A New Function-based Computerized Ionospheric Tomography Algorithm

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Abstract

A new function-based computerized ionospheric tomography (CIT) algorithm for beacon measurements from satellite is proposed in this paper. It employs relative TEC measurements as input source data, and spherical harmonic function (SHF) and empirical orthogonal functions (EOF) as the basis function to represent spatial and temporal variation of ionosphere. Truncated singular value decomposition (TSVD) regularization is proposed for solving the ill-posed matrix inversion problem. The accuracy and reliability of this algorithm has been verified by simulated results.

1. Introduction

At the end of the 1980s, Austin proposed for the first time that the CIT technology could be used to reconstruct two-dimensional distribution of ionospheric electron density with absolute Total Electron Content (TEC) observations [1]. Since the phase difference at the start time of the measurement of the beacon observation is known only up to 2π, an unknown constant is contained in the TEC data derived from two coherent radio signals by using the differential Doppler method. And only relative TEC measurements can be derived from satellites beacon measurements. Aiming to obtain the absolute TEC from beacon observations, multi-stations method was proposed by R.Leitinger[2] which is helpful to solve the offset of relative TEC, but it may lead to about 15% error of the TEC measurements under some assumptions on the symmetry of the ionosphere in different directions.

Beside the above-mentioned, ionospheric tomography also poses some extra physical limitations in the performance of the tomographic algorithms. Due to these limitations, conventional tomographic imaging methods have to be modified. Priori information about the ionosphere should be added to the CIT algorithm [1,3]. The function-based Computerized Ionospheric Tomography Algorithm is one of the representative algorithms which can involve the priori information about the ionosphere and provide spatial and temporal distribution of the ionosphere.

2. Methodology

The linear integral of the electron density can be obtained from the difference between phase shifts in the two radio signals. The linear integral is called absolute TEC, and defined by

$$TEC = \frac{f_r c}{80.62\pi} \frac{m_1^2 m_2^2}{m_2^2 - m_1^2}\left(\Phi(t) + \Phi_0\right) = \int_p N_e ds \tag{1}$$

Where N_e is the electron density distribution, p is ray path between the satellite and receiver, $\Phi(t)$ is differential phase shift of Doppler., Φ_0 is the unknown constant of phase integration, f_r is reference frequency, m_1 and m_2 are ratios of frequencies of receiving signal and f_r.

In function-based approach to tomographic reconstruction of the electron density, the electron density is expressed as a set of horizontal and vertical base functions. SHF and EOF have been used as basis functions for modeling the horizontal and vertical variation of the electron density respectively

$$N_e = \sum_{k=1}^{K}\sum_{n=0}^{N}\sum_{m=0}^{n} \overline{P}_{nm}[\cos(\varphi)]\left\{a_{nm}\sin(m\lambda) + b_{nm}\cos(m\lambda)\right\} \mathrm{EOF}_k(h) \tag{2}$$

Where φ and λ represent latitude and longitude respectively, h is the height. $\overline{P}$ is the normalized Legendre function, m, n, k is number of SHF and EOF coefficients. EOF can be obtained from empirical ionospheric model or the measurements which are related to the electron density. A set of absolute TEC measurements on the line-of-sight paths linked from a beacon satellite to ground-based beacon receivers can form the following matrix as Equation:

$$\mathbf{y} = \mathbf{H}\mathbf{x} \tag{3}$$

Where x is unknown tomography coefficients, while the whole set of absolute TEC measurements are expressed by a column vector **y**. **H** is a forward operator which contains the basis functions generated using EOF and SHF expansions. Because the constant of phase integration is unknown, only relative TEC could be calculated directly. We have modified the procedure so that it can now employ relative TEC data to CIT. In this treatment a reference measurement for each TBB receiver is subtracted from the TEC measurements to remove the unknown integration constant:

$$\mathbf{b} = \mathbf{y} - \mathbf{y_0} = (\mathbf{H} - \mathbf{H}_0)\mathbf{x} = \mathbf{A}\mathbf{x} \tag{4}$$

Where **b** is the difference between different samples of relative TEC data, $\mathbf{y}_0$ is the measurement of the designated satellite for the given receiver (typically the smallest TEC measurement). The forward operator **A** consists of both direct forward operator **H** and reference operator $\mathbf{H_0}$ as described in Eq.(4). Non-uniqueness of the reconstructions owing to limited angle measurements or non-optimal receiver will lead to ill-posed problems of the matrix **A**. To come up with a stable solution for the model parameters, application of regularization techniques is inevitable. The TSVD regularization technique proposed by J.J.Yu[4] is used to solve the problem in CIT in this paper. According to the geometric theorem of singular value decomposition (SVD), the design matrix $\boldsymbol{A}$ can be expressed as follows:

$$\boldsymbol{A} = \boldsymbol{U\Sigma V}^{\mathrm{T}} = \sum_{i=1}^{n} u_i \sigma_i v_i^{\mathrm{T}} \tag{5}$$

$$\boldsymbol{X}_{\mathrm{TSVD}} = \sum_{i=1}^{k} \frac{\langle u_i, \boldsymbol{b}\rangle}{\sigma_i} v_i \tag{6}$$

in Eq.(5), $\boldsymbol{U}=(u_1,u_2,...u_n)$ and $\boldsymbol{V}=(v_1,v_2,...v_n)$ are orthonormal matrices which include the left and the right singular vectors of matrix $\boldsymbol{A}$ respectively, and $\boldsymbol{\Sigma}=\mathrm{diag}(\sigma_1,\sigma_2,...,\sigma_n)$ is a diagonal matrix in which the diagonal elements are singular of the design matrix. $\boldsymbol{X}_{\mathrm{TSVD}}$ is the solution of the Eq.(4) with TSVD regularization. The optimal truncated parameter k can be estimated by Generalized Cross Validation (GCV) method.

3. Results and discussion

Seven stations over China region is considered for the analysis. In this simulation, the latitude ranged from 10°N to 50°N along the longitude 115°E. The height ranged from 100km to 800km with step of 25km. The discretized spacing in latitude is 0.5°. Time interval ranged from UT00:00~21:00 in steps of 3 hours. Relative TEC measurements are obtained by integrating electron density values along the receiver-satellite line of sight at given time using Eq.(1). Independent distributed random noise with zero mean and a standard deviation of 0.3TECU is added to the simulated TEC since precision of beacon TEC measurements is usually better than 0.3TECU. The 10.7 cm radio flux $F_{10.7}$ index was setting to 75.0 and Consultative Committee on International Radio (CCIR) map of ionospheric characteristics was selected in IRI model [5]. Figure 1 shows temporal and spatial variation of the "Simulated-truth" electron density obtained by IRI model. It can be seen obviously that Equatorial Ionization Anomaly (EIA) crest region appears near at latitude 20°N and the maximum electron density occurred at universal time 06:00(with local time of 14:00 at longitude 115°), as has been indicated by many authors[1,2]. Figure 2 shows the tomographic electron density obtained by the algorithm proposed in this paper. We can see that the tomographic value show good coherence with the “truth value” both at temporal and spatial variation. The tomographic value has been compared with "simulated truth" value for all the tomographic epochs of the simulation. And corresponding correlation coefficient and root-mean-square error (RMSE) are also calculated. The proposed algorithm has a good performance in reconstruction of the variation of ionosphere as indicated by RMSE(3.4×10^{10}el.m^{-3}) and correlation coefficient (0.98) .

4. Conclusion

A new function-based ionospheric tomographic algorithm has been proposed in this paper. The important advantage of the algorithm is that it can reconstruct the ionospheric electron density with relative TEC measurements and requires no estimation of the integral constant. Function-based algorithm with EOF and SHF representing the variations in horizontal and vertical direction is adopted in this proposed algorithm as prior information of ionosphere, while TSVD regularization selected to deal with the ill-pose matrix inversion problem of the CIT. Simulated results indicated that this algorithm has good accuracy and reliability in two-dimensional electron density reconstruction. The algorithm can be further validated by using more real beacon measurements of different seasons or years in the future.

5. Acknowledgments

This research is supported by International Science & Technology Cooperation Program of China, (Grant No. 2011DFA-22270) and “China Seismo-Electromagnetic Satellite (CSES)” project.

6. References

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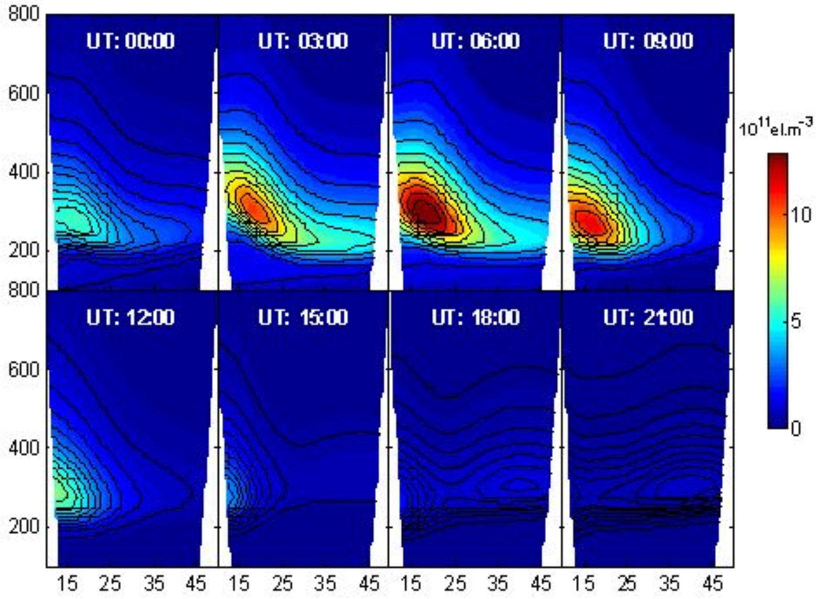

Figure 1. IRI Electron density profile on UT00:00-21:00 ($F_{10.7}$=75.0, Longitude = 115°E)

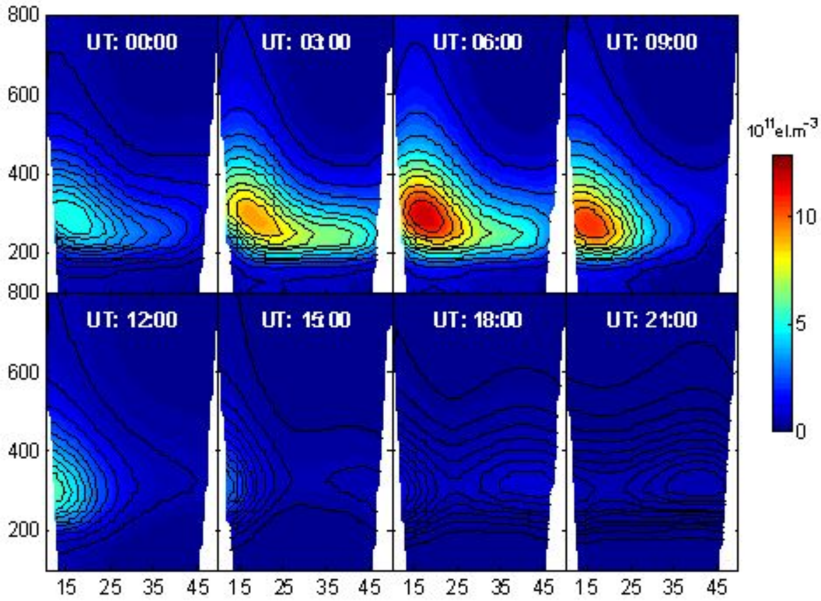

Figure 2. Electron density profile obtained by proposed CIT algorithm