A Three-Dimensional Microwave Imaging Technique Combining Inverse Equivalent Current and Finite Element Methods

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Abstract

An approach combining boundary integral and finite element methods for imaging of three-dimensional dielectric objects is presented. Based on the equivalence principle, the problem is decomposed into two parts: a radiation and a cavity problem. While the radiation problem is linear and stable, the latter one is non-linear and ill-posed. A boundary integral formulation is utilized to solve the radiation problem and the cavity problem is solved via a finite element method. Since this part of the problem is nonlinear and ill-posed, a Gauss-Newton approach equipped with a Krylov subspace method is employed to reconstruct the material properties in the sense of regularization. Promising reconstructions are obtained.

1 Introduction

Microwave imaging is a very important topic in electrical engineering due to its wide range of applications in the areas of material test and measurement, non-destructive-testing, and biomedical applications, etc. One can find successful methods for the reconstruction of material properties in the literature. The majority of the methods are based on integral equations [1]. The basic advantage of such inversion techniques is that the domain of interest can be kept to a minimum. However, using integral equation formulations puts some restrictions on the background media and it is also computationally expensive. In order to overcome these drawbacks, finite element based inversion techniques have been proposed [2]. While there is no restriction for the background medium for these techniques, the domain of interest must be chosen greater than that of inversion techniques based on integral equations.

In this framework, an inversion technique exploiting the equivalence principle is introduced for the imaging of three-dimensional objects in free-space. The problem is decomposed into two parts; an exterior and an interior problem. Since the exterior problem is formulated by the boundary integral method, the reconstruction domain can be kept to a minimum. This reduces the computational complexity significantly since the interior problem is nonlinear and, therefore, requires iterative solution. For the interior problem, a finite element formulation is used. Therefore, there is no restriction on the allowed background media. In other words, the proposed technique combines the advantages of integral equation and finite element based methods and eliminates their drawbacks. Furthermore, Cauchy data reconstructed on the closed surface is used to estimate the noise level which is needed for regularization and an initial guess. Since the solution of the integral equation is computationally expensive, the multilevel fast multipole method (MLFMM) is utilized in order to reduce the computational burden of the exterior problem both for computation time and storage. The method is tested against synthetic data and promising results are reported.

A non-magnetic, isotropic material D is assumed to be illuminated by a number of transmitters producing incident fields $\boldsymbol{E}^{\mathrm{inc}}$. The object D has the unknown complex relative permittivity $\varepsilon_{\mathrm{r}}(\boldsymbol{r})$ with respect to an (in)homogeneous background. Let us define the total electric field $\boldsymbol{E}$ and the scattered electric field $\boldsymbol{E}^{\mathrm{s}} = \boldsymbol{E} - \boldsymbol{E}^{\mathrm{inc}}$, which is nothing but the contribution of the inhomogeneous object D to the total electric field. One can introduce a volume V with a surface $S := \partial V$ including D. The medium external to S is assumed to be homogeneous. Measurement points and sources are assumed to locate outside the domain V. Now, the inverse problem considered in this study can be defined as the reconstruction of the complex relative permittivity by using the measured (given) scattered fields. We make use of the equivalence principle to decompose the problem into two parts: an exterior and an interior problem. To this aim, the field inside the volume V is set to the null-field and unknown electric $\boldsymbol{J}^{\mathrm{s}}$ and magnetic $\boldsymbol{M}^{\mathrm{s}}$ surface currents are introduced on the surface S. These currents produce the same scattered electric fields as the object D in the region external to V. This part of the problem is a *radiation problem* and forms the exterior problem. In the interior problem, these surface currents $\boldsymbol{J}^{\mathrm{s}}$ and $\boldsymbol{M}^{\mathrm{s}}$ serve as boundary conditions and the electromagnetic problem inside V will be addressed as a *cavity problem*.

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2 The Radiation Problem

One of the most appropriate techniques to solve radiation problems is the electric field integral equation which can be used to represent the scattered electric field as

$$\boldsymbol{E}^{\mathrm{s}}(\boldsymbol{r}) = -\mathcal{L}(\eta_0 \boldsymbol{J}^{\mathrm{s}}) + \mathcal{K}(\boldsymbol{M}^{\mathrm{s}}) \quad \boldsymbol{r} \in \mathbb{R}^3 \setminus \overline{V} \tag{1}$$

where η_0 is the wave impedance of the homogeneous background medium. $\mathcal{L}$ and $\mathcal{K}$ are integral operators and their explicit expression can be found in [p.464, 3]. It is known that the integral equation (1) has analytical kernels, therefore, it is an ill-posed problem. From the physical point of view, non-radiating currents producing null fields outside V but non-zero fields inside V lie in the null space of the integral equation. In order to obtain a stable solution, the non-radiating currents must be eliminated. The boundary conditions in $\boldsymbol{J}^{\mathrm{s}} = \boldsymbol{n} \times \boldsymbol{H}^{\mathrm{s}}$ and $\boldsymbol{M}^{\mathrm{s}} = \boldsymbol{E}^{\mathrm{s}} \times \boldsymbol{n}$ can be invoked to eliminate the non-radiating currents from the solution of (1). Here, $\boldsymbol{n}$ denotes unit normal vector directed outward. Using these boundary conditions and the jump relation, one can obtain the Calderon operator in compact form [4]

$$\mathcal{P}_{\mathrm{c}}(\eta_0 \boldsymbol{J}^{\mathrm{s}}, \boldsymbol{M}^{\mathrm{s}}) = 0. \tag{2}$$

The operator $\mathcal{P}_{\mathrm{c}}$ forces the currents to produce the null-field inside V. It corresponds to forcing the non-radiating currents to be zero. The currents in (1) and (2) are expanded in terms of Rao-Wilton-Glisson basis functions on the triangular surface mesh. While equation (1) is tested by appropriate Dirac-delta distributions, an Galerkin approach is used to weight the Calderon operator. Let us denote any solution by $\boldsymbol{X}$ and matrices inherited from (1) and (2) by $\boldsymbol{A}_{ext}$ and $\boldsymbol{P}_c$, respectively. The $\boldsymbol{b}$ vector is obtained by testing the scattered electric field. Then, one can couple (1) with (2) and obtain a stable equation system

$$\left(\boldsymbol{A}_{\mathrm{ext}}^{*}\boldsymbol{A}_{\mathrm{ext}} + \gamma \boldsymbol{P}_{\mathrm{c}}^{*}\boldsymbol{P}_{\mathrm{c}}\right)\boldsymbol{X} = \boldsymbol{b}\,. \tag{3}$$

Here, γ is introduced as a scaling factor. $*$ sign is used to denote adjoint (complex conjugate transpose) of the associated matrices. It should be noted that $\boldsymbol{A}_{\mathrm{ext}}^{*}\boldsymbol{A}_{\mathrm{ext}}$ is ill-conditioned and $\boldsymbol{P}_{\mathrm{c}}^{*}\boldsymbol{P}_{\mathrm{c}}$ is a positive-definite matrix. In this sense, equation (3) and γ resemble a regularized equation and its regularization parameter. However, equation (3) is not a regularized one due to the fact that it is not a damped equation. In other words, non-zero null-spaces of $\boldsymbol{A}_{\mathrm{ext}}^{*}\boldsymbol{A}_{\mathrm{ext}}$ and $\boldsymbol{P}_{\mathrm{c}}^{*}\boldsymbol{P}_{\mathrm{c}}$ do not coincide and both matrices must be satisfied. MLFMM is employed to reduce the computational solution complexity [5].

3 Cavity problem

Once the surface currents $\boldsymbol{J}^{\mathrm{s}}$ and $\boldsymbol{M}^{\mathrm{s}}$ have been computed, the inverse problem is reduced to a cavity problem with inhomogeneous boundary condition. The domain V is discretized by tetrahedral meshes. Curl-conforming lowest order edge elements are used to discretize $\boldsymbol{E}$ and the complex relative permittivity ε_{r} is expanded in terms of pulse basis functions. The corresponding column vectors including unknown expansion coefficients are denoted by $\boldsymbol{E}^{\mathrm{sim}}$ and $\boldsymbol{\varepsilon}_{\mathrm{r}}$ for the electric field and the relative permittivity, respectively. After following a finite element method [3], the nonlinear system of equations

$$\left[\boldsymbol{R} + \boldsymbol{K}(\boldsymbol{\varepsilon}_{\mathrm{r}})\right]\boldsymbol{E}^{\mathrm{sim}} = \boldsymbol{f} \tag{4}$$

is obtained. Here, $\boldsymbol{R}$ and $\boldsymbol{K}$ denote the stiffness and the mass matrices. $\boldsymbol{f}$ is a known vector inherited by the boundary conditions. Now, one can introduce the cost functional

$$C(\boldsymbol{\varepsilon}_{\mathrm{r}}) = \sum_{s} ||\boldsymbol{d}_s(\boldsymbol{\varepsilon}_{\mathrm{r}})||_S^2 = \sum_{s} ||\ -\boldsymbol{M}_{\mathrm{s}}^{\mathrm{s}} + \boldsymbol{n} \times \boldsymbol{E}_{\mathrm{s}}^{\mathrm{inc}} - \boldsymbol{P}\boldsymbol{E}_{s}^{\mathrm{sim}}||_S^2 \tag{5}$$

where $\boldsymbol{d}_s(\boldsymbol{\varepsilon}_{\mathrm{r}})$ is the discrepancy between the surface electric field reconstructed by solving the radiation problem and the simulated electric field obtained by the interior problem. $\boldsymbol{P}$ denotes the projection matrix which restricts the electric fields in the volume V to the tangential components of the electric fields on the surface S. The subscript s denotes the index of the incident wave. Minimization of the cost functional (5) is both nonlinear and ill-conditioned (due to the presence of the projection matrix). A regularized solution is obtained by employing a Gauss-Newton technique equipped with a Krylov subspace method for regularization [6]. The Lorentz reciprocity theorem is exploited to estimate an initial guess and the noise level which is needed for the regularization based on the Krylov subspace method.

4 Results

A homogeneous lossy L-shaped material having relative permittivity $\varepsilon_{\mathrm{r}} = 1.8 - j1.0$ was considered to be placed in free-space. The long and the short edges of the material were $80\,\mathrm{mm}$ and $40\,\mathrm{mm}$. 36 plane waves operating at $5\,\mathrm{GHz}$ were considered to illuminate the object. The number of measurements points were 648 and the synthetic datasets were contaminated by adding 10% noise. The reconstruction domain was chosen as a cube with dimension $100\,\mathrm{mm}$. The number of unknowns for the surface currents, the electric fields and the material parameters were 8.604, 16.141 and 11.947, respectively. The reconstructed magnetic surface current is shown in comparison to the true profile in figure 1. After reconstructing the surface

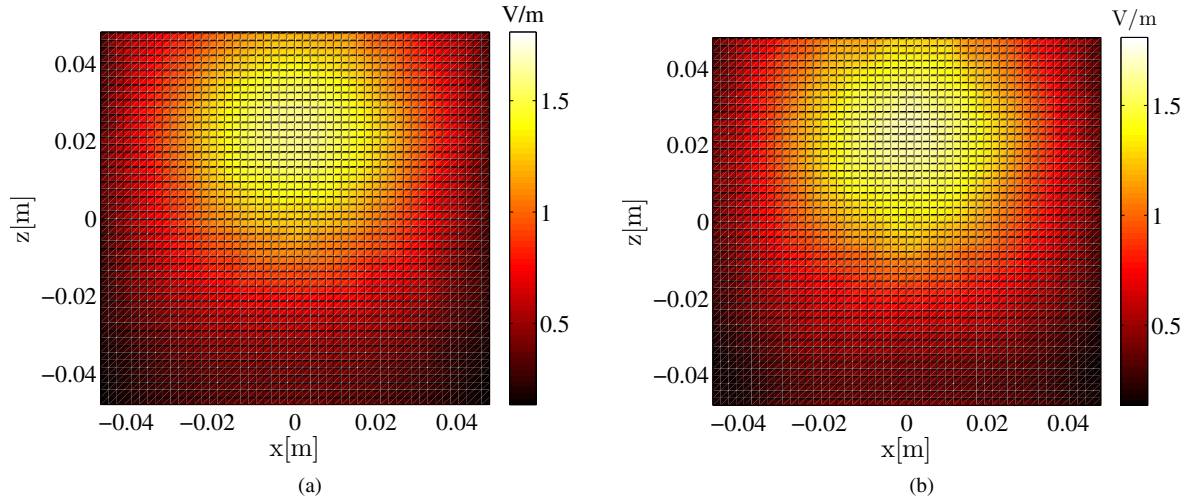


Figure 1: Comparison of magnitude of magnetic currents (a) true and b) reconstruction)

currents, the second part of the method was run. The real and the imaginary parts of the reconstructed relative permittivity are presented in figure 2. Figure 3 shows the true profile from a different view and an isosurface plot of the magnitude of the

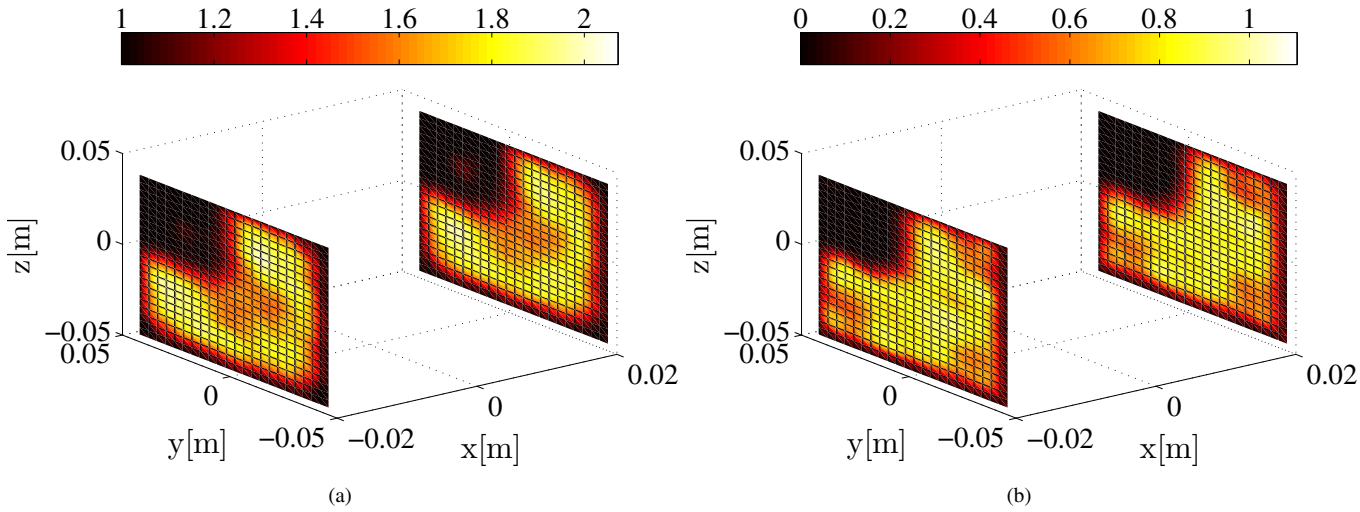
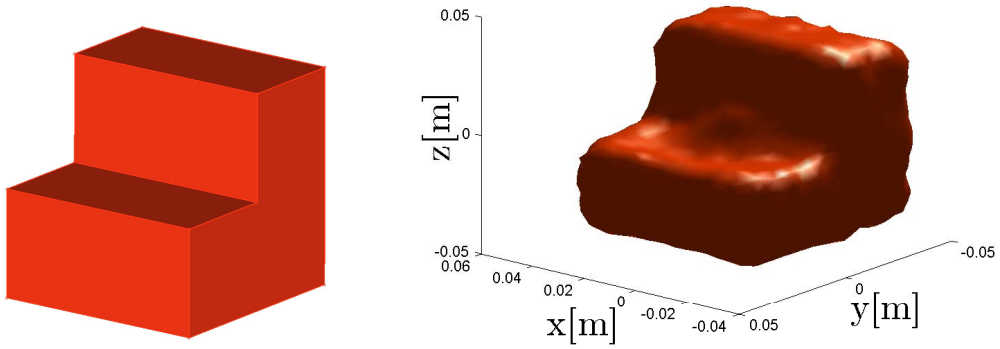


Figure 2: a) The real part and b) the imaginary part (multiplied by -1) of the reconstructed relative permittivity distribution

reconstructed relative permittivity with level 1.5. Through figures 1, 2 and 3, one can conclude that the surface currents, the shape of the material and both the real and the imaginary parts of the relative permittivity were estimated successfully by the method. The computations were performed on one core of a PC having 3.30 GHz clock speed and 8 GB RAM. The computation



(a) (b)

Figure 3: a) True profile and b) isosurface plot of the magnitude of the reconstructed permittivity

times for surface current reconstruction of a dataset (one incident wave) and profile reconstruction (all incident waves) were $49\,\mathrm{sec}$ in average and $48\,\mathrm{sec}$, respectively.

5 Conclusion

An inverse scattering technique exploiting the equivalence principle was addressed. The proposed method is based on the boundary integral method and the finite element method and retains their advantages while it eliminates their drawbacks. The Calderon identity was used to eliminate the non-radiating currents from the solution of the boundary integral equation method. The computational burden of the integral equation was decreased by using the multilevel fast multipole method. In the cavity problem, the Gauss-Newton method equipped with a Krylov subspace method was employed to reconstruct the material properties. Promising results were reported.

6 References

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