due to the severe measurement conditions, such as the large distance between the non-cooperative targets and radar. As shown in the experimental results in [5], the classification performance dramatically deteriorates under the low SNR condition. Since the SNR directly relates to the distance between the target and radar for a given noise power and radar power, the noise robustness of a classification method is an important factor in increasing the classification distance between the target and radar in the real application. Based on the sparse representation theory, a simple noise reduction method is proposed in [5]. Nevertheless, this method requires the prior information about SNR. If the SNR is not estimated accurately, the classification performance will decrease.

The research reported here seeks an alternative approach to denoise the complex-valued low resolution radar echo based on Complex Probabilistic Principal Component Analysis (CPPCA) model to distinguish the moving vehicle and walking human. In the real battlefield, since the vehicles and persons usually undertake the different tasks, it is important to discriminate these two kinds of targets[4].

2. NOISE REDUCTION PREPROCESSING APPROACH

The PCA model can construct the orthogonal signal and noise subspaces for a set of given data, and realize the noise reduction via reconstructing the data within the signal subspace and discarding the noise subspace. The Probabilistic Principal Component Analysis (PPCA) model[6] developed a probabilistic model for not only the signal but also the noise components. An obvious advantage of PPCA over PCA is that some adaptive model selection methods can be applied for data denoising under the probabilistic formulation. The conventional PPCA model is developed in the real-value domain, while the radar target echoes include the inphase and quadrature components, which are complex-valued data. In order to apply PPCA model for radar echo denoising, we should first extend the real-valued PPCA model in [6] to the complex-value domain.

Given a received low-resolution echo $\mathbf{x} \in \mathbb{C}^{M\times 1}$, it can be modeled via CPPCA as

$$\mathbf{x} = \mathbf{W}\mathbf{y} + \boldsymbol{\mu} + \boldsymbol{\varepsilon} \tag{1}$$

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In (1), $\mathbf{W} \in \mathbb{C}^{M\times d}$ with $d < M$ denotes the orthogonal projection matrix with $\boldsymbol{\omega}_p^H \boldsymbol{\omega}_q = c$ ($p,q = 1,2,\cdots,d$), where $c = 0$ for $p \neq q$ and $c \neq 0$ for $p = q$, *i.e.*, $\mathbf{W}^H\mathbf{W}$ is a diagonal matrix; $\mathbf{y} \in \mathbb{C}^{d\times 1}$ denotes the latent variables, and $\mathbf{y} \sim \text{CNorm}(0,\ \mathbf{I}_d)$ with $\text{CNorm}(\cdot)$ representing the complex Gaussian distribution with $p(\mathbf{y}\,|\,\boldsymbol{\mu}_0,\boldsymbol{\Sigma}_0) = \frac{1}{(\pi)^K\,|\,\boldsymbol{\Sigma}_0\,|}\exp\left\{-(\mathbf{y}-\boldsymbol{\mu}_0)^H \Sigma_0^{-1}(\mathbf{y}-\boldsymbol{\mu}_0)\right\}$; $\boldsymbol{\mu} \in \mathbb{C}^{M\times 1}$ denotes the mean vector of the observed data; $\boldsymbol{\varepsilon} \in \mathbb{C}^{M\times 1}$ denotes the noise component, which is independent from the signal component $\mathbf{y}$, and $\boldsymbol{\varepsilon} \sim \text{CNorm}\left(0,\ \delta^2\mathbf{I}_M\right)$ .Then the radar echo $\mathbf{x}$ should follow the complex Gaussian distribution, $\mathbf{x} \sim \text{CNorm}\left(\boldsymbol{\mu}, \mathbf{C} = \mathbf{W}\mathbf{W}^H + \delta^2\mathbf{I}_M\right)$. The corresponding density is

$$p(\mathbf{x}) = \left(1/(\pi)^M\,\delta^2\mathbf{I}_M + \mathbf{W}\mathbf{W}^H\right)\cdot\exp\left[-(\mathbf{x}-\boldsymbol{\mu})^H(\delta^2\mathbf{I}_M + \mathbf{W}\mathbf{W}^H)^{-1}(\mathbf{x}-\boldsymbol{\mu})\right]. \tag{2}$$

The complex echo from a range cell is the summation of the complex-valued scattering responses from all scatterers in this range cell. According to the central limit theorem, the summation of many independent variables approximately follows the Gaussian distribution. Therefore, for the sake of simplicity, it is usually reasonable to assume that the real and imaginary parts of the complex echo from a range cell respectively follow the Gaussian distributions, and the complex-valued variable follows the complex Gaussian distribution.

Based on (1), the log-likelihood function for a set of observed data $\{x_n\}_{n=1}^N$ is

$$L(\boldsymbol{\Phi}) = \sum_{i=1}^{N}\log\left(p\left(\mathbf{x}_i\right)\right) = -N\left(\log|\,\delta^2\mathbf{I}_M + \mathbf{W}\mathbf{W}^H\,| + Tr\left(\left(\delta^2\mathbf{I}_M + \mathbf{W}\mathbf{W}^H\right)^{-1}\mathbf{S}\right) + M\log\pi\right) \tag{3}$$

with

$$\mathbf{S} = (1/N)\sum_{n=1}^{N}\left(\mathbf{x}_n - \boldsymbol{\mu}\right)\left(\mathbf{x}_n - \boldsymbol{\mu}\right)^H \tag{4}$$

where $\boldsymbol{\Phi} = \left\{\boldsymbol{\mu},\delta^2,\mathbf{W}\right\}$ denotes the model parameters, N is the number of measurements in the data set, $Tr(\bullet)$ represents to get the trace of the matrix inside parentheses, $\mathbf{S}$ denotes the covariance matrix of measurements.

To get the maximum likelihood estimator for $\mathbf{W}$, we let the derivation of the log-likelihood with respect to $\mathbf{W}^*$, *i.e.*, the conjugate complex derivative, be a zero matrix. According to [7], we can get

$$\partial L/\partial\mathbf{W}^* = -N\left(\partial\log|\,\mathbf{C}\,|/\partial\mathbf{W}^* + \partial Tr\left(\mathbf{C}^{-1}\mathbf{S}\right)/\partial\mathbf{W}^*\right) = -N\left(\mathbf{C}^{-1}\mathbf{S}\mathbf{C}^{-1}\mathbf{W} - \mathbf{C}^{-1}\mathbf{W}\right) = 0. \tag{5}$$

Since $\mathbf{W} \neq \mathbf{0}$, if $\mathbf{C} = \mathbf{S}$, we can substitute $\mathbf{S} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H$, where the matrixes $\mathbf{U} = \left[\mathbf{u}_1,\mathbf{u}_2,\cdots,\mathbf{u}_M\right]$ and $\boldsymbol{\Lambda} = Diag(\lambda_1,\lambda_2,\cdots,\lambda_d)$ are respectively composed of the eigenvectors $\left\{\mathbf{u}_q\right\}_{q=1}^M$ and eigenvalues $\left\{\lambda_q\right\}_{q=1}^M$ of $\mathbf{S}$ with $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$, and $\mathbf{C} = \mathbf{W}\mathbf{W}^H + \delta^2\mathbf{I}_M$, then

$$\mathbf{W}_{ML} = \mathbf{U}_d\left(\boldsymbol{\Lambda}_d - \delta^2\right)^{1/2}\mathbf{T} \tag{6}$$

where $\mathbf{U}_d = \left[\mathbf{u}_1,\mathbf{u}_2,\cdots,\mathbf{u}_d\right]$ and $\boldsymbol{\Lambda}_d = Diag(\lambda_1,\lambda_2,\cdots,\lambda_d)$, $\mathbf{T}$ is an arbitrary rotation matrix, and can be selected as an identity matrix $\mathbf{I}_d$ in the real application for the sake of simplicity. If $\mathbf{C} \neq \mathbf{S}$, after the derivation similar to that in [6], the estimator of $\mathbf{W}_{ML}$ is exactly same as (6). We can easily get the maximum likelihood estimators for the other two parameters u and δ^2 as

$$\mathbf{u}_{ML} = (1/N)\sum_{n=1}^{N}\mathbf{x}_n \quad , \quad \delta_{ML}^2 = \left(1/M-d\right)\sum_{q=d+1}^{M}\lambda_q \tag{7}$$

As shown in (6) and (7), there is still a parameter to be determined, *i.e.*, d . The adaptive model selection method proposed in [8], Bayesian Information Criterion (BIC), can be applied to estimate d . The general formula of BIC is defined as[8]

$$d^* = \arg\min BIC(d) = \arg\min\left(-2L(\boldsymbol{\Phi}) + \log N\cdot Q(d)\right) \tag{8}$$

where $\boldsymbol{\Phi}$ denotes the model parameters, $L(\boldsymbol{\Phi})$ denotes the log-likelihood of the observed data set, $Q(d)$ denotes the freedom of the model parameters. For our CPPCA model, $\boldsymbol{\Phi} = \left\{\boldsymbol{\mu},\delta^2,\mathbf{W}\right\}$, $L(\boldsymbol{\Phi})$ has been derived in (3). Then the BIC formula for our CPPCA model with ignoring the terms independent on d is

$$d^* = \arg\min\left(2N\log|\,\mathbf{C}_{ML}\,| + 2N\text{Trace}\left(\mathbf{C}_{ML}^{-1}\mathbf{S}\right) + 2\log N\cdot\left(Md - d\left(d-1\right)/2\right)\right) \tag{9}$$

where $\mathbf{C}_{ML} = \delta_{ML}^2\mathbf{I}_M + \mathbf{W}_{ML}\mathbf{W}_{ML}^H$ is dependent on d .

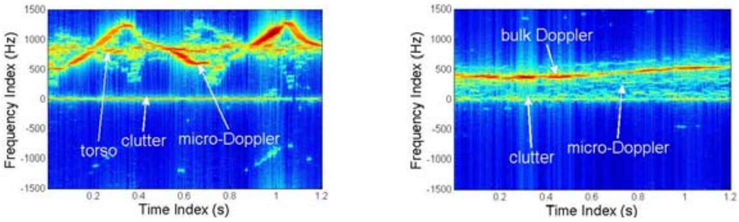
After all the parameters are estimated, the noise reduction for a noised x_n can be achieved via reconstructing the echo within the BIC-determined signal subspace and discarding the residual noise subspace.

$$\tilde{\mathbf{x}}_n = \mathbf{U}_{d^*}\mathbf{U}_{d^*}^H \mathbf{x}_n \tag{10}$$

As discussed above, the CPPCA-BIC based noise reduction method doesn't require the noise power or SNR prior information during the denoising.

3. FEATURE EXTRACTION FROM TIME-FREQUENCY SPECTROGRAM OF MICRO-DOPPLER SIGNATURE

The measured data of the walking human and moving vehicle are collected with a low resolution radar, which works at Ka waveband and under the SNR condition about 35dB. Single person or tracked vehicle moves away or towards radar in the measurement experiment. Since the time-frequency analysis shows how the instantaneous frequencies of a signal vary with time, it is a good tool to analyze the micro-Doppler characteristic of a target. Figure 1 shows the time-frequency spectrograms of some measured echoes, where we use the short time Fourier transform (STFT). The torso-motion, micro-motion and clutter components are indicated in Figures 1 (a) for the data of walking human; while the bulk-motion, micro-motion and clutter components are indicated in Figures 1 (b) for the data of moving vehicle.



(a) (b)

Fig. 1. Time-frequency spectrograms of some measured echoes under SNR about 35dB: (a) walking human; (b) moving vehicle.

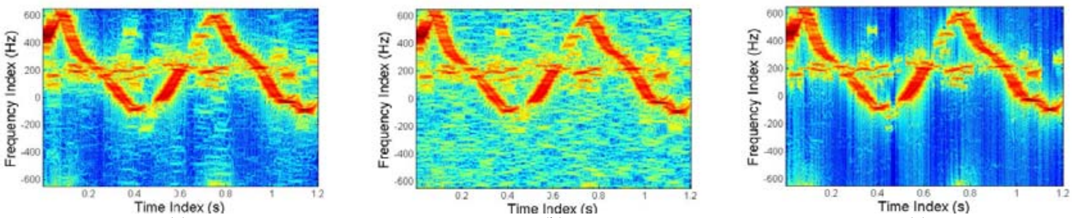
As shown in Figure 1, we can find some differences in the time-frequency spectrograms between the two kinds of ground moving targets. Here we use the following three features to depict the distinctions.

•Feature 1: $F(1) = -\sum_{t=1}^{T}\sum_{f=1}^{F} p_{t,f}\log(p_{t,f})$ with $p_{t,f} = \left(S(t,f)\big/\sum_{t'=1}^{T}\sum_{f'=1}^{F} S(t',f')\right)$, which represents the entropy of the time-frequency spectrogram $S(t,f)$. The time-frequency spectrogram of a signal shows the energy distributions of all instantaneous frequency components.

•Feature 2: $F(2) = -\sum_{f=1}^{F}\rho_f\log(\rho_f)$ with $\rho_f = G(f)\big/\sum_{f'=1}^{F} G(f')$, $G(f) = \mathrm{FFT}(g(t))$ and $g(t) = \sum_{f=1}^{F} f\cdot|S(t,f)|^2\big/\sum_{f=1}^{F}|S(t,f)|^2$, where $\mathrm{FFT}(\cdot)$ denotes the fast Fourier transform. $F(2)$ represents the entropy of the average Fourier spectrum $G(f)$. Since $g(t) = \sum_{f=1}^{F} f\cdot|S(t,f)|^2\big/\sum_{f=1}^{F}|S(t,f)|^2$ can represent the average instantaneous frequency of the target signal, we can get all of the instantaneous frequency components within the time interval $[1,T]$ via $G(f)$.

• Feature 3: $F(3) = \max(\boldsymbol{\rho})$ represents the maximum value of the normalized average Fourier spectrum $\boldsymbol{\rho} = \{\rho_f\}_{f=1}^{F}$. The more the energy of $\boldsymbol{\rho}$ gathers, the larger the maximum value is.

4. EXPERIMENTAL RESULTS



(a) (b) (c)

Fig. 2. Time-frequency spectrograms of the measured echoes from a walking human: (a) SNR ≈ 35dB; (b) SNR=10dB; (c) denoised result from (b) via CPPCA-BIC.

Figure 2 shows a denoising example of the walking human. The time-frequency spectrum of the walking human in Figure 2 (a) is under the high SNR condition of about 35dB and that in Figure 2 (b) is under the SNR condition of 10dB. Figure 2 (c) depicts the time-frequency spectrum of the denoised signal from Figure (b) via the CPPCA-BIC based noise reduction method. We can see that the proposed noise reduction method works well not only in removing the noise component but also in reconstructing the target signal components including the torso-Doppler and micro-Doppler components. Similar results can be obtained for other measured data including those from moving vehicle.

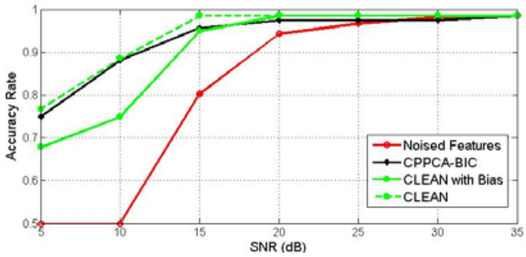


Fig. 3. Variation of the classification accuracy rates of the measured data with the test SNR via Noised Features, CPPCA-BIC, CLEAN with Bias and CLEAN.

Since the measured dataset are collected under high SNR condition, a part of the measured data are used as the training data; while the rest of the measured data as the test data, which are added simulated noise corresponding to the different SNR cases. The Support Vector Machine (SVM)[9] with Gaussian kernel is chosen as the classifier. The parameter of Gaussian kernel is tuned in the range [0, 5] for each classification method, and we select the best classification result from those of the different parameter settings for the following analysis and comparison. Figure 3 depicts the classification accuracies versus SNR, generated via the four classification schemes, including classification with the CPPCA-BIC based noise reduction method, classification with the CLEAN based noise reduction method and the accurate SNR information[5], classification with the CLEAN based noise reduction method and the ±3dB biased SNR information, and classification without noise reduction. For simplicity, the above four classification schemes are referred to as *CPPCA-BIC*, *CLEAN*, *CLEAN with Bias* and *Noised Features* respectively in Figure 3. When the SNR≥25dB, the influence of noise is not so significant, and the four methods yield the similar classification accuracies; when SNR≤25dB, the three noise-robust methods outperform *Noised Features*; when SNR<15dB, the biases on SNR affect the performance of *CLEAN*; *CLEAN* and *CPPCA-BIC* yield the similar classification results for all cases except for SNR≈15dB. Therefore, *CPPCA-BIC* is more robust in the real application due to no SNR prior required. If we assume the classification rate threshold for the real application is 80%, *CPPCA-BIC* can work under SNR≥7dB, while *Noised Features* requires SNR≥15dB. According to the radar equation, in real application the 8dB advantage in SNR will bring an increase about 60% in the classification distance between radar and target.

5. CONCLUSION

In this paper, we proposed a noise-robust radar target classification scheme based on CPPCA model with BIC to discriminate the moving vehicle and walking human. Compared with the existing CLEAN based noise reduction method[5], the CPPCA-BIC based method can work without SNR prior information. Experimental results based on measured data show that the proposed classification scheme can obtain good classification and denoising performance under the relatively low test SNR condition. In the real application, the advantage in SNR can effectively extend the classification distance between the target and radar.

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