Propagation of Radial Gaussian-Schell Model Beam Array in Non-Kolmogorov Turbulence

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Abstract

The propagation spreading of improved radial Gaussian-Schell model beam array in the non-Kolmogorov turbulence is investigated. Influences of ring radius and generalized exponent on the beam spreading are studied. An optimum ring radius for minimum root-mean-square beam width is suggested.

1. Introduction

It is very important to study the optical propagation in the atmospheric turbulence for many practical applications such as the remote sensing, imaging and communication systems. For a long time, the Kolmogorov model has been widely accepted and used to investigate the influences of the turbulence on the optical propagation. However, recent experimental results have shown great deviation from the predictions of Kolmogorov model. Then, a non-Kolmogorov model is presented [1,2], which reduces to Kolmogorov model with the generalized exponent $\alpha = 3.67$. In practice the laser beam arrays have attracted much attention because of their wide application in the high energy weapons and the inertial confinement fusion. The effective radius of curvature of the beam and radial/linear beam array is studied [3-5]. In this paper, we present an improved radial beam array and study the effects of the generalized exponent and the ring radius on the beam spreading of the improved radial beam array. Moreover, the improved radial beam array is compared with the radial beam array by using the minimum root-mean-square (rms) beam width, corresponding to the optimum ring radius.

2. Analytical Formulae

We assume that a radial beam array consists of N equal beams, which are located symmetrically on a ring with the radius r_0 and the separation angle between two adjacent beams is $2\pi/N$, as shown in Fig. 1(a), and an improved radial beam array constructed by adding a laser beam in the center of the radial beam array, as shown in Fig. 1(b). According to the same procedure [3], for the coherent combination, the rms beam widths of the two beam arrays propagating through the non-Kolmogorov turbulence are both expressed as

$$w = (A + Bz^2/k^2 + 8/3Tz^3)^{1/2},$$

by the definition $w = \left[2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x^2+y^2)\langle I(x,y,z)\rangle \mathrm{d}x\mathrm{d}y \Big/ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle I(x,y,z)\rangle \mathrm{d}x\mathrm{d}y\right]^{1/2}$, where

$$A = \frac{1}{2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\left[2w_0^2 + (a_m+a_n)^2 + (b_m+b_n)^2\right]S_{mn} \Big/ \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} S_{mn},$$

$$B = 2\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\left(\frac{1}{w_0^2}+\frac{1}{\sigma_0^2}\right)\left\{2-\left(\frac{1}{w_0^2}+\frac{1}{\sigma_0^2}\right)\left[(a_m-a_n)^2+(b_m-b_n)^2\right]\right\}S_{mn} \Big/ \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} S_{mn},$$

$$S_{mn} = exp\left\{-\left(\frac{1}{2w_0^2}+\frac{1}{2\sigma_0^2}\right)\left[(a_m-a_n)^2+(b_m-b_n)^2\right]\right\},$$

and

$$T = \pi^2\int_0^{\infty}\kappa^3\,\phi_n(\kappa,\alpha)\mathrm{d}\kappa,$$

in which $a_i = r_0\cos\theta_i, b_i = r_0\sin\theta_i$ ($i = 0,1,\cdots,N-1$) for the radial beam array, and $a_i = r_0\cos\theta_i, b_i = r_0\sin\theta_i$ ($i = 0,1,\cdots,N-2$), $a_{N-1} = 0, b_{N-1} = 0$ for the improved radial beam array with $\theta_i = i\cdot 2\pi/N$.

Considering the inner-scale and outer-scale effects, the non-Kolmogorov spectrum is expressed as [6]

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$$\Phi_n(\kappa,\alpha) = A(\alpha)\tilde{C}_n^2 \cdot \exp(-\kappa^2/\kappa_m^2)(\kappa^2+\kappa_0^2)^{-\alpha/2},\ \ 0 \le \kappa < \infty,\ \ 3 < \alpha < 4,$$

where $A(\alpha) = \Gamma(\alpha-1)\cdot\cos(\alpha\pi/2)/(4\pi^2)$, $\kappa_0 = 2\pi/L_0$ and $\kappa_m = c(\alpha)/l_0$, in which $c(\alpha) = \{\Gamma[(5-\alpha)/2]\cdot A(\alpha)\cdot 2\pi/3\}^{1/(\alpha-5)}$, l_0 is the inner scale and L_0 is the outer scale.

3. The Optimum Ring Radius

In numerical simulations, for simplicity, the parameters $z = 10$ km, $\lambda = 850$ nm, $w_0 = 0.01$ m, $\sigma_0 = 0.01$ m, $\tilde{C}_n^2 = 1\times10^{-15}\mathrm{m}^{3-\alpha}$, $L_0 = 1$ m and $l_0 = 0.01$ m are kept fixed. The influences of r_0 on the rms beam width w of the radial Gaussian-Schell model (GSM) beam array and the improved GSM radial beam array are depicted in Fig. 2 with $N = 10$ and $\alpha = 3.8$. It indicates that for the two beam arrays ($N \ge 2$), there is an optimum ring radius r_{0m} which leads to a minimum w and r_{0m} depends strongly on the beam number N.

4. Comparison of the Two Beam Arrays

Fig. 3 plots the rms beam width w as the function of r_0 for different N with $\alpha = 3.8$. The result indicates that when N is relatively small, the minimum w, corresponding to the optimum ring radius r_{0m}, of the radial GSM beam array is smaller than that of the improved radial GSM beam array, such as $N = 4$. However, when N is large enough, the minimum w of the radial GSM beam array is larger than that of the improved radial GSM beam array, such as $N = 6$. Besides, the influence of the generalized exponent α on w is depicted in Fig. 4 with $N = 10$ and $r_0 = 0.01$ m. It shows that w increases initially and then decreases with the increase in α. Further, for $N = 10$ and $r_0 = 0.01$ m, w of the improved radial GSM beam array is always smaller than that of the radial GSM beam array.

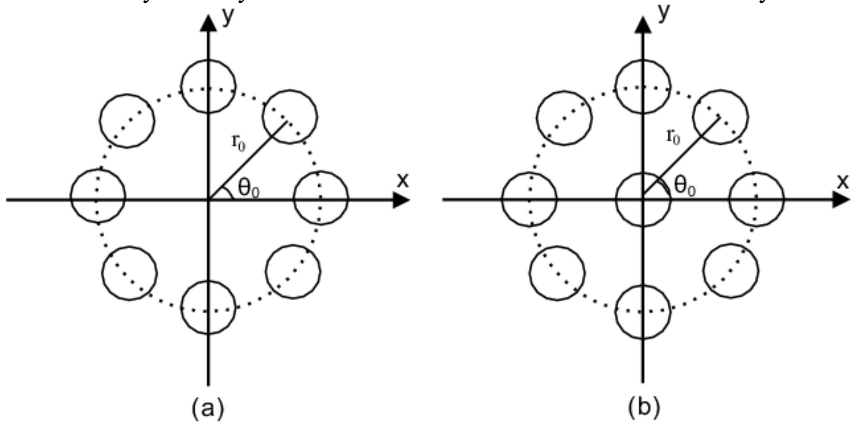

Fig. 1. The schematic diagrams of (a) the radial beam array and (b) the improved radial beam array.

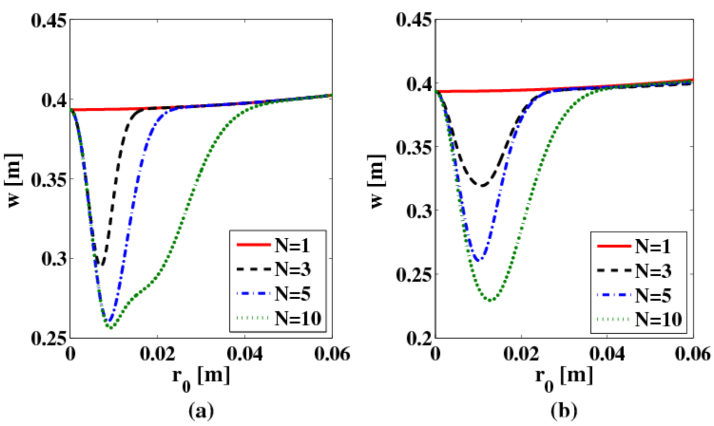

Fig. 2. The rms beam width w as the function of r_0 for different N with $\alpha = 3.8$ (a) for the radial GSM beam array and (b) for the improved radial GSM beam array.

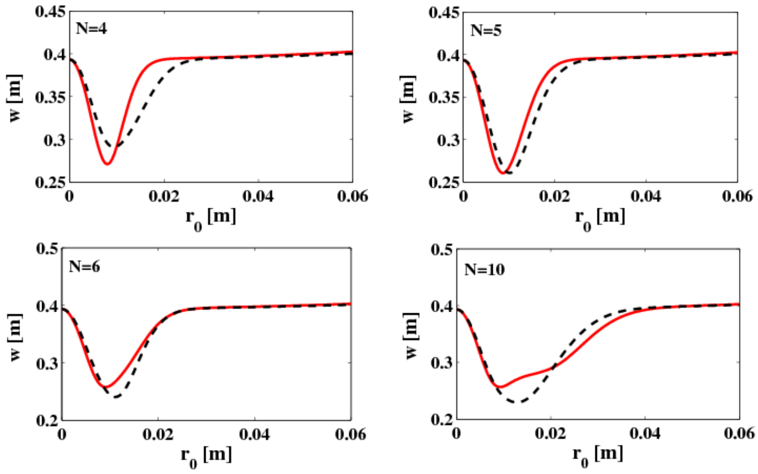

Fig. 3. The rms beam width w as the function of r_0 for different N with $\alpha = 3.8$ for the radial GSM beam array (red solid line) and the improved radial GSM beam array (black dashed line).

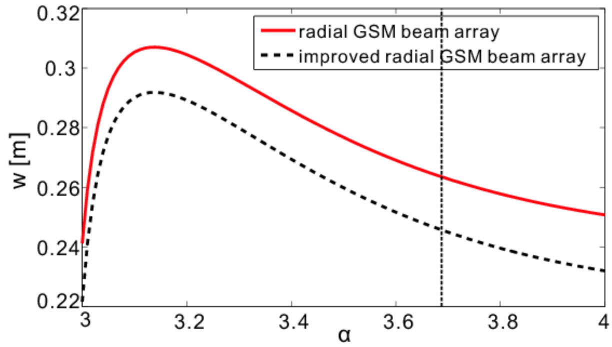

Fig. 4. The rms beam width w as the function of the generalized exponent α with $N = 10,\ r_0 = 0.01m$.

5. Conclusion

In summary, the influences of the ring radius and the generalized exponent α on the rms beam width w of the radial GSM beam array and the improved GSM beam array propagating in non-Kolmogorov turbulence are investigated. For the two beam arrays, there is an optimum ring radius which leads to a minimum w. Besides, we obtain that when N is relatively small, the beam quality of the radial GSM beam array is better than that of the improved radial GSM beam array and when N is large enough, the beam quality of the improved radial GSM beam array is better than that of the radial GSM beam array by comparing the minimum w, corresponding to the optimum ring radius r_{0m}. Further, w has great dependence on α, which indicates that the spreading and focusing ability of the beam array propagating in non-Kolmogorov turbulence is different from that in Kolmogorov turbulence. Therefore, a specific treatment is required when non-Kolmogorov turbulence exists.

6. Acknowledgments

This work is supported by the National Science Fund for Distinguished Young Scholars of China (Grant No. 6 1225003), National Natural Science Foundation of China (Grant No. 61101005), and the Project-sponsored by SRF for ROCS, SEM.

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