A Simplified Model of Dual-frequency Impedance Transformers for Complex Frequency-dependent Impedance Loads

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Abstract

A simplified model is presented for matching two frequency-dependent complex impedances at two arbitrary frequencies. For the simplest topology form, the proposed impedance transformer consists of only a single transmission line with a shunt stub. The design equations which are the function of the frequency ratio and complex termination impedances are derived analytically. The results of numerical simulations and experiments demonstrate the validity of the proposed transformer.

1. Introduction

The impedance transformers, or the impedance-matching networks, are one of the key building blocks in RF/microwave circuits [1]. For minimum power wastage, most microwave circuits need employ impedance transformers to match the different impedances. With the development of mobile communication, devices are required to work at two frequencies in many cases [2]. In 2002, a paper discovered a two-section transformer that can match at a frequency and its first harmonic [3]. Then Monzon derived its closed-form equations in [4], and references [5, 6] extended this architecture to two arbitrary frequencies. The above transformers can only match two real impedances. Given the input and output impedances of most microwave components are complex values, reference [7] extends the application scope of dual-frequency transformer from two resistances to two complex impedances. Furthermore the input or output impedances of microwave components usually vary with frequencies. Therefore the methods of matching a frequency-dependent complex impedance load to 50Ω have been published in [8]-[10]. Finally [11] proposed a dual-frequency transformer which can match two complex frequency-dependent impedances with four sections transmission lines.

In consideration of circuit complexity, the simplest design is generally the most preferable for the engineering solutions. In this paper, the simplified topology of dual-band frequency-dependent complex impedances transformer is proposed. Compared with four-section transmission lines transformer in [11], this design which is composed of only a single transmission line and a shunt stub is more concise in form. The design equations are derived with computer-aided method. Numerical examples and experiments are performed for verifications.

2. Design Equations

The two possible topologies for the proposed dual-frequency transformer are illustrated in Figure 1. They consist of a transmission line section with a shunt stub either at the source end [Figure 1(a)] or at the load end [Figure 1(b)]. Here we take the structure of Figure 1(a) for example. The electrical lengths of transmission lines are θ_a and θ_b at the first frequency f_1 while the characteristic impedances are Z_a and Z_b, respectively. Assume the frequency ratio of two design frequencies f_2 to f_1 is m ($m \geq 1$). Therefore, the electrical lengths at f_2 should be $m*\theta_i$, i=*a, b*. The source and load impedances are $Z_{o1} = R_{o1} + jX_{o1}$, $Z_{L1} = R_{L1} + jX_{L1}$ at f_1 and $Z_{o2} = R_{o2} + jX_{o2}$, $Z_{L2} = R_{L2} + jX_{L2}$ at f_2, respectively. Y_{in1}, Y_{in2} and Y_{down} are the input admittances looking from the position defined in Figure 1(a). The expressions of them are as follows:

$$Y_{in2} = \frac{Z_a + jZ_L \tan(\theta_a)}{Z_a * [Z_L + jZ_a \tan(\theta_a)]}, \tag{1}$$

$$Y_{down} = \begin{cases} j\tan(\theta_b)/Z_b & \text{open stub} \\ -j\cot(\theta_b)/Z_b & \text{shorted stub} \end{cases}, \tag{2}$$

$$Y_{in1} = Y_{in2} + Y_{down}. \tag{3}$$

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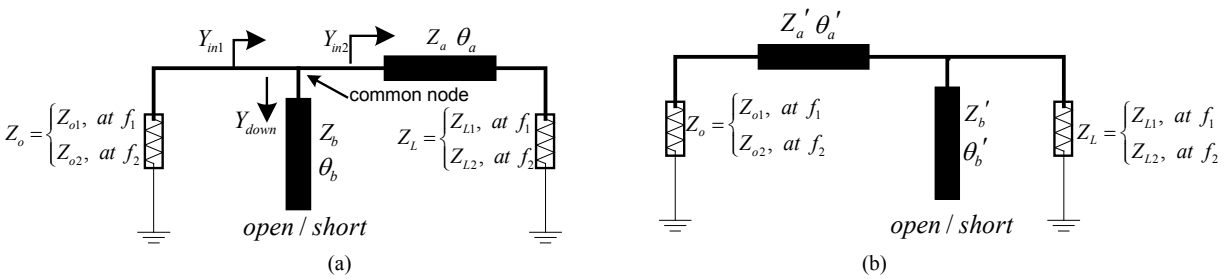

Figure 1 Configuration of the proposed transformer.

According to the theory of transmission line, the impedance matching condition can be simply described as: conj(Y_{in1})=1/Z_o, where conj(*) represents the conjugate function. Due to Z_b and tan(θ_b) are real number, Y_{down} is a pure imaginary value. Then according to the equation (3), the real part of Y_{in2} should equal to the real part of Y_{in1}. We can obtain the equations (4):

$$\mathrm{Re}\left(\frac{Z_a + jZ_L \tan(\theta_a)}{Z_a * [Z_L + jZ_a \tan(\theta_a)]}\right) = \mathrm{Re}(1/Z_o). \tag{4}$$

And substitute the values of Z_L and Z_o into (4). It will become:

$$\frac{R_L[1+\tan^2(\theta_a)]}{R_L^{\ 2} + [X_L + Z_a \tan(\theta_a)]^2} = \frac{R_o}{R_o^{\ 2} + X_o^{\ 2}}. \tag{5}$$

It is important that the formula (5) must be satisfied at the two design frequencies simultaneously. We can solve out the expression of Z_a:

$$\begin{cases} Z_a = \dfrac{1}{\tan(\theta_a)}\left[\pm\sqrt{\dfrac{R_{L1}[1+\tan^2(\theta_a)](R_{o1}^{\ 2} + X_{o1}^{\ 2})}{R_{o1}} - R_{L1}^{\ 2}} - X_{L1}\right] \\ Z_a = \dfrac{1}{\tan(m\theta_a)}\left[\pm\sqrt{\dfrac{R_{L2}[1+\tan^2(m\theta_a)](R_{o2}^{\ 2} + X_{o2}^{\ 2})}{R_{o2}} - R_{L2}^{\ 2}} - X_{L2}\right] \end{cases}. \tag{6}$$

In (6), the goal is to calculate the values of Z_a, θ_a. Unfortunately, the nonlinear (6) is a transcendental equation set which cannot be solved out unless m is known. For different m, the result of θ_a has different expressions. However for some specific application, i.e. m is given, we can solve it using some mathematical tools, such as Matlab and Mathematica. When the shunt stub is open, the relationship among the imaginary parts of Y_{in2}, Y_{down} and Z_o can be expressed in the formula (7), and when the stub is shorted, it can be expressed in the formula (8):

$$\begin{cases} \dfrac{\tan(\theta_b)}{Z_b} = -(I_{left1} + I_{right1}) \\ \dfrac{\tan(m\theta_b)}{Z_b} = -(I_{left2} + I_{right2}) \end{cases}, \quad \text{(open stub)} \tag{7}$$

$$\begin{cases} -\dfrac{\cot(\theta_b)}{Z_b} = -(I_{left1} + I_{right1}) \\ -\dfrac{\cot(m\theta_b)}{Z_b} = -(I_{left2} + I_{right2}) \end{cases}, \quad \text{(shorted stub)} \tag{8}$$

in which I_{leftj} and I_{rightj} are the imaginary parts of the admittances 1/Z_o and Y_{in2}, which are defined in Appendix. The subscript j=1, 2 denotes the two design frequencies f_1 and f_2, respectively. Once Z_a and θ_a are settled, the right sides of (7) and (8) are constant. Z_b and θ_b can be derived. Hence the design of the transformer has been completed. It should be noticed that the values of θ_b and θ_a should be chosen carefully to assure desired parameter values are available for engineering implementation, which means: Z_a >0 and Z_b >0. Just as the analysis in [8], we can choose the type of stub, i.e. open or short, according to the sign of "$I_{\mathrm{leftj}} + I_{\mathrm{rightj}}$". The design with the stub at the load side shown in Figure 1(b) can be calculated similarly. Exchanging Z_L and Z_o in the equations (6) and (7) or (8), we can obtain the design formulas for Figure 1(b).

3. Experiment

In this section, a dual-frequency transformer which matches two complex frequency-dependent impedances is fabricated and measured. The prototype is made on a 1-mm-thick Rogers 4350 substrate with a dielectric constant of 3.48. The Agilent N5230C vector network analyzer is used to measure the reflection coefficient. The transformer works at 1GHz and 2GHz. The source and load ends are series connected with a 2.2pF capacity and a 5.6nH inductance, respectively. Thus the impedances of both terminals are unequal complex values and vary with frequency. According to the models of capacity and inductance of the Murata Co., the source and load impedance are listed in Table I. Calculated with the values in Table I, the source shorted stub should be employed here. Table II shows the required parameters of the transformer.

Figure 2 shows the photograph of the fabricated transformer. And the comparison between the simulated and measured results is illustrated in Figure 3, in which the center frequencies are slightly shifted to 0.942 and 1.88GHz. Deviation between the measured and simulated results is caused by the parasitic effects arisen from via holes and the inaccurate parameters of models and substrate. It can be observed that return losses are better than 23dB at the two center frequencies and the measurement results match well with the simulation.

TABLE I The Source and Load Impedances of the prototype(Ω)

f_1		f_2	
source	load	source	load
50.247-j67.807	50.651+j34.193	50.32-j27.099	51.077+j75.274

TABLE II The Parameters of the Fabricated Transformer

Z_a(Ω)	θ_{a1} (rad)	Z_b(short) (Ω)	θ_{b1} (rad)
78.26	1.349	49.61	1

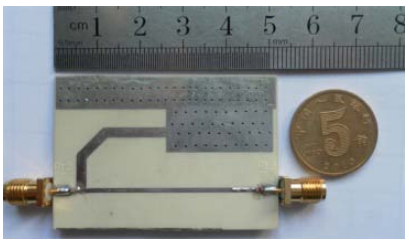

Figure 2 The photograph of the fabricated transformer.

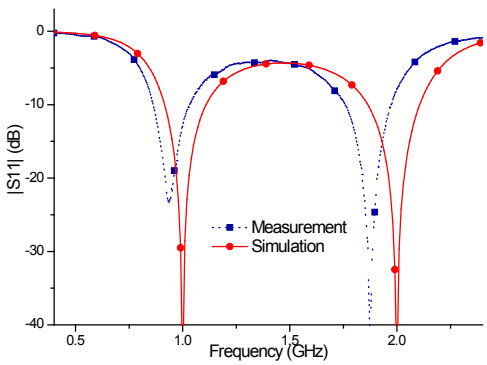

Figure 3 The comparison between the simulated and measured results.

4. Conclusion

A simplified impedance transformer has been proposed to match two frequency-dependent complex impedances at two bands simultaneously. The design equations for this transformer are derived analytically. Numerical simulations and experiment of complex impedance matching network verify the novel method. The presented transformer simplifies the dual-frequency match problem of frequency-dependent complex impedances and can be used widely in dual-frequency microwave circuits and systems.

5. Acknowledgments

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7. Appendix: Variable Definition of Equation (7) and (8)

$$I_{left1} = -\frac{X_{01}}{R_{01}^{\ 2} + X_{01}^{\ 2}} \tag{9}$$

$$I_{right1} = \frac{R_{L1}^{\ 2}\tan(\theta_a) - Z_a^{\ 2}\tan(\theta_a) + X_{L1}Z_a\tan^2(\theta_a) - X_L Z_{a1} + X_L^{\ 2}\tan(\theta_a)}{Z_a\{R_{L1}^{\ 2} + [X_{L1} + Z_a\tan(\theta_a)]^2\}} \tag{10}$$

$$I_{left2} = -\frac{X_{02}}{R_{02}^{\ 2} + X_{02}^{\ 2}} \tag{11}$$

$$I_{right2} = \frac{R_{L2}^{\ 2}\tan(m\theta_a) - Z_a^{\ 2}\tan(m\theta_a) + X_{L2}Z_a\tan^2(m\theta_a) - X_{L2}Z_a + X_{L2}^{\ 2}\tan(m\theta_a)}{Z_a\{R_{L2}^{\ 2} + [X_{L2} + Z_a\tan(m\theta_a)]^2\}} \tag{12}$$