

# Wave emissions from the lower-hybrid to the electron cyclotron frequency due to reflected ions within the front of quasiperpendicular supercritical shocks

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A hallmark of supercritical shocks in collisionless plasmas is the presence of a substantial population of ions reflected off of the steep shock front. These ions carry a substantial amount of energy and are fundamental to the transformation of directed kinetic energy into thermal energy, a tenet of shock physics. For quasi-perpendicular geometries the reflected ions' velocity, as seen in the normal incidence frame (NIF), is in large part directed at  $90^\circ$  to the magnetic field  $\mathbf{B}_0$ . The relative drift between the population of reflected ions and the electrons enables the excitation of several microinstabilities in the frequency range from the lower-hybrid up to the electron cyclotron.

Here we investigate the wave emissions possible under various propagation angles for the condition where the ion beam is directed at  $90^\circ$  to  $\mathbf{B}_0$ . Therefore the plasma considered is characterized by a double anisotropy: one defined by the direction of  $\mathbf{B}_0$ , the other by that of the reflected beam. For the sake of clarity let us choose cartesian coordinates such that  $\mathbf{B}_0$  points in the  $\hat{z}$ -direction and the perpendicular component of the wave propagation vector  $\mathbf{k}$  points in the  $\hat{x}$ -direction, namely  $k_\perp = k_x$  and  $k_\parallel = k_z$ . Furthermore, let  $\Psi$  be the plane defined by  $\hat{z}$  and the beam velocity  $\mathbf{V}_b$ . Note that  $\Psi$  needs not to coincide with the plane  $[x, z]$ . There are therefore two main angles playing a role: 1) the angle  $\theta_{bk}$  between the wave vector and  $\mathbf{B}_0$ , 2) the angle  $\psi_{vk}$  between the wave vector and the plane  $\Psi$  containing the beam. As a result the  $3 \times 3$  dielectric tensor is full with terms that are distinct, except for the symmetric pair  $Q_{xz} = Q_{zx}$ .

In this context the most important wave type is the oblique whistler. It is a continuation from the fast magnetosonic branch present at lower frequencies. Its polarization is mixed with both electrostatic and electromagnetic components. While the electromagnetic part is of course right-hand polarized, the electrostatic part becomes more important as the angle  $\theta_{bk}$  increases [1]. A good approximate expression for the real part of the whistler dispersion relation can be obtained from the low frequency dispersion relation given in [2] for a cold plasma model. It may be written as

$$\omega(k, \theta_{bk}) = \frac{kV_A}{[1 + (kc/\omega_{pe})^2]^{1/2}} \left[ 1 + \cos^2 \theta_{bk} \frac{(kc/\omega_{pi})^2}{1 + (kc/\omega_{pe})^2} \right]^{1/2} \quad (1)$$

where  $V_A$  is the Alfvén velocity and the denominators include a correction for short wavelengths that are comparable to the electron inertia length. For frequencies from the lower-hybrid and up, our regime of interest here, and if one excludes propagation angles very close to  $90^\circ$  such that  $\cos \theta_{bk} < \mu^{1/2}$ , then the wavevector is large enough to satisfy  $(kc/\omega_{pi})^2 \mu^{1/2} \cos \theta_{bk} > 1$ . Here  $\mu = m/M$  is the electron to ion mass ratio. Hence, the  $\cos^2 \theta_{bk}$  term dominates the square bracket on the right-hand-side of (1) and thus the angle  $\theta_{bk}$  directly controls the frequency. For a given wavenumber, waves in any direction on a cone of angle  $\theta_{bk}$  have the same frequency. On the other hand, the angle  $\theta_{vk}$  determines the projection of the beam velocity on the wavevector and thus the waves which resonate with the beam, according to

$$\omega = k \sin \theta_{bk} V_b \cos \theta_{vk}. \quad (2)$$

Hence the angle  $\theta_{vk}$  is an element controlling the possible growth of the waves.

Figure 1 shows the results of the dispersion equation (1) over a few orders of magnitude for the simpler case where  $\theta_{vk} = 0$ . The various dispersion relations are marked by solid lines while the dashed lines indicate the associated Doppler frequency of the ion beam following equation (2). A quasi-parallel whistler wave is shown in blue with label 10 for  $\theta_{bk} = 10^\circ$ . For this wave there is no intersection with the dashed blue line. Therefore, no instability involving quasi-parallel whistlers is expected. On the other hand, an oblique whistler which is shown in green with label 45 for  $\theta_{bk} = 45^\circ$  intersects the dashed green line at point A. Hence, one can expect the emission of oblique whistlers destabilized by the reflected ion beam. Such oblique whistlers have indeed been measured by Polar in Earth's shock front [3]. Their instability was studied theoretically in [4]. The green lines exhibit a second intersection at higher frequencies at point B. For such short wavelengths ( $kc/\omega_{pi} > 40$ ), though, the whistlers are heavily damped by the electrons and we do not expect an instability to be possible. A whistler wave which propagates at  $90^\circ$  and saturates at the lower-hybrid frequency ( $\omega/\Omega_i = 40$ ) is shown in red. Since it

does not intersect the dashed red line, no instability with whistlers in the perpendicular direction can be expected. However, the dashed red line intersects the electron Bernstein mode ( $\omega/\Omega_i \sim 1600$ ) at point C. This corresponds to a true instability which has been studied in great detail [5] and the associated waves observed by the Wind and STEREO satellites [6].

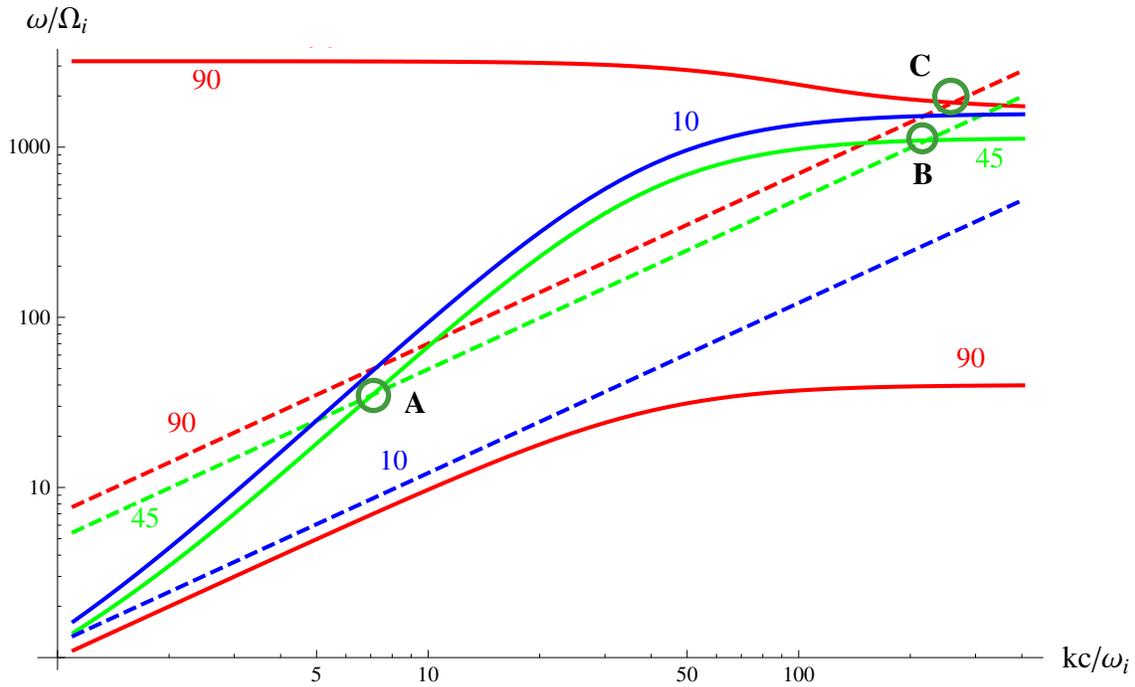


Figure 1: Results of the dispersion equation (1) for various angles:  $\theta_{bk} = 10^\circ$  in blue,  $\theta_{bk} = 45^\circ$  in green, and  $\theta_{bk} = 90^\circ$  in red. See description in the text.

Presently, since we use a kinetic plasma model, as mentioned above, we are confronted to a full  $3 \times 3$  dielectric tensor in order to compute the dispersion relation. Numerical solutions for parameter values typical of the solar wind will be presented. In addition, particular attention will be paid to the group velocity of the unstable waves. As the unstable region is inhomogeneous and finite, our results show that the most important emissions need not be the ones with the fastest growth rates yet the ones which can grow the most, hence the role of the group velocity.

## References

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