Modified Wheeler Cap Method for Measurement of Antenna Efficiency

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Abstract

A new non-circuit model method to predict the antenna efficiency via means of Wheeler cap measurements is proposed. The method uses an interpolated prediction of the quality factor of the antenna in the radiating and non-radiating modes with the help of Wheeler caps. The method works for narrow, multiple, and wide band antennas with a complete immunity to the effect of the cavity modes. Also, the semi-analytical approach in dealing with the reflection coefficient embraces an insightful view to individual resonators in the measurements. The proposed procedure provides a simpler, more accurate, and straightforward method to measure the antenna efficiency than other Wheeler cap based alternatives. The results of the proposed methods are verified with a commercial full wave solver and measurements.

1. Introduction

The antenna efficiency is one of the important parameters of an antenna, but it is difficult to measure [1]. The Wheeler cap method, proposed in [2], is a simple method to measure the radiation efficiency of small antennas. It assumes a simple resonator circuit model to the antenna (parallel or series RLC circuit). The efficiency is evaluated by performing two separate measurements of the reflection coefficient of the antenna under test. The first measurement is for the antenna in free-space, where radiation is allowed. Thus, the radiation resistance combined with the losses in the circuit model can be evaluated. The second measurement is for the antenna under test inside a sealed metallic Wheeler cap that is specially designed not to interact with the antenna near fields [2]. It only prevents the antenna to leak the radiated power by shortening out the radiation resistance in the circuit model. Using this method, the radiation efficiency for an antenna can be evaluated by separating the radiation from Ohmic losses in the antenna based on the assumed circuit model, which varies from an antenna type to another. An advanced approach to the Wheeler cap method has been presented in [3] to extend the usage of the Wheeler cap method to other complicated antennas. An accurate circuit model of the antenna, based on higher order transformer circuit, is used to determine the radiation resistance and losses from the different reflection coefficient measurements. This method extends successfully the Wheeler cap method capability to cover wide and multiple band antennas, but with an overhead of numerical optimization to fit the antenna responses to the circuit model.

Alternatively, the Generalized Wheeler Cap method (GWC) is a quite reliable method for measuring the efficiency of small antennas [4]. The method featured an adequate procedure to measure the radiation efficiency of single mode antenna using large cavities (i.e. cavity modes are significantly involved in the measuring frequency band of interest). The method is based on 2-port network analysis using S-parameters without going through the details of the circuit model of the antenna under test. The GWC algorithm has been validated with different antenna types in [5] presenting the accuracy and capability of the GWC method to overcome the presence of cavity modes in the measuring frequency band. This method introduces a sort of averaging to the measured reflection of the antenna inside the Wheeler cap; resulting in suppressing the effect of cavity modes. The method has some challenges to compute the efficiency of wide and multiple band antennas, which are mainly the effects of excessive number of cavity modes within the antenna frequency band of operation. This causes significant deterioration of prediction of the antennas efficiency.

In this work, a semi-analytical approach is used to obtain the quality factor curve of the antenna response in free-space and inside the Wheeler cap based on the invariant definition of Q for reflection coefficient. The obtained quality factors are used directly to compute the antenna efficiency by separating the radiation from Ohmic losses in the antenna without going through developing exact circuit models of the antennas under test. Another incentive view to the obtained expressions of the reflection coefficient is the ability to precisely determine the resonances of the structure under test (antenna resonance(s) and cavity modes) and their quality factors. The proposed technique is compared to the well-establish GWC method for accuracy with different antennas. The results are verified with the HFSS commercial package [6] and measurements.
2. The Quality Factor Method

The process starts by fitting the antenna measured reflection coefficient in both the free space and inside the Wheeler cap to the rational function $\Gamma$, which is a function of the complex frequency $s = \sigma + j\omega$, as presented in [7] viz

$$\Gamma(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + 1},$$

where $a_n$ and $b_n$ are constants evaluated by the least square fitting of each reflection coefficient to the function. The fitting polynomial orders and frequency band segments are chosen (manually or adaptively) to satisfy a specific tolerance criterion (e.g. 0.1%) between the measured and fitted curves. Then, the quality factor can be evaluated using the invariant formula in [8] without assuming a circuit model for the antenna as

$$Q_0 = \frac{f_0 |d\Gamma(s)/df|}{1 - |\Gamma|^2},$$

where $f_0$ is, by definition, the resonance frequency of the unloaded resonator.

The procedure continues by assuming that the Wheeler cap doesn’t add much loss to the measurement setup, so the computed quality factor of the antenna inside the Wheeler cap represents the losses inside the antenna body only. Moreover, the antenna efficiency can be computed in terms of the quality factor using a simple expression that can be easily derived [9] as

$$\eta_r = 1 - \frac{Q_a}{Q_w},$$

where $\eta_r$ is the radiation efficiency of the antenna, $Q_a$ and $Q_w$ are the computed quality factors of the antenna in free-space and inside the Wheeler cap, respectively. The total efficiency of the antenna can be calculated by introducing the effect of the antenna matching $\Gamma_a$ as $\eta_{tr} = (1 - |\Gamma_a|^2)\eta_r$.

The main benefit of evaluating the quality factor through this expression is that it accurately accommodates the measured response, even if it is under-sampled from the measurements. Therefore, it can give a better prediction of the quality factor compared to other methods, or even by numerically differentiating the expression in Eq. (2). Another potentially important property of the fitted expression is the ability to accurately determine resonances of the measured structure. Exploring the poles and zeros of the fitted reflection coefficient rational function, the resonances of the device under test appear clearly in this expression. In case of measuring the antenna response in free-space, the resultant poles and zeros contain the antenna resonance frequency(ies) within the in-band frequency range along with other numerical artifact resonances outside the measurement band. Also, the response of the antenna inside the Wheeler cap has pairs of poles and zeros related to the antenna resonance(s) (slightly shifted), cavity modes, and other artifact resonance.

3. Results and Discussions

The following subsections provide verification and measurements cases to prove the introduced concept.

3.1 An Electrically Small Antenna

Since the original Wheeler cap method is developed to measure the efficiency of small antennas, it is important to examine the applicability of the proposed technique on this antenna type. An EZ electrically small antenna, as proposed in [10] with dimensions $H = 8.3$ mm, $H_1 = 1.778$ mm, $L_s = 18$ mm, $W_s = 1.524$ mm, $W_a = 1.5$ mm, $W_v = 1.016$ mm, and $H_v = 14.732$ mm is considered. The substrate is 0.78 mm thick and has $\varepsilon_r = 2.2$ is shown in Fig. 1(a). A relatively large Wheeler cap is used with this antenna of square cross section and dimensions ($170 \times 170 \times 150$ mm$^3$). Through fitting the antenna response in free-space into the rational function in Eq. (1), the expression has a pole in the studied frequency band that occurs exactly at 1.373GHz and a quality factor of 43, which complies with the reported results in [10] for the resonance of the used antenna. Alternatively, the fitted response inside the Wheeler cap shows correspondent close poles and zeros that represent individual resonances in the structure under test as presented in Table. 1. Through the tabulated poles and zeros, it is obvious that the antenna resonance suffers from a slight frequency shift due to loading. Also, cavity modes are there as well with relatively high quality factors. Fig. 1 presents the simulation results of the antenna efficiency obtained from the full wave analysis and computed through the proposed quality factor and GWC methods. The results are almost identical between both methods, which embrace the ability of the proposed technique to obtain the same results of previous techniques in their ‘efficient’ domain. The measured results of the proposed technique with the EZ small antenna are presented in Fig. 1(c). The results are quite off from the
simulated results due to the sensitivity of the EZ antenna to the small air gap between the antenna and the ground plane. This causes reduction to the radiation resistance, thus deteriorates the matching and lowers the efficiency. However, the quality factor method and the GWC method almost agree on the same value of the antenna efficiency.

![Antenna geometry image](image)

Fig. 1. The EZ small antenna geometry, $S_{11}$, and antenna efficiency, (b) Simulation using HFSS, and (c) Measurement.

### 3.2 A Dual Mode Dielectric Resonator Antenna

Obtaining the efficiency of a wide band antenna is quite challenging for the GWC method. This is due to averaging the antenna losses with cavity modes losses in the Wheeler cap. A U-shaped DRA is used to illustrate this behavior with dimensions: $L_a = 30 \, \text{mm}$, $W_a = 5 \, \text{mm}$, $H_a = 18 \, \text{mm}$, $H_1 = 8 \, \text{mm}$, $H_p = 11 \, \text{mm}$, $W_1 = 10 \, \text{mm}$, and $D = 12.5 \, \text{mm}$, and $\varepsilon_r = 10.2$ as shown in Fig. 2(a). The simulated and measured efficiency results are shown in Fig. 2. The used Wheeler cap has dimensions ($100 \times 100 \times 60 \, \text{mm}^3$). The results show that the proposed method gives more accurate results than the GWC method. It is worth mentioning that the obtained results for the Q factor method are almost the same regardless of the measurement frequency resolution. However, the GWC method is highly dependent on the accuracy of representing the cavity modes in the test setup. It requires them to be accurately represented to operate correctly (as seen in the lower band), but as frequency increases, the demand of frequency resolution increases to accommodate closer cavity modes. This is a challenging requirement that is not necessary possible either computationally or practically.

![Antenna geometry image](image)

Fig. 2. The U-shaped DRA geometry and $S_{11}$ and antenna efficiency. (a) antenna geometry (b) simulations using HFSS, and (c) measurements.

### 3.3 A Wide Band Antenna

Another example of a wide bandwidth antenna can be shown by using the multilayer compact circular DRA introduced in [11]. The antenna geometry is shown in Fig. 3(a) with dimensions: $R_s = 7 \, \text{mm}$ and $A = 3.7 \, \text{mm}$. The multilayer structure has the bottom layer with $h_2 = 2.5 \, \text{mm}$ and $\varepsilon_r = 6.15$, middle layer with $h_2 = 3.35 \, \text{mm}$ and $\varepsilon_r = 2.33$, and the top layer with $h_1 = 2.5 \, \text{mm}$, and $\varepsilon_r = 10.2$. The Wheeler cap dimensions are $(45 \times 45 \times 30 \, \text{mm}^3)$. The simulated
and measured results are shown in Fig. 3. It is clear from the presented results that the quality factor method gives more accurate results to the antenna efficiency with more tolerance to the errors caused by cavity modes. This is due to the use of the rational function fitting and the fact that cavity modes effects are miniaturized when combined through the quality factor of the antenna. It is important to elaborate the challenges in sealing the Wheeler cap to the antenna ground plane using conducting tape at this frequency band. To overcome this problem and maybe going further in frequency bands, a contact-less Wheeler cap is under developments to avoid such problems.

Fig. 3. The multilayer wide band compact circular DRA geometry and $S_{11}$ and antenna efficiency. (a) antenna geometry, (b) Simulation, and (c) Measurements.

4. Conclusion

An efficient method to predict the antenna efficiency was introduced using Wheeler caps. The proposed method made use of a semi-analytical approach to obtain the quality factor of the antenna in radiating and non-radiating modes using the Wheeler cap to improve the accuracy. The proposed procedure exhibited cavity modes tolerable behavior for different antennas when compared to the previously used methods in a simple and straightforward approach. The results of the proposed technique were verified with a commercial full wave solver and measurement.

5. References