Eliminating Effects of Multipath Propagation on Diagnostics of Inhomogeneous Media Using Spatial Processing of Field

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Abstract

By using spatial processing of field based on the double weighted Fourier transform (DWFT) method, we suggest eliminating effects of multipath propagation on results of field measurements during signal propagation in a smoothly inhomogeneous medium. We explore the possibility of eliminating effects of multipath propagation from results of scattered-field measurements during signal focusing by a local plasma irregularity and refraction by several local irregularities. We present results of the numerical simulation of phase after the DWFT spatial processing and without it for multipath propagation of radio signal.

1. Introduction

During electromagnetic-wave propagation in inhomogeneous media, a signal may propagate along various paths joining points of emission and reception. This multipath effect followed by strong amplitude variations [1] is one of the most serious problems of systems of communication, navigation and diagnostics of inhomogeneous media. It needs to be taken into account for investigating radio-wave propagation in the Earth's troposphere and ionosphere, radio-wave propagation through interplanetary and near-Sun plasma, propagation of light in the turbulent atmosphere and optical fibre, and propagation of sound in the ocean. As for the multipath-effect mechanism, it may be of various forms, for example, may be determined by wave reflection from surrounding objects or wave refraction and diffraction in an inhomogeneous medium. When light passes through the focusing lens, in the focal plane a caustic and a multipath effect are formed (Fig. 1 a). These phenomena may also be observed during electromagnetic-wave refraction by two or several local irregularities (Fig. 1 b).

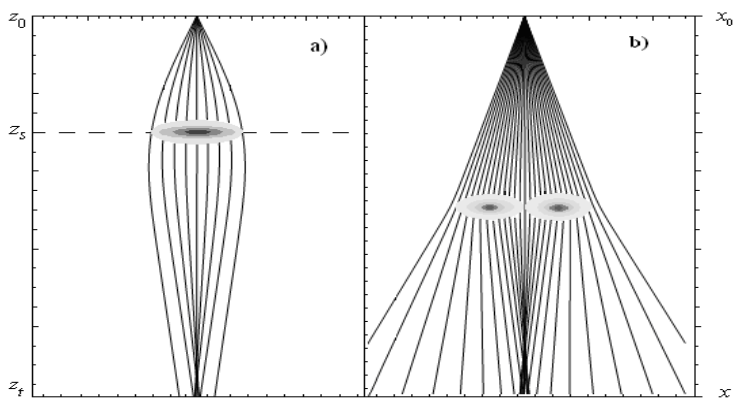

Fig. 1. The ray pattern: a) focusing by the lens, b) refraction by two irregularities.

Describing wave fields in the region of appearance of multipath effect in the context of one or another mathematical model is a difficult task. For example, in conditions of multipath propagation, the wave amplitude in geometrical optics (GO) approximation abruptly increases on caustics (enveloping rays) [1], which points out the necessity to use in field description GO-approximation generalisations accounting for diffraction effects of caustic type such as the Maslov method, interference integral method, etc. [2]. However, these methods demand a priori knowledge about ray structure, which is rarely possible in diagnostic tasks. The Rytov approximation does not allow us to describe fields in conditions of strong amplitude variations, this method inapplicable for field description in conditions of multipath propagation [3]. The phase-screen approximation describes fields in the region of multipath propagation and caustics, but the weakness of this model is the necessity to set screen parameters [4].

The authors of [5-6] proposed an integral representation of the field of a wave, scattered by irregularity, in the form of double weighted Fourier transform (DWFT). With a small length of a randomly inhomogeneous layer, the integral DWFT representation goes into results of the phase-screen method in the small-angle approximation [7], i.e. it helps us in describing the field in the region of multipath propagation, including vicinities of caustics. This integral expression produced a spatial processing of field measurements [6] in transverse receiver/transmitter coordinates. As is shown in [6-7], the DWFT

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processing provides the super-Fresnel resolution without information on coordinates of irregularities in conditions of strong and weak phase variations. Consider next the possibility of eliminating multipath effects in diagnostics of inhomogeneous media, using the spatial processing of the DWFT field.

2. DWFT and its wave-field inversion

Let a source and a receiver be at points $\mathbf{r_0} = (z_0, x_0, y_0) = (z_0, \boldsymbol{\rho}_0)$ and $\mathbf{r} = (z_t, x, y) = (z_t, \boldsymbol{\rho})$, where $\boldsymbol{\rho}_0 = (x_0, y_0)$ and $\boldsymbol{\rho} = (x, y)$ are two-dimensional vectors in planes $z = z_0$ and $z = z_t$. Between the source and the receiver is an inhomogeneous medium [6]. In this case, the wave field in the DWFT method is as follows [5–7]:

$$U(\boldsymbol{\rho},\boldsymbol{\rho}_0) = \frac{-Ak^2}{4\pi^3 Z^3}\exp\left[ik\left(Z + (\boldsymbol{\rho}+\boldsymbol{\rho}_0)^2/2Z\right)\right]\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\xi d^2\xi_0 \exp\left(ik\left[2(\xi\xi_0 - \xi\boldsymbol{\rho}_0 - \xi_0\boldsymbol{\rho})/Z + \Phi(\xi,\xi_0,z_t,z_0)\right]\right), \tag{1}$$

where $Z = z_t - z_0$ is the distance between the planes with the source and receiver; A_0 is the incident spherical wave amplitude; $k = \omega / c$, $\omega = 2\pi f$ is the radiation frequency; c is the velocity of light in free space; $\Phi(\xi, \xi_0, z_t, z_0)$ is the linear integral from $\tilde{\varepsilon}(\mathbf{r})$ calculated from formula

$$\Phi(\xi,\xi_0) = 1/2\int_{-\infty}^{+\infty}\tilde{\varepsilon}(\xi(z'-z_0)/Z + \xi_0(z_t - z')/Z, z')dz, \tag{2}$$

$\tilde{\varepsilon}(\mathbf{r}) = \varepsilon(\mathbf{r}) - 1$ is the permittivity variation $\varepsilon(\mathbf{r})$.

By applying integral operator (3) to the field $U(\boldsymbol{\rho},\boldsymbol{\rho}_0)$

$$\tilde{U}\left(\boldsymbol{\rho}^*,\boldsymbol{\rho}_0^*\right) = \int_{-\infty}^{\infty}\int_{\infty}^{\infty} d^2\rho d^2\rho_0 U(\boldsymbol{\rho},\boldsymbol{\rho}_0)\exp\frac{ik}{Z}\left\{-Z^2 + 2(\boldsymbol{\rho}^*\boldsymbol{\rho}_0 + \boldsymbol{\rho}_0^*\boldsymbol{\rho}) - (\boldsymbol{\rho}+\boldsymbol{\rho}_0)^2/2\right\}, \tag{3}$$

we obtain

$$\tilde{U}(\boldsymbol{\rho}^*,\boldsymbol{\rho}_0^*) = \hat{L}\left[U(\boldsymbol{\rho},\boldsymbol{\rho}_0)\right] = -\frac{A_0\pi Z}{4k^2}\exp\left\{ik\left[2\boldsymbol{\rho}^*\boldsymbol{\rho}_0^*/Z + \Phi(\boldsymbol{\rho}^*,\boldsymbol{\rho}_0^*)\right]\right\}. \tag{4}$$

Thus, after processing (3), we can find phase (5)

$$k\tilde{\Phi}(\boldsymbol{\rho}^*,\boldsymbol{\rho}_0^*) = \arg\left[\tilde{U}(\boldsymbol{\rho}^*,\boldsymbol{\rho}_0^*)\right] - 2\boldsymbol{\rho}^*\boldsymbol{\rho}_0^*/Z = \frac{k}{2}\int_{-\infty}^{\infty}\tilde{\varepsilon}\left[\boldsymbol{\rho}^*(z'-z_0)/Z + \boldsymbol{\rho}_0^*(z_t - z')/Z, z'\right]dz'. \tag{5}$$

The phase $k\tilde{\Phi}(\boldsymbol{\rho}^*,\boldsymbol{\rho}_0^*)$ in (5) has the same form as in the GO approximation. It is significant that wave field (4) after spatial DWFT processing (3) has no amplitude fluctuations which appear because of wave focusing by irregularities [8].

To analyze the multipath effect on the resolution capability of the DWFT method, we should substitute the direct problem solution $U(\boldsymbol{\rho},\boldsymbol{\rho}_0)$ in (3). Unfortunately there is no strict solution to the problem of spherical wave propagation with local irregularity. So, it is worthwhile taking an approximated solution for the processed field $U(\boldsymbol{\rho},\boldsymbol{\rho}_0)$ in (3). To examine the resolution capability of the inverse DWFT, for the processed field model we employ the solution derived with the phase screen method [3].

3. Numerical simulation results

Let local irregularities be between a source and a receiver located in the planes $z = z_0$ and $z = z_t$ respectively (Fig. 1a, b). In the case when irregularities are located in the vicinity of the plane $z = z_s$, the wave field with consideration of the multipath effect may be found in the phase-screen approximation [6–7]

$$U(\boldsymbol{\rho},\boldsymbol{\rho}_0) = \frac{ikA_0\exp\{ikZ\}}{8\pi^2(z_t - z_s)(z_s - z_0)}\int_{-\infty}^{\infty} d^2\rho_s \exp\left\{ik\left(\frac{(\boldsymbol{\rho}_s - \boldsymbol{\rho}_0)^2}{2(z_s - z_0)} + \frac{(\boldsymbol{\rho} - \boldsymbol{\rho}_s)^2}{2(z_t - z_s)} + \Phi_s(\boldsymbol{\rho}_s)\right)\right\}. \tag{6}$$

If transverse sizes of the irregularities exceed the Fresnel radius, we may use the GO approximation. This is equivalent to the application of the stationary-phase method to the calculation of field expression (6). Stationary-phase points $\boldsymbol{\rho}_{sc}$ can be determined from equation

$$\partial\left(\tilde{\Phi}_s(\boldsymbol{\rho}_s) + (\boldsymbol{\rho}_s - \boldsymbol{\rho}_0)^2/\left(2(z_s - z_0)\right) + (\boldsymbol{\rho} - \boldsymbol{\rho}_s)^2/\left(2(z_t - z_s)\right)\right)/\partial\boldsymbol{\rho}_s = 0. \tag{7}$$

As a model of inhomogeneous-medium permittivity we will take a sum of Gaussian functions:

$$\tilde{\varepsilon}(\boldsymbol{\rho}, z) = \varepsilon_{mi} \sum_{i=1}^{N} \exp\left(-\left[(\boldsymbol{\rho} - \boldsymbol{\rho}_{mi})^2 + (z - z_{mi})^2 \right] / \left(2 l_i^2 \right) \right). \quad (8)$$

For model (8), we substitute the $\tilde{\Phi}_S(\boldsymbol{\rho}_S)$ calculation result in (7) and obtain the dependence of the transverse coordinate $\boldsymbol{\rho}$ on $\boldsymbol{\rho}_{sc}$

$$\boldsymbol{\rho}\left(\boldsymbol{\rho}_{sc}\right) = \boldsymbol{\rho}_{sc} \frac{z_t - z_0}{z_S - z_0} - \boldsymbol{\rho}_0 \frac{z_t - z_S}{z_S - z_0} - (z_t - z_S) \sqrt{\frac{\pi}{2}} \varepsilon_{mi} \sum_{i=1}^{N} \frac{\boldsymbol{\rho}_{sc} - \boldsymbol{\rho}_{mi}}{l_i} \exp\left(- \frac{\left(\boldsymbol{\rho}_{sc} - \boldsymbol{\rho}_{mi} \right)^2}{2 l_i^2} \right). \quad (9)$$

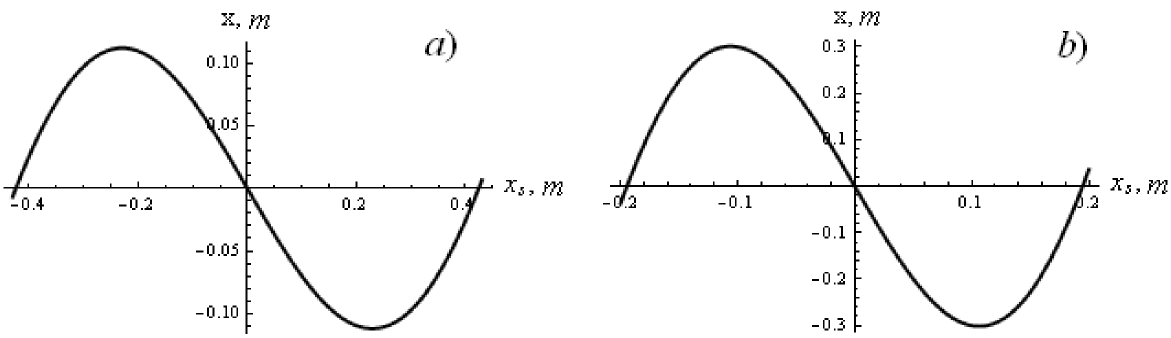

Fig. 2. The dependence of the transverse ray coordinate at the observation point on the coordinate of ray output from the screen for the focusing irregularity (a) and for two irregularities located symmetrically about the z axis (b).

Figure 2a presents results of calculations of (9) for laboratory plasma at $y = 0$ for $N = 1$, $x_m = z_m = 0$, $\varepsilon_m = 0{,}14$, $l = 40cm$, $z_t = -8m$, $z_0 = 8m$. As $\varepsilon_m > 0$, this case corresponds to propagation in a lens-like medium. The figure implies that the multipath effect in the plane $z = z_t$ is observable in the range of $-0{,}11m < x < 0{,}11m$ enclosed by points of intersection of the caustic plane and the plane $z = z_t$. These points match extremes of the curve $\boldsymbol{\rho}\left(\boldsymbol{\rho}_{sc}\right)$. In the region of multipath propagation, to each point at a given value x three rays arrive – one direct and two refracted.

Figure 2b presents results of calculations of (9) for $N = 2$, $z_{m1,2} = y = 0$, $\varepsilon_{m1} = \varepsilon_{m2} = -0{,}17$, $x_{m1} = -26cm$, $x_{m2} = 26cm$, $l_1 = l_2 = 15cm$, $z_t = -5m$, $z_0 = 5m$. In this case, $\varepsilon_{m1,2} < 0$ and there is refraction by two irregularities (Fig. 1b). Here, as during focusing by the lens, the multipath effect arises in the range of $-0{,}3m < x < 0{,}3m$ where three rays arrive – one direct ray and two rays reflected from two irregularities. Note the qualitative similarity between these cases; this is associated with the similarity between ray patterns in both of the situations in the vicinity of the Z axis.

Consider now how DWFT spatial processing (3) influences results of scattered-field measurements in case of multipath effect. Note that in real conditions the integration in (3) may be realised only in finite terms. In simulation, sizes of the integration domain in (3) were selected by us from conditions obtained by [7]. Moreover, the simulation of spatial processing (3) requires solving the direct problem $U(\boldsymbol{\rho}, \boldsymbol{\rho}_0)$. Here as the processed-field model we use solution (6). Explore the possibility of eliminating multipath effects through spatial processing of field (3) for cases of field focusing by the lens (Fig. 1a and Fig. 2a) and refraction by two local irregularities (Fig. 1b and Fig. 2b).

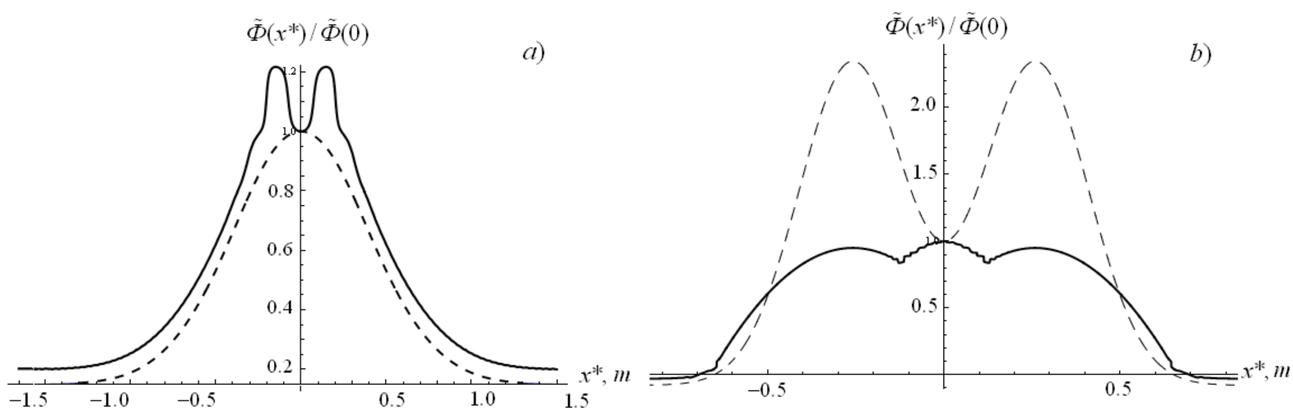

Fig. 3. The normalised phase $\tilde{\Phi}(x^*)/\tilde{\Phi}(0)=\tilde{\Phi}(x^*,x^*,0,0)/\tilde{\Phi}(0,0)$ without processing (solid line) and after the DWFT processing (dashed line) for a) one focusing irregularity and b) refraction by two local irregularities.

Figure 3 presents results of calculations of the phase $\tilde{\Phi}(x^*)/\tilde{\Phi}(0)=\tilde{\Phi}(x^*,x^*,0,0)/\tilde{\Phi}(0,0)$ as a function of $x^*=x_0{}^*$ in the cross-section $y^*=y_0{}^*=0$ at the following simulation parameters λ=2,5cm, $x_m=z_m=0$, $\varepsilon_m=0{,}14$, $l=40cm$, $z_t=-8m$, $z_0=8m$, $a_F=31cm$, Fresnel radius does not exceed sizes of the irregularity. The figure 3a implies that after the spatial processing (solid curve) the irregularity produces a wider response with two maxima, which complicates the determination of parameters of the irregularity under study. The application of the DWFT processing (dashed line) allowed us to eliminate the multipath effect and determine the real profile of the irregularity. Figure 3b illustrates the phase for the case of refraction by two local irregularities at $\lambda=2mm$, $\varepsilon_{m1}=\varepsilon_{m2}=-0{,}17$, $x_{m1}=-26cm$, $x_{m2}=26cm$, $l_1=l_2=15cm$, $z_t=-5m$, $z_0=5m$. In this case, the Fresnel radius $a_F=7{,}2cm$ does not exceed sizes of the irregularity. The phase without processing (solid line) prohibits the determination of two Gaussian irregularities because of appearance of the multipath effect. The DWFT spatial processing (dashed line) allowed us to eliminate the multipath effect.

3. Conclusion

Our study demonstrates that effects of multipath propagation on results of diagnostics of inhomogeneous media can be eliminated using an additional spatial processing of the DWFT field. This simplifies considerably the search for physical characteristics of inhomogeneous media. The numerical simulation results show that the application of the DWFT processing even in conditions of strong phase fluctuations during wave scattering by one or several different-scale irregularities makes it possible to eliminate multipath effects.

4. Acknowledgments

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