Fast Irregular Electromagnetic Field Transformation Utilizing Spatial or Spectral Equivalent Sources Representations Together with Hierarchical Radiation Operator Evaluation

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Abstract

Arbitrary high-frequency electromagnetic fields sampled by arbitrary field probes are efficiently transformed by hierarchical plane wave based evaluations of the radiation operator for equivalent sources representing an antenna or a scatterer. The equivalent sources can be either electric and magnetic surface current densities in the spatial domain or it is possible to directly work with various spectral domain representations of these sources on varying levels of the radiation operator hierarchy. The resulting inverse problems are efficiently solved by an iterative equation solver. Large scale radiation and scattering applications for synthetic and measured field data are investigated.

1. Introduction

Electromagnetic field measurements in arbitrary environments become more and more important in order to be able to fully utilize the potential of modern communications and sensor technologies. Originally field measurements have been performed under controlled conditions in the far-field or the near-field of the field sources and especially the measurement configurations for near-fields have been based on regular sampling of canonical measurement surfaces in order to facilitate efficient canonical field transformation techniques [1]. More flexibility is usually provided by equivalent current techniques [2][3], which become, however, computationally very demanding for electrically large source configurations. By employing acceleration approaches such as those according to the hierarchical plane waved based principles known form the multilevel fast multipole method (MLFMM) [3][4][5][6] very efficient inverse equivalent current transformation approaches can be realized. If the equivalent sources are not needed in the spatial domain, even better efficiency can be achieved by using an appropriate spectral representation in form of plane waves [7] or localized spherical multipoles. Based on these techniques, an efficient and flexible approach for radiation or scattering field transformations is presented and its performance is demonstrated by large scale transformation examples.

2. Formulation

Consider a device under test (DUT) as shown in Figure 1, which can either be a radiating antenna or a scattering object illuminated by an incident field. The field is measured at arbitrary sample locations with arbitrary measurement probes. As illustrated in the figure, the unknown object is replaced by a finite set of equivalent sources whose amplitudes shall be determined from the measured field samples. The determination of the equivalent sources' amplitudes is performed as an inverse problem solution. Once the equivalent sources' amplitudes are known, far-fields and near-fields outside the source region can be computed.

Important for such a measurement based approach is the accurate consideration of the influence of the measurement probe, which is assumed to be a finite sized antenna with a certain radiation characteristic. Its influence on the measurement process can either be considered by a spatial weighting integral according to

$$U(\vec{r}_M) = \iiint\limits_{V_{Probe}} \vec{w}_{Probe}(\vec{r} - \vec{r}_M) \cdot \vec{E}(\vec{r}) dV$$

or by spectral weighting in form of

$$U(\vec{r}_M) = \oiint \widetilde{\vec{w}}_{Probe}(-\vec{k}) \cdot \widetilde{\vec{E}}(\vec{k}) d\hat{k}^2 \cdot$$

$\vec{w}_{Probe}(\vec{r})$ is the spatial probe weighting function (an equivalent electric current density for the probe in transmit mode for a reciprocal probe), $\vec{r}_M$ is the location of the measurement sample, and $\vec{E}(\vec{r})$ is the electric field due to the antenna or the scattering object. The tilde indicates the spectral or *k*-space representation of a quantity, which is e.g. for the probe given as

$$\tilde{\vec{w}}_{Probe}(\vec{k}) = \iiint_{V_{Probe}} \vec{w}_{Probe}(\vec{r}-\vec{r}_M)\,\mathrm{e}^{j\vec{k}\cdot(\vec{r}-\vec{r}_M)}dV$$

where V_{Probe} is the volume of the probe with nonzero spatial weighting function. The complete forward operator can also be expressed in the spatial domain as

$$U(\vec{r}_M) = \iiint_{V_{Probe}} \vec{w}_{Probe}(\vec{r}-\vec{r}_M)\cdot\iiint_{V_{AUT}}\vec{\vec{G}}_J^E(\vec{r},\vec{r}')\cdot\vec{J}(\vec{r}')\,dV'\,dV$$

or in the spectral domain in form of

$$U(\vec{r}_M) = -j\frac{\omega\mu}{4\pi}\oiint\tilde{\vec{w}}_{Probe}(-\vec{k})\cdot\,T_L(\vec{k},\vec{r}_M)\,(\vec{I}-\hat{k}\hat{k})\cdot\tilde{\vec{J}}(\vec{k})\,d\hat{k}^2\cdot$$

Since electromagnetic fields must be solutions of Maxwell's equations, the spectral representation can be written as a two-dimensional integral. This integral can be performed over the Ewald sphere resulting in a spectral representation with propagating plane waves and a diagonal translation operator $T_L(\vec{k},\vec{r}_M)$, as it is common in the multilevel fast multipole method (MLFMM) [3][4][5][6].
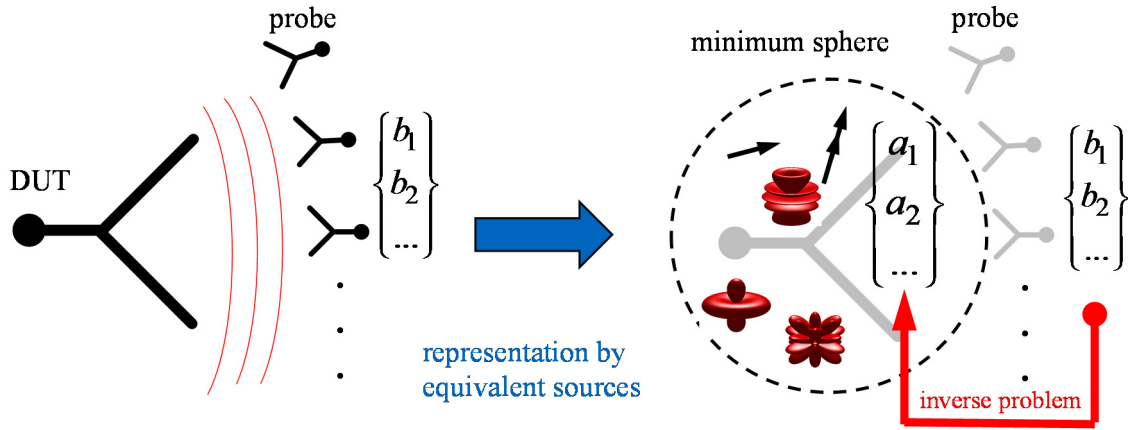

Figure 1: Measurement configuration and inverse problem formulation based on equivalent radiation or scattering sources.

$\vec{J}(\vec{r})$ are equivalent electric current sources representing the radiation or scattering behavior of the device under test (DUT), which are expanded by a set of basis functions, either in the spatial domain or in the spectral domain. Working with a spatial domain expansion, surface current densities on a surface around the DUT are sufficient, since we work with time harmonic fields (time factor $\mathrm{e}^{j\omega t}$). In the implementation of the transformation algorithm, electric and magnetic surface current densities on triangular meshes are supported according to

$$\vec{J}_A(\vec{r}') = \sum_p J_p\vec{\beta}_p(\vec{r}') \qquad\qquad \vec{M}_A(\vec{r}') = \sum_q M_q\vec{\beta}_q(\vec{r}')$$

where the $\vec{\beta}_n$ are vector basis functions on the utilized triangular surface mesh. The transformation process consists of inverting the forward operator in order to determine the unknown expansion coefficients J_p and M_q. This is done based on a multilevel hierarchical evaluation of the forward operator, e.g. see [3][4][5][6][7]. The multilevel procedure is flexible in the way that sources and probes can be treated on different levels and instead of employing the spatial current expansion, it is also possible to directly work with a spectral representation of the sources with respect to the origin of their associated boxes of the octree structure. The spectral expansion can be in form of plane wave samples on the Ewald sphere [7] or the plane wave spectra can be expanded in terms of spherical harmonics as already utilized in [6] as an intermediate step. All three expansion techniques allow for spatial localization and thus the consideration of a priori information about the shape of the DUT. However, true diagnostics information about the DUT is usually obtained by inspecting the resulting surface currents of the spatial expansion.

3. Results

In a first application scenario, spherically measured near-field data for a Rohde & Schwarz double ridged waveguide antenna HF907 [8] is considered. The near-field samples have been collected by an NSI near-field measurement system [9] using a measurement radius of 3 m. The operating frequency is 17.15 GHz for which the electric size of the antenna structure is pretty large (about 17 λ). Figure 2 shows the obtained electric and magnetic equivalent surface current densities on a rectangular box around the antenna structure. The surface mesh consists of 368 680 triangles.
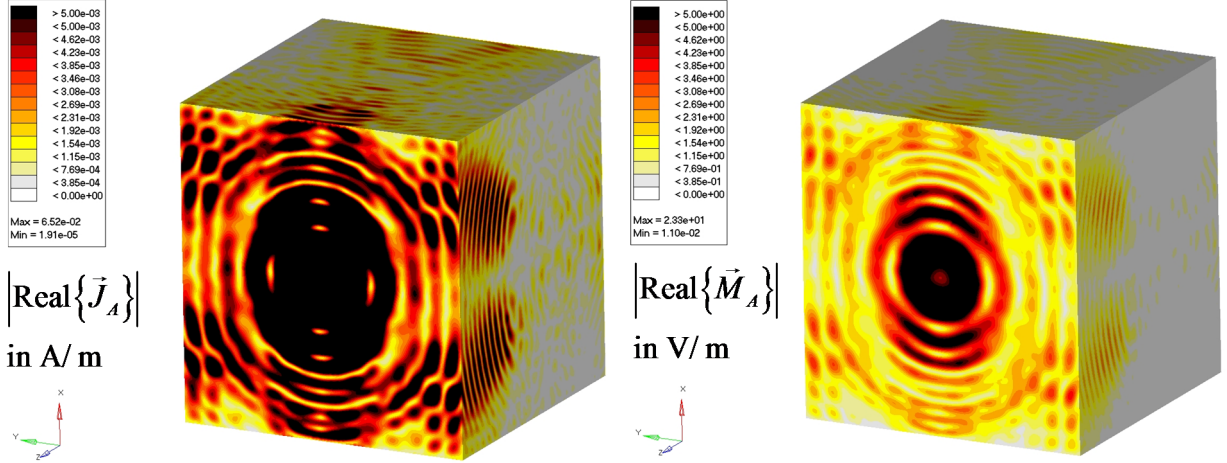

Figure 2: Electric and magnetic equivalent surface current densities on a box around the HF907 antenna for f=17.15 GHz.

A transformed far-field cut of the antenna computed with our "fast irregular antenna field transformation algorithm (FIAFTA)" in the E-plane with spatial equivalent currents according to Figure 2 and with spherical harmonics in the finest octree boxes with an edge length of 0.5 wavelength. In Figure 3 both solutions are compared with the corresponding data obtained with the NSI transformation algorithm [9]. The achieved agreement, i.e. the error level is very satisfactory.
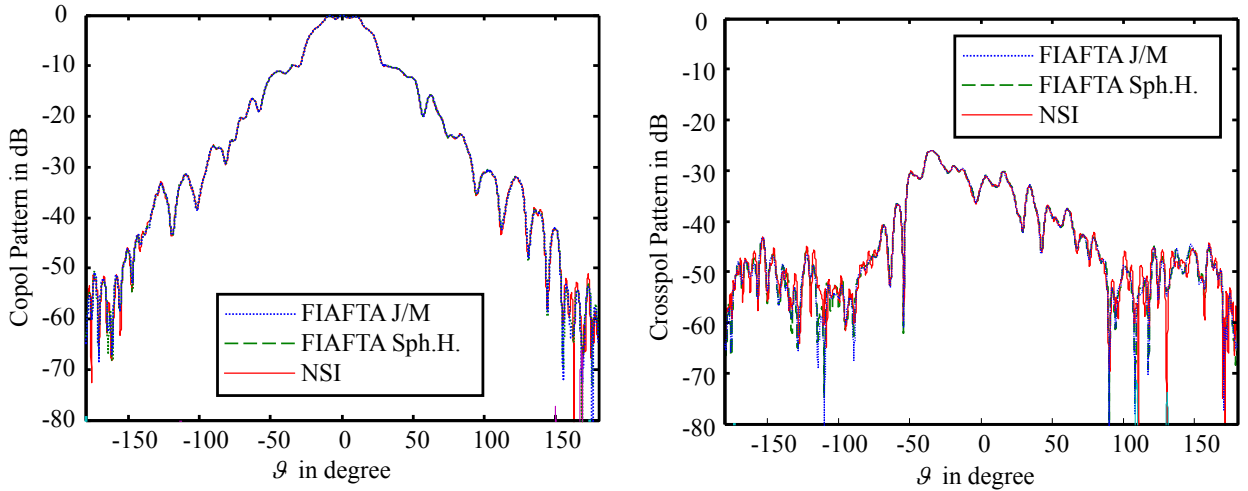

Figure 3: Comparison of transformed far-field cuts of HF907 antenna in the E-plane for a frequency of 17.15 GHz. FIAFTA transformed results with spatial currents and spherical harmonics expansion are compared with NSI transformed results.

The second transformation scenario is for plane wave scattering at a trihedral reflector with a plane wave normally incident with respect to the aperture. Reference data for this problem was computed by a method of moments integral equation solver from which also the near-field samples in a plane above the aperture of the trihedral were computed on a strongly irregular grid. The trihedral configuration together with the transformation results are shown in Figure 4. 2D FFT transformed results assuming planar regular sampling are also shown and give pretty good results near the main lobe but produce severe errors off the main lobe whereas our transformation result shows a constant low error level.
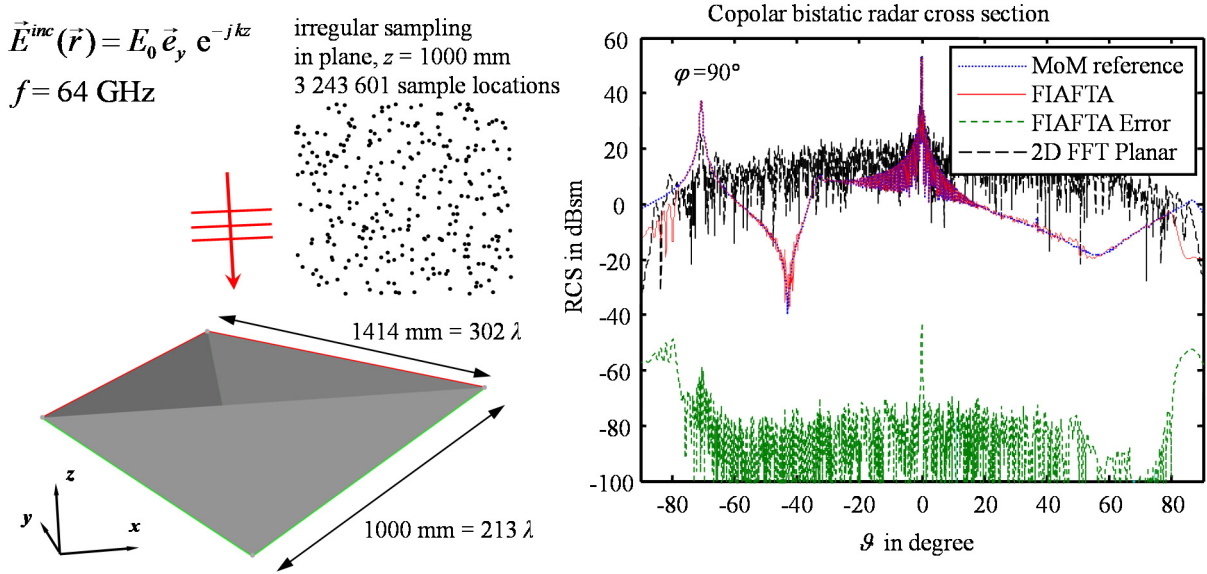
Figure 4: Bistatic radar cross section (RCS) of a trihedral reflector obtained from irregular near-field samples. FIAFTA transformed results with spherical harmonics expansion and 2D FFT transformed results (assuming regular sampling) are compared with method of moments (MoM) reference results.

5. Conclusion

A hierarchical plane wave based field transformation algorithm with various equivalent sources representations has been discussed and demonstrated for antenna field as well as scattering field transformations. The algorithm provides high flexibility, excellent efficiency, as well as excellent diagnostics capabilities for large scale irregular measurements with arbitrary (of course known) field probes.

7. References

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