Direction Finding of Coherent Signals with Mutual Coupling Error Based on Matrix Transformation

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In order to estimate the direction of arrival (DOA) of coherent signals with mutual coupling error, a mutual coupling self-calibration method based on matrix transformation is proposed. First, the real noise subspace is obtained by performing decorrelation; then a novel mutual coupling matrix (MCM) transformation method is given, MCM is expressed by mutual coupling coefficients and position matrix, and array manifold is transformed, decoupling is accomplished by the angle information of signals independent of mutual coupling coefficients; at last, DOA can be estimated by peak searching utilizing the modified rank reduction estimation method. The simulation results of the algorithm illuminate the effectiveness of the proposed method.

Keywords: DOA estimation, mutual coupling error, coherent signal, matrix transformation

I. Introduction

Direction of Arrival estimation by array has been a hot research topic in the field of signal processing. With good DOA estimation performance in ideal direction finding conditions, the multiple signal classification (MUSIC) algorithm [1] is widely used. However many coherent signals exist in space because of multipath propagation, intentional interference, et al. DOA estimation performance of traditional MUSIC algorithm for coherent signals reduce sharply. This problem has been paid much attention, and plenty of research methods [2-3] have sprung up in recent years. Unfortunately, the high DOA estimation performance of these methods relies on accuracy expression for the array manifold. Actually, array perturbations such as mutual coupling of elements, complex gain errors, and sensors position errors are often inevitable in practice. So it is needed to calibrate [4] and estimate [5] array parameters imminently. Especially mutual coupling errors will exist with high frequency of incidence signals and small space between sensors; it will destroy the array manifold, and impact on the estimation performance for direction finding seriously.

So it is important to solve the problem that how to obtain accuracy DOA estimation for coherent signals in present of mutual coupling errors. For this case, a DOA estimation method based on mix signals is developed in [8]. Using the data of incoherent signals, the parameters of mutual coupling are firstly estimated, in turn, the DOA of coherent signal is obtained with mutual coupling errors known. In fact this kind of algorithm does not essentially solve these problems. The methods in [9] make use of traditional spatial smoothing algorithm for decorrelation, and use middle array receiving data only to decouple, however it has lost the array aperture. Spatial smoothing algorithm is also utilized for decorrelation in [10], a decoupling method based on rank-reduction (RARE) estimator is given, but the performance of DOA estimation is unsatisfied because of more pseudo-peaks.

For the problem that it is difficult to estimate DOA of coherent signals in the presence of mutual coupling errors, an effective mutual coupling self-calibration method is proposed. The modify MUSIC (MMUSIC) [2] is applied to do decorrelation; we can express MCM by mutual coupling coefficients and position matrix, thus array manifold can be transformed, then utilizing improved RARE algorithm, the DOA can be estimated by peak searching.

II. System Model

Consider a uniform linear array (ULA) consisting of M identical sensors with space $d \le \lambda/2$ between neighboring sensors. There are N uncorrelated narrowband far-field signals $\{s_n(t)\}_{n=1}^{N}$ arriving at array from directions $\{\theta\}_{n=1}^{N}$. Assuming that noise is independent and identically distributed Gaussian process with zero mean. Signals and noises are uncorrelated. The output of the array is

$$\boldsymbol{X}(t) = \boldsymbol{A}(\theta)\boldsymbol{S}(t) + \boldsymbol{N}(t) \tag{1}$$

where $\boldsymbol{A}(\theta) = [\boldsymbol{a}(\theta_1) \quad \boldsymbol{a}(\theta_2) \quad \cdots \quad \boldsymbol{a}(\theta_N)]$ is $M \times N$ array manifold matrix; $\boldsymbol{S}(t)$ is $N \times 1$ vector of signals, received ; $\boldsymbol{N}(t)$ is $M \times 1$ noise vector.

Considering sensors mutual coupling errors, array manifold matrix can be given as

$$\tilde{\boldsymbol{a}}(\theta) = \boldsymbol{C}\boldsymbol{a}(\theta) \tag{2}$$

where $\boldsymbol{C}$ is $M \times M$ mutual coupling matrix (MCM). For the ULA, MCM can be modeled by a banded symmetric Toeplitzs structure, the first row is $\boldsymbol{c}$, that $\boldsymbol{c} = [c_1, c_2, \cdots, c_p, 0, \cdots, 0]$, $0 < |c_p| < |c_{p-1}| < \cdots < c_1 = 1$, where p is the number of non-zero complex coefficients. So the data received by array will be rewritten by

$$\tilde{\boldsymbol{X}}(t) = \boldsymbol{C}\boldsymbol{A}(\theta)\boldsymbol{S}(t) + \boldsymbol{N}(t) \tag{3}$$

The covariance matrix of data received is expressed as

$$\tilde{\boldsymbol{R}} = E[\tilde{\boldsymbol{X}}(t)\tilde{\boldsymbol{X}}^{\mathrm{H}}(t)] = \boldsymbol{C}\boldsymbol{A}\boldsymbol{R}_{\boldsymbol{S}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{C}^{\mathrm{H}} + \sigma^2\boldsymbol{I} \tag{4}$$

where $\boldsymbol{R}_{\boldsymbol{S}} = E[\boldsymbol{S}(t)\boldsymbol{S}(t)^{\mathrm{H}}]$ is covariance matrix of

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signals, $\boldsymbol{I}$ is identity matrix and σ^2 is noise power, $(\bullet)^{\mathrm{H}}$ expresses conjugate transpose operation, $E(\bullet)$ respects statistic expectation, then, eigenvalue decompose (EDV) of $\tilde{\boldsymbol{R}}$

$$\tilde{\boldsymbol{R}} = \boldsymbol{U}_S \boldsymbol{\Sigma}_S \boldsymbol{U}_S^{\mathrm{H}} + \boldsymbol{U}_V \boldsymbol{\Sigma}_V \boldsymbol{U}_V^{\mathrm{H}} \tag{5}$$

where, $\boldsymbol{U}_S$ is the column span of $\boldsymbol{CA}$ responding to signal subspace, $\boldsymbol{U}_V$ is noise subspace matrix; $\boldsymbol{\Sigma}_S$ and $\boldsymbol{\Sigma}_V$ are diagonal matrix associating with signal power and noise power, respectively.

Based on the subspace theories, using (6), we can estimate the DOAs of signals.

$$\tilde{\boldsymbol{a}}^{\mathrm{H}}(\theta)\boldsymbol{U}_V\boldsymbol{U}_V^{\mathrm{H}}\tilde{\boldsymbol{a}}(\theta)=0 \tag{6}$$

However, it should be noted that $\boldsymbol{R_S}$ is a singular matrix, which means the rank of it is less than N in the condition of multiple coherent incident signals. In this case, $\boldsymbol{U}_S$ and $\boldsymbol{U}_V$ are mixed together, we can't obtain the $\boldsymbol{U}_V$ by EVD of covariance matrix directly. Even worse, the array manifold contains unknown mutual coupling coefficients in presence of mutual coupling errors. So these factors will lead to the fact that the DOA estimation directly by formula (6) will invalid. Now, it is particularly important to modify the covariance matrix and restore the rank of it by utilizing decorrelation algorithms. Moreover, decoupling is another essential condition, it needs to analyze the characteristics of MCM structure, and transform the array manifold. In this way, the DOAs of signals can be obtained correctly.

Ⅲ Introduction of The Proposed Method

The proposed method can realize DOA estimation under the condition of coherent signals by decorrelation and decoupling, the decoupling method is the focus of this paper.

The MMUSIC algorithm [2] is used to do decorrelation. Compared with widely used spatial smoothing algorithm, MMUSIC algorithm does not sacrifice the aperture array, and has high estimation accuracy. MMUSIC algorithm is realized through the processing of covariance matrix mainly. The covariance matrix after processing is define as $\boldsymbol{R}_m$, let

$$\boldsymbol{R}_m = \boldsymbol{R} + \boldsymbol{J}\boldsymbol{R}^*\boldsymbol{J} \tag{7}$$

where, $(\bullet)^*$ expresses conjugate operation, and $\boldsymbol{J}$ is permutation matrix with all anti-diagonal elements are 1.

In practice, we just can obtain an estimation of the covariance matrix $\hat{\tilde{\boldsymbol{R}}}$ because of mutual coupling errors and infinitely sampling data, then $\hat{\tilde{\boldsymbol{R}}}_m$ can be derived by (7). Obviously, it is easy to get the real $\hat{\tilde{\boldsymbol{U}}}_V$ with EVD of $\hat{\tilde{\boldsymbol{R}}}_m$, decorrelation is achieved.

Now, let's focus on the problem of decoupling. As known that MCM is composed of p mutual coupling coefficients distributed in p different locations. Hence, we can give the transformation matrix corresponding with each mutual coupling coefficient one by one. First, $\boldsymbol{C}$ can be quantified as

$$\boldsymbol{C} = \sum_{q=1}^{p} c_q \boldsymbol{E}_q \tag{8}$$

$$\boldsymbol{E}_q(i,j) = \delta(\boldsymbol{C}(i,j) - c_q) \tag{9}$$

$$\delta(k) = \begin{cases} 1, & k = 0 \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

the $c_q (q = 1, \cdots, p)$ of (10) are mutual coupling coefficients of MCM, and $\boldsymbol{E}_q(i,j)$ is position matrix which implies the locations of mutual coupling coefficients in MCM. Using this crucial observation, we define $M \times p$ $\boldsymbol{W}(\theta)$ as array manifold transformation matrix, then array manifold can be represented by

$$\tilde{\boldsymbol{a}}(\theta) = \boldsymbol{C}\boldsymbol{a}(\theta) = \sum_{\mathrm{q}=1}^{p} c_q \boldsymbol{E}_q \boldsymbol{a}(\theta) = \sum_{\mathrm{q}=1}^{p} c_q \boldsymbol{W}_q(\theta) \tag{11}$$

where, $\boldsymbol{W}_q(\theta) = \boldsymbol{E}_q \boldsymbol{a}(\theta)$ is the column vector of $\boldsymbol{W}(\theta)$. Furthermore, $\tilde{\boldsymbol{a}}(\theta)$ can be expressed in matrix form,

$$\begin{aligned} \tilde{\boldsymbol{a}}(\theta) &= [\boldsymbol{W}_1(\theta), \boldsymbol{W}_2(\theta), \cdots, \boldsymbol{W}_p(\theta)]\,\bar{\boldsymbol{c}} \\ &= \boldsymbol{W}(\theta)\,\bar{\boldsymbol{c}} \end{aligned} \tag{12}$$

where $\bar{\boldsymbol{c}} = [c_1, c_2, \cdots, c_p]$. For the ULA, each element of $\boldsymbol{W}(\theta)$ is defined by

$$\begin{aligned} &W_{i,j}(\theta) = \begin{cases} a_i & j = 1 \\ a_{i+j-1} + a_{i+1-j} & 2 \le j \le p \end{cases} \\ &\quad i \in (1, \cdots, M),\ j \in (1, \cdots, p) \\ &a_0 = a_M = 0, a_{-1} = a_{M+1} = 0, \cdots, a_{-(p-1)} = a_{M+(p-1)} = 0 \end{aligned} \tag{13}$$

MCM $\boldsymbol{C}$ has different forms in different array structure, but we can also transform array manifold in accordance with the above ideas, which proves broad applicability of the proposed decoupling method.

Utilizing the above derivation results, substituting (12) into (6), we have

$$\bar{\boldsymbol{c}}^{\mathrm{H}}\boldsymbol{W}(\theta)^{\mathrm{H}}\hat{\boldsymbol{U}}_{V_T}\hat{\boldsymbol{U}}_{V_T}^{\mathrm{H}}\boldsymbol{W}(\theta)\bar{\boldsymbol{c}} = 0 \tag{14}$$

Assuming that $\boldsymbol{Q}(\theta) = \boldsymbol{W}(\theta)^{\mathrm{H}}\hat{\boldsymbol{U}}_{V_T}\hat{\boldsymbol{U}}_{V_T}^{\mathrm{H}}\boldsymbol{W}(\theta)$ is an $p \times p$ matrix, thus, (14) can be rewritten by

$$\bar{\boldsymbol{c}}^{\mathrm{H}}\boldsymbol{Q}(\theta)^{\mathrm{H}}\bar{\boldsymbol{c}} = 0 \tag{15}$$

From (15), it is easy to see that $\boldsymbol{Q}(\theta)$ is independent of $\bar{\boldsymbol{c}}$ completely. According to $\bar{\boldsymbol{c}} \neq 0$, only under the condition of the $\mathrm{rank}[\boldsymbol{Q}(\theta)] < p$, (15) can be well established. It means $\boldsymbol{Q}(\theta)$ is not a full-rank matrix if and only if $\theta_n \in \{\theta\}_{n=1}^{N}$. Without loss of generality, traditional RARE algorithm of mutual coupling calibration put this problem into the following minimum estimation

$$\hat{\theta} = \arg\min_{\theta}\{\det[\boldsymbol{Q}(\theta)]\} \tag{16}$$

or

$$\hat{\theta} = \arg\min_{\theta}\{\lambda_{\min}[\boldsymbol{Q}(\theta)]\} \tag{17}$$

where $\det[\bullet]$ and $\lambda_{\min}[\bullet]$ denote the determinant operator and the smallest eigenvalue operator, respectively. Unfortunately both methods have their advantages and disadvantages, the method of (16) has less calculation load, but it is easy to form a pseudo-peak, which make estimation stability unsatisfactory; the method of (17) can get better estimate performance, however EDV in each searching angle is required which leads to higher complexity. Hence in order to solve above problems, the RARE algorithm should be improved, we first calculate the trace of matrix $\boldsymbol{Q}(\theta)$. We know that

$$\mathrm{tr}[\boldsymbol{Q}(\theta)] = \sum_{k=1}^{q}\lambda_k \tag{18}$$

where, $\lambda_k (k = 1,\cdots,q)$ are eigenvalues of $\boldsymbol{Q}(\theta)$. In fact, we also know that

$$\det[\boldsymbol{Q}(\theta)] = \prod_{k=1}^{q}\lambda_k \tag{19}$$

In consequence, $\mathrm{tr}[\boldsymbol{Q}(\theta)]$ and $\det[\boldsymbol{Q}(\theta)]$ will obtain the minimum value as long as $\theta_n \in \{\theta\}_{n=1}^{N}$, So in this paper the product of $\mathrm{tr}[\boldsymbol{Q}(\theta)]$ and $\det[\boldsymbol{Q}(\theta)]$ is defined as the DOA estimation function to estimate DOAs, which means

$$\hat{\theta} = \arg\min_{\theta}\{\mathrm{tr}[\boldsymbol{Q}(\theta)]\bullet\det[\boldsymbol{Q}(\theta)]\} \tag{20}$$

thus, the spectral function can be given by

$$P(\theta) = \frac{1}{\mathrm{tr}[\boldsymbol{Q}(\theta)]\bullet\det[\boldsymbol{Q}(\theta)]} \tag{21}$$

in this way, DOAs of signals can be obtained by spectral peak searching.

After the processing above, pseudo-peaks can be inhibited effectively, at the same time the matrix trace operation just is an add operation which do not increase more computation. All of these insure that the novel method, we proposed, has good DOA estimation performance.

Ⅳ Simulation

In the following examples, several simulation experiments are shown to evaluate the effective and superior performance of the new method, with a comparison against traditional forward/backward spatial smoothing based on rank reduce method (RARE-FBSS) used in [10], and the method in [9]. These methods are chosen because they are based on a similar data and able to solve the same problem as the proposed method. Moreover, the results of MMUSIC with mutual coupling coefficients are also obtained.

We consider there is a ULA consisting 9 sensors with $d = \lambda / 2$. And two coherent signals arrive from $0°$ and $20°$. Set the first row of MCM as

$$\bar{c} = [1, 0.6578 + 0.2394\mathrm{i}, 0.2598 - 0.1500\mathrm{i}, 0, \cdots, 0]^{\mathrm{T}}, p = 3$$

Consider. The root mean squared error (RMSE) and the successful probability of the DOA estimation are used as the performance measure. Here RMSE expresses as

$$\mathrm{RMSE} = \sqrt{\sum_{i=1}^{K}\sum_{n=1}^{N}(\theta_i - \hat{\theta}_{i,n})^2 \Big/ (NK)} \tag{22}$$

where, N is the number of signals; K is total of Monte Carlo simulation experiments, θ_n and $\hat{\theta}_{n,i}$ are the true value and estimated value by the nth signal and the ith experiment, respectively. The successful probability is considered as K_s / K, where K_s is sum of the successful experiments, in which DOA estimation RMSE are all less than $1°$.

Example A: RMSE Performance. The RMSE for estimating DOA are obtained by RARE-FBSS method, method in [9], the proposed method and MMUSIC method known MC. Fig. 1 shows the RMSE of DOA estimation in different SNR after Monte-Carlo experiments, and in each experiment the snapshots number is 200. RMSE performance affected by the snapshots in SNR=10dB is displayed in Fig. 2.

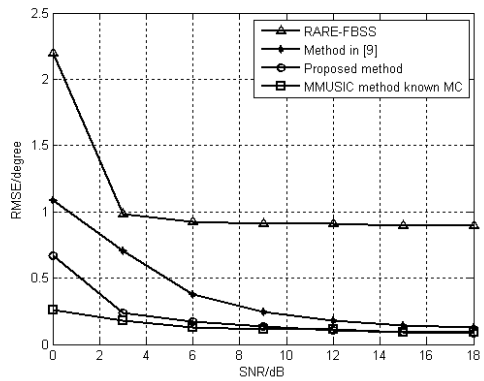

Fig. 1. RMSE of DOA estimation versus different SNR.

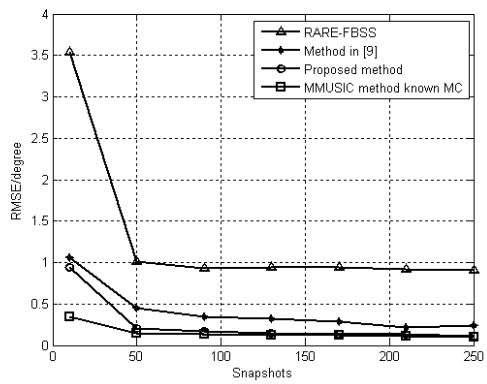

Fig. 2. RMSE of DOA estimation versus different snapshots number.

From simulation results we can see that the RMSEs of

all methods become lower as the SNR or the number of snapshots increases, moreover it will be stable when the SNR and the number of snapshots reach a certain threshold. Besides, it can be observed that the RMSE performance given by the new method is better than other methods, which is mainly because that the information of all sensors is utilized in the method. So there is no array aperture loss. There is one more point, the estimation performance of the proposed method is much close to MMUSIC algorithm with known uncertainties under SNR> 10 dB and snapshot numbers N>150, which illustrates high estimation precision of proposed method.

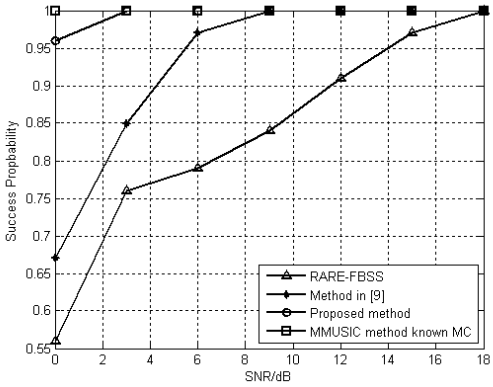

Fig. 3. Successful probability versus different SNR.

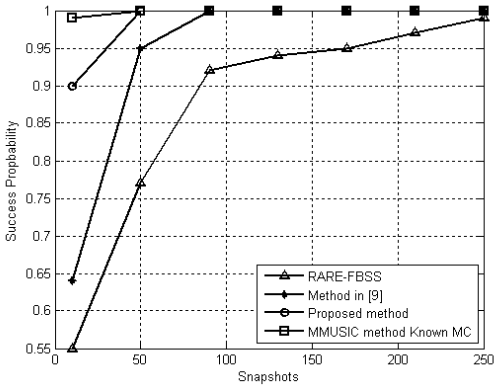

Fig. 4. Successful probability versus different snapshot number.

Example B: Successful probability statistics. Successful probability statistics of 100 Monte-Carlo experiments by the above methods versus different SNR in 200 snapshots and versus different snapshot number in 10dB are given in Fig. 3 and Fig. 4, respectively. Both figures clearly indicate that success probability of all methods become higher when SNR and the snapshot number increase, before they reach a certain threshold. And the successful probability of the proposed method is higher than other methods. Especially the successful probability of the proposed method can achieve 100% in large snapshot, when SNR > 3 dB. When snapshot number is 50, the successful probability approach 100% in SNR=10 dB. All of them show that the proposed method is effectiveness in application of DOA estimation.

Ⅴ Conclusion

In order to estimate the DOA of coherent signals with mutual coupling error, an efficient DOA estimation method which is based on the MMSUC decorrelation and improved RARE decoupling with matrix transformation has been proposed. The method does not need spatial smoothing, without loss of array aperture and inhibit pseudo-peaks, thus high estimation precision and good reliability can be achieved. Simulation experiments have indicated excellent performance in direction finding of the proposed method, which indicates. Thereby indicating it has broad application prospects.

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