Frequency Hopping Inverse Synthetic Aperture Radar Imaging
With Relative Coherent Processing and Compressed Sensing

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Abstract

An imaging algorithm with relative coherent processing (RCP) and compressed sensing (CS) was proposed and used for frequency hopping inverse synthetic aperture radar (FH-ISAR) imaging. It can be used to enhance the quality of FH-ISAR images. The basic concept of FH-ISAR and its imaging principle was detailed analyzed and the main formulas were given. Simulations were executed and all results proved that, the algorithm was correct and effective; the precision of range alignment was greatly improved with relative coherent processing, and the image blurring caused by frequency hopping and target maneuvering were reduced by compressed sensing. It has a great promising in FH-ISAR imaging.

1. Introduction

Inverse synthetic aperture radar (ISAR) imaging is an effective way to acquire high resolution images of targets at long range and it is an irreplaceable tool in the task of non-cooperative target recognition (NCTR). One drawback of ISAR is that higher order target motion, such as ship’s pitching and rolling in rough seas and aircraft’s maneuvering, causes time-varying Doppler into radar signal. This time-varying Doppler causes severe image blurring in cross-range during the ISAR imaging processing and severely deteriorates the target’s image. Therefore, in order to fulfill the goal of NCTR, methods must be developed to compensate for the undesired motion. Another drawback of traditional ISAR is that fixed operation frequency is sensitive to electronic jamming and it is hardly to operation under circumstance with severe electronic jamming. Inverse synthetic aperture radar with frequency hopping signals can greatly improve the anti-jamming property of ISAR. However, time-varying Doppler caused by target maneuvering and Doppler incoherence caused by frequency hopping from pulse to pulse further deteriorates the severe image blurring in cross-range and it is difficult to implement it in real applications. In this paper, an imaging algorithm with relative coherent processing (RCP) and compressed sensing (CS) was proposed and used for frequency hopping inverse synthetic aperture radar (FH-ISAR) imaging. Its basic principles were detailed analyzed. The main formulas were deduced, computer simulations were executed and the effect of this algorithm was illustrated and analyzed. All results proved that, the algorithm proposed in this paper is not only capable, but also effective, and the image quality was greatly improved without any deteriorations of spatial resolution.

2. Concept of Frequency Hopping Inverse Synthetic Aperture Radar

Frequency hopping inverse synthetic aperture radar (FH-ISAR) is an ISAR with frequency hopping signals from pulse to pulse, and its return signal without any error can be represented as:

\[ s(x,t) = \sum_{k=1}^{N_x} A_k \exp \left\{ -j \frac{4\pi f_c(t)}{c} \left[ R_0 + x_k \cos(\theta(t)) + y_k \sin(\theta(t)) \right] \right\} \]

where \( x \) is the range bin, \( N_x \) is the number of scatterers in the range bin \( x \), \( A_k \) is the magnitude of the radar return signal, \( R_0 \) is the distance to the rotational center of the target which is constant during the coherent processing interval, \((x_k, y_k)\) is the down-range and cross-range distance from the target’s rotational center to the \( k \)th point scatterer, and \( f_c(t) \) varies with the slow time \( t \) is the operation carrier frequency coded by frequency hopping code, it is different from pulse to pulse, and can be expressed as:

\[ f_c(t) = f_0 + c(t) \cdot \Delta f_c \]

where \( f_0 \) is the fixed reference operation frequency, \( \Delta f_c \) is the smallest frequency hopping step, and \( c(t) \) is the random frequency hopping code. In conventionally, the slow time \( t \) can be expressed as:

\[ t = n \cdot PRT, n = 1,2,\ldots,N_p \]

where \( PRT \) is the pulse repetition time, \( N_p \) is the pulse number during coherent processing interval (CPI). So, the operation frequency and return signal can be expressed as:

\[ f_c(t) = f_c(n) = f_0 + c(n) \cdot \Delta f_c, n = 1,2,3,\ldots,N_p \]
\[ s(x, n) = \sum_{j=1}^{N} A_j \exp \left\{- j \frac{4\pi f_j(n)}{c} \left[ R_n + x_j \cos(\theta(n)) + y_j \sin(\theta(n)) \right] \right\} \]  \hspace{1cm} (5)

### 3. FH-ISAR Imaging with Relative Coherent Processing and Compressed Sensing

For conventional ISAR with fixed operation frequency, namely, \( f_j(n) = f_\theta = \text{const} \), the image of target can be easily obtained by two domain FFT operations along down-range and cross-range of return signal after precision movement compensations. However, it is hard to implement such an algorithm for FH-ISAR imaging and there are several difficult in FH-ISAR imaging. One difficult is that higher order target motion, observed when targets are maneuvering, introduces time-varying Doppler into the radar signal. It is this time-varying Doppler that causes severe image blurring in cross-range during the signal processing stage and effectively renders the target’s image unrecognizable even to the most experienced ISAR operators. Another difficult is that operation frequency hopping from pulse to pulse causes Doppler incoherence, and further deteriorates the severe image blurring in cross-range. So, in order to obtain clearly images of interested target, the precision of movement compensation must be improved and energy accumulation method must be detailed researched. Fortunately, Relative coherent processing (RCP) proposed in [1] can be used to enhance the precision of movement compensation and the recently booming compressed sensing technologies can be used for cross range imaging and an imaging algorithm with relative coherent processing (RCP) and compressed sensing was proposed and used for frequency hopping inverse synthetic aperture radar (ISAR) imaging in this paper.

Generally, ISAR imaging can also be expressed as a signal filtering procedure with a band pass filter. In such a process, the band pass filter can be changed by frequency shift the processed signal to a different band, so series of results with different center frequency can be obtained; then, all these results can be synthesized further to obtain a final result. This processing method was called relative coherent processing (RCP)[1]. Because of the coherent accumulation of main lobe of the PSF and non-coherent accumulation of side lobes of PSF, the image contrast can be improved to some extent without any deterioration of spatial resolution with RCP[1]. For simplicity, the principle of relative coherent processing (RCP) was ignored in this section, and we just focused on its application in FH-ISAR for enhancing the precision of range alignment. Suppose the range compressed signal of FH-ISAR echoes as

\[ r(t) = \exp\left\{2\pi f_j(n)(t - \tau_d)\right\} \left\{\sin[\pi \Omega (t - \tau_d)/2]\right\} \]  \hspace{1cm} (6)

where \( \tau \) is the fast time, \( \tau_d \) is the time delay, and \( \tau_d = 2R(n)/c \). Then, the signal after range compression with RCP can be expressed as \[ ]:

\[ \sum_{j=1}^{N} = N \exp\left\{- j\pi \Omega \tau_d \right\} \sin(c[kT][t - 2R(n)/c]) \]  \hspace{1cm} (7)

The amplitude of equation (7) can be expressed as:

\[ \sum_{j=1}^{N} = N \sin(c[kT][t - 2R(n)/c]) \sin(c[kT]) \]  \hspace{1cm} (8)

In equation (7), the factor \( \sin(c[kT][t - 2R(n)/c]) \) and factor \( 1/\sin(c[kT]) \) has the same main lobe width and the same zero crossing periods. However, compared to the factor \( \sin(c(kT)) \), the main lobe width and zero crossing period of factor \( \sin(c[kT][t - 2R(n)/c]) \) were \( 1/N \) times of the former result. It is obvious that the factor \( \sin(c(kT)) \) determined the final main lobe width of the imaging result, and the main lobe width of the final result with RCP is \( 1/N \) times of the conventional imaging result. It’s evident that, after RCP, energy of the same targets had a further coherent accumulation, the main lobe of range response has sharpened, and the scattering centers were clearly enhanced. The range resolution after RCP was improved to some extent, and an amplitude gain equals \( N \), power gain equals \( N^2 \) can be obtained, so the image contrast was greatly improved, and such that it is plays an important role in enhancing range alignment precision in FH-ISAR imaging.

After range RCP, the scattering centers were clearly enhanced. It is important for FH-ISAR imaging, because the range alignment can easily be realized and the precision such a processing can be greatly improved and guaranteed. After range alignment, the initial phase correction processing, such as DCT, PGA, and so on, was applied to the echo, then, we comes to the most important step of FH-ISAR imaging, namely, cross-range imaging. As all we know, due to the frequency hopping from pulse to pulse, the cross-range signal of the \( k \) th scatterer can be simply expressed as:

\[ s_x^k(n) = A_k \exp\left\{- j\frac{\pi \cdot f_j(n)}{c} R_\theta(n) \right\} \]  \hspace{1cm} (9)

Suppose the movement error was compensated totally, the cross-range signal can be simply expressed as:

\[ s_x^k(n) = A_k \exp\left\{- j\frac{\pi \cdot f_j(n)}{c} y_j \sin(\theta(n)) \right\} \]  \hspace{1cm} (10)

where \( \theta(n) = \theta_\theta + \Omega \cdot n \cdot PRT \), and \( \Omega \) is the constant angular velocity of the radar target from which the Doppler cross-range location of the scatterer is extracted, then, the cross-range signal can be further expressed as:

\[ s_x^k(n) = A_k \exp\left\{- j\frac{\pi \cdot f_j(n)}{c} y_j \sin(\theta_\theta + \Omega \cdot n \cdot PRT) \right\} \]  \hspace{1cm} (11)
If \( \theta(n) \) is assumed to be small, then 
\[
\sin \theta(n) = \theta(n) + \frac{\Omega \cdot PRT}{c} \cdot n,
\]
the cross-range signal can be further expressed as: 
\[
s_r^k(n) = A_k \exp \left[ -j4\pi \frac{f(n)}{c} y_1 \theta_1 - j4\pi \frac{f(n)}{c} \Omega PRT \cdot n \cdot x_1 \right]
\]
(12)

It is clearly that, the cross-range imaging can be realized with FFT operations when 
\( f_1(n) = f_0 = \text{const} \). However, it is not true for FH-ISAR, with which 
\( f_1(n) = f_0 + c(n) \cdot \Delta f, n = 1,2,3,....,N_p \), there exists severe cross-range image blurring when 
the conventional azimuth FFT method was used for cross-range imaging of FH-ISAR even all the movement errors were 
compensated at all, because frequency hopping from pulse to pulse causes severe Doppler incoherence in FH-ISAR. In 
order to obtain clear target images, new imaging method was needed. Suppose the real operational frequency number 
is \( M_{f_1} \), which is a limited number and the frequency hopping code \( c(n) = \{ 0,1,2,....,M_{f_1} \} \). Based on this simple fact, it is 
important to realize that the whole cross-range signal can be divided into several parts with different operation frequency 
and different sparse apertures, with each part has the same specific operation frequency, and can be used for cross-range 
imaging with compressed sensing (CS) technology. By taking full advantage of the sparsity of target’s reflectivity and 
the direct information sampling property of compressed sensing, the cross-range imaging of FH-SAR can be easily 
realized successfully.

Ignoring the initial phase, the \( m^{th} \) part of the cross-range signal with specific operation frequency expressed as: 
\[
s_r^k(m) = A_m \exp \left[ -j4\pi \frac{f_0}{c} \Omega PRT \cdot n_m \cdot x_1 \right]
\]
(13)

where \( n_m \) is a non-empty orthogonal sub-set divide of set \( \{1,2,3,4,....,N_p\} \), which was decided by the specific 
operational frequency hopping code \( c(n) = \{ 1,2,3,....,N_p \} \). 

Suppose the length of sequence \( n_m \) is \( M \), then, the \( m^{th} \) part of the cross-range signal can be further expressed as:
\[
\begin{bmatrix}
0 \\
0 \\
. \\
0 \\
A_k \\
0
\end{bmatrix}_{N_p \times 1} = \Psi_{N_p \times N_{f_1}} \Phi_{M \times N_{f_1}}
\]
(14)

where \( \Psi \) is the matrix of Fourier transform, \( \Phi \) is the measure matrix, with \( \Phi(i, n_m) = 1, i = 1,2,3,....,M \), and it is 
obviously that Equation (14) is one of the standard models for CS applications. By using standard OMP algorithm, a 
series of sparse imaging results can be obtained for each part of the FH-ISAR cross-range signal with specific operation 
frequency. So the whole FH-ISAR imaging algorithm with RCP and CS can be illustrated as Fig.1.

\[\text{Table.1 Simulation Parameters}\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Transmit Power</td>
<td>5000 W</td>
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<tr>
<td>Smallest Frequency Hopping Step</td>
<td>400 MHz</td>
</tr>
<tr>
<td>Reference Carrier Frequency</td>
<td>5.3 GHz</td>
</tr>
<tr>
<td>Signal Bandwidth</td>
<td>400 MHz</td>
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<tr>
<td>Pulse Width</td>
<td>1 us</td>
</tr>
<tr>
<td>Pulse Repetition Frequency</td>
<td>500 Hz</td>
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<tr>
<td>Sampling Frequency</td>
<td>500 MHz</td>
</tr>
<tr>
<td>Frequency Number</td>
<td>10</td>
</tr>
<tr>
<td>Frequency Hopping Bandwidth</td>
<td>( 5 \times 400 ) MHz</td>
</tr>
<tr>
<td>Real Operation Frequency</td>
<td>5.4, 5.1, 5.5</td>
</tr>
<tr>
<td>5.9, 6.3, 6.7</td>
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<tr>
<td>7.1, 7.5, 7.9</td>
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<tr>
<td>8.3 GHz</td>
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<td>Pulse Number</td>
<td>1024</td>
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\[\text{Fig.1. FH-ISAR imaging algorithm with RCP and CS}\]

\[\text{4. Imaging Simulations}\]

In order to validate the capability of imaging algorithm, simulations were executed. The simulation parameters 
were given in table.1. The target model and its maneuvering law were depicted in Fig.2. The processing results were 
given in Fig.3, Fig.4. It is obvious that the main lobe of the range compression response was greatly sharpened after RCP,
and it’s evident that the scattering centers were clearly enhanced; therefore the precision of range envelope alignment was improved. So, the final imaging results were very clear, and the imaging blurring due to frequency hopping was completely reduced.

(a) Target model            (b) Maneuvering flying track
Fig.2. Target model and its maneuvering flying track

Echo after conventional range compression and range alignment
Azimuth Cell
Range Cell
100  200  300  400  500  600  700  800  900  1000
10  20  30  40  50  60  70  80  90  100

Echo after range compression with RCP and range alignment
Azimuth Cell
Range Cell
100  200  300  400  500  600  700  800  900  1000
20  40  60  80  100  120  140  160  180

(a) Conventional range compression       (b) Range compression with RCP
Fig.3. Range alignment comparison

(a) Result with RD Algorithm              (b) Result with RCP and FFT               (c) Result with RCP and CS
Fig.4. Imaging results comparison

5. Conclusion

All research results proved that the imaging algorithm with relative coherent processing (RCP) and compressed sensing for frequency hopping inverse synthetic aperture radar (FH-ISAR) proposed in this paper was not only capable, but also effective. It can be used to improve the precision of range alignment and reduce the image blurring caused by frequency hopping. It has a great promising in FH-ISAR imaging.

6. References


