Design of Cognitive Radio Sequences with Low Aperiodic Autocorrelations and Low Peak-to-Average Power Ratio

Su Hu[1], Zilong Liu[2], Gang Wu[1] and Shaoqian Li[1]
[1] National Key Laboratory of Science and Technology on Communications
University of Electronic Science and Technology of China (UESTC)
Email: {husu, wugang99, lsq}@uestc.edu.cn

[2] School of Electrical and Electronic Engineering
Nanyang Technological University, Singapore
Email: zilongliu@ntu.edu.sg

Abstract

A cognitive radio (CR) sequence refers to a sequence under spectral hole constraint in a CR system. In this work, we present a new numerical algorithm for generating CR sequences with low peak-to-average power ratio (PAPR) and low aperiodic auto-correlation functions (AACF). The resultant CR sequences are useful in channel estimations and synchronization in CR communications.

I. Introduction

Recently, cognitive radio (CR) is considered to be a promising paradigm to provide the capability of using or sharing the spectrum in an opportunistic manner to solve the scarcity of available spectrum. The spectrum opportunity is defined as spectrum holes that are not being used by the designated primary users at a particular time in a particular geographic area [1]-[3]. The non-contiguous multicarrier code division multiple access (MC-CDMA) provides the capability of reducing the effects of dispersive fading, due to the advantages of both multicarrier transmission and spectrum spreading [4]-[6].

An interesting problem in CR systems is how to design sequences with good auto-correlations and low peak-to-average power ratio (PAPR) so as to support channel estimation and synchronization. Note that traditional sequences cannot be applied directly into CR systems, because their design generally assumes the availability of the whole spectral band (rather than certain non-contiguous spectral bands in a CR system as specified by the spectrum hole constraint) for every sequence. From now on, a sequence which satisfies the spectrum hole constraint in a CR system is also called a "CR sequence", to distinguish it from the traditional sequences with no spectrum hole constraint. Conversely, the correlation property of traditional sequences will be damaged/lost if a spectrum hole constraint is imposed by the spectrum nulling, e.g., polyphase sequences with ideal impulse-like auto-correlations [7]-[10].

The main contribution of this work is a new numerical algorithm for the generation of length-N CR sequences with low PAPR and low aperiodic auto-correlations. Our proposed algorithm makes use of the following sequence property: a sequence with low aperiodic auto-correlations should also possess low periodic auto-correlations. This allows us to find suboptimal frequency-domain magnitudes of the desired CR sequence through optimizing the periodic auto-correlations (which is a convex optimization problem), thus leading to a lower complexity algorithm compared with that in [11].

II. Design of Cognitive Radio Sequences with Low AACF and Low PAPR

For a length-N complex-valued sequence $\mathbf{b} = [b_0, b_1, \cdots, b_{N-1}]^{\mathrm{T}}$, define its AACF and PAPR as follows.

$$\begin{aligned} C_{\mathbf{b}}(\tau) &= \sum_{n=0}^{N-\tau-1} b_n b_{n+\tau}^*,\ 0 \leq \tau \leq N-1. \\ \mathrm{PAPR}(\mathbf{b}) &= \frac{\max\limits_{0\leq n\leq N-1} |b_n|^2}{(1/N)\sum\limits_{n=0}^{N-1} |b_n|^2}. \end{aligned} \tag{1}$$

Consider a CR system where the entire spectrum is divided into N subcarriers. Further, denote by $\Omega \subset [0, 1, \cdots, N-1]$ the *spectrum hole constraint* which specifies all unavailable subcarrier positions. Let $\mathbf{B} = [B_0, B_1, \cdots, B_k, \cdots, B_{N-1}]^{\mathrm{T}}$ be a *frequency-domain* sequence which is obtained by $\mathbf{B} = \mathcal{F}_N \mathbf{b}$, where $\mathcal{F}_N$ is a DFT matrix of order N.

978-1-4673-5225-3/14/$31.00 ©2014 IEEE

Definition 1: $\mathbf{b}$ is called a "CR sequence" if $\mathbf{B}$ satisfies the spectrum hole constraint, i.e., $B_k = 0$ if $k \in \Omega$.

For simplicity, we suppose $\|\mathbf{b}\|_2^2 = N$, i.e., $\sum_{n=0}^{N-1} |b_n|^2 = N$. By Parseval's theorem, we have

$$\|\mathbf{B}\|_2^2 = \mathbf{b}^{\mathrm{H}} \mathcal{F}_N^{\mathrm{H}} \mathcal{F}_N \mathbf{b} = N. \tag{2}$$

To optimize the AACF of $\mathbf{b}$, we recall the key idea of the CAN (cyclic algorithm new) in [12] proposed by Stoica *et al*: the correlation sidelobes of $\mathbf{b}$ vanish if the $2N$-point DFT of $[\mathbf{b}^{\mathrm{T}}, \mathbf{0}_N]^{\mathrm{T}}$ have identical magnitude of $1/\sqrt{2}$. Hence, a CR sequence $\mathbf{b}$ with low AACF sidelobes can be obtained by solving the following problem:

$$\begin{aligned} \min_{\mathbf{B},\mathbf{P}} \quad & \mathcal{J}_1(\mathbf{B},\mathbf{P}) = \left\| \mathcal{F}_{2N} \begin{bmatrix} \mathcal{F}_N^{\mathrm{H}} \mathbf{B} \\ \mathbf{0}_N^{\mathrm{T}} \end{bmatrix} - \mathbf{P} \right\|_2^2, \\ \text{s.t.} \quad & (1): B_k = 0 \text{ if } k \in \Omega; \\ & (2): |P_k| = \frac{1}{\sqrt{2}}, \quad k = 0, 1, \cdots, 2N-1, \end{aligned} \tag{3}$$

where $\mathbf{P} = [P_0, P_1, \cdots, P_{2N-1}]^{\mathrm{T}}$.

On the other hand, to optimize the PAPR of $\mathbf{b}$, we wish to solve the following problem:

$$\begin{aligned} \min_{\mathbf{B},\mathbf{p}} \quad & \mathcal{J}_2(\mathbf{B},\mathbf{p}) = \left\| \mathcal{F}_N^{\mathrm{H}} \mathbf{B} - \mathbf{p} \right\|_2^2, \\ \text{s.t.} \quad & (1): B_k = 0 \text{ if } k \in \Omega; \\ & (2): |p_k| = 1, \quad k = 0, 1, \cdots, N-1, \end{aligned} \tag{4}$$

where $\mathbf{p} = [p_0, p_1, \cdots, p_{N-1}]^{\mathrm{T}}$.

By introducing a penalty factor $\lambda \in [0, 1]$ which controls the relative weighting of $\mathcal{J}_1$ and $\mathcal{J}_2$, our optimization problem can now be formulated as follows:

$$\begin{aligned} \min_{\mathbf{B},\mathbf{P},\mathbf{p}} \quad & \mathcal{J}(\mathbf{B},\mathbf{P},\mathbf{p}) = \lambda \mathcal{J}_1(\mathbf{B},\mathbf{P}) + (1-\lambda) \mathcal{J}_2(\mathbf{B},\mathbf{p}), \\ \text{s.t.} \quad & (1): B_k = 0 \text{ if } k \in \Omega; \\ & (2): |P_k| = \frac{1}{\sqrt{2}}, \quad k = 0, 1, \cdots, 2N-1; \\ & (3): |p_k| = 1, \quad k = 0, 1, \cdots, N-1. \end{aligned} \tag{5}$$

As an example, let $N = 64$, $\Omega = \{14, 15, \cdots, 19\} \cup \{40, 41, \cdots, 47\}$ and $\lambda = 0.15$. The resultant CR sequence magnitudes (time- and frequency-) and the AACF are shown in Fig. 1.

References

[1] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Select. Areas Commun.*, vol.23, no.2, pp.201-220, 2005

[2] T. Yucck and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Comm. Survey Tutorials*, vol. 11, no.1, pp.116-130, 2009

[3] I. Budiarjo, H. Nikookar, and L. P. Ligthart, "Cognitive radio modulation techniques," *IEEE Signal Process. Mag.*, vol.25, no.6, pp.24-34, 2008

[4] E. Azarnasab, R. Kempter, N. Patwari, and B. Farhang-Boroujeny, "Filterbank multicarrier and multicarrier CDMA for cognitive radio systems," in Proc. *IEEE Int. Conf. Cogn. Radio Oriented Wireless Netw. Commun.*, Orlando, FL, Aug. 2007, pp. 472C481.

[5] Q. Qi, L.B. Milstein, and D.R. Vaman, "Cognitive radio based multiuser resource allocation in mobile ad hoc networks using multicarrier CDMA modulation," *IEEE J. Sel. Areas Commun.*, vol.26, no.1, pp70-82, 2008

[6] Z. Wu, P. Ratazzi, V. D. Chakravarthy, and H. Lang, "Performance evaluation of adaptive non-contiguous MC-CDMA and non-contiguous CI/MC-CDMA for dynamic spectrum access," in Proc. *IEEE Int. Conf. Cogn. Radio Oriented Wireless Netw. Commun.*, Singapore, May 2008, pp. 1C6.

[7] R. L. Frank and S. A. Zadoff, "Phase shift pulse with good periodic correlation properties," *IRE Trans. Inf. Theory*, vol. IT-8, pp. 381-382, Oct. 1962.

[8] D. C. Zhu, "Polyphase codes with good periodic correlation properties," *IEEE Trans. Inf. Theory*, vol. IT-18, pp. 531-533, July 1972.

[9] B. M. Popovic, "Generalized chirp-like polyphase sequences with optimum correlation properties," *IEEE Trans. Inf. Theory*, vol. 38, pp. 1406-1409, July 1992.

[10] W. H. Mow, "A Unified Construction of Perfect Polyphase Sequences," in *Proc. 1995 IEEE International Symposium on Information Theory (ISIT '95)*, Canada, Sept. 1995, pp. 459.

[11] L.-S. Tsai, W.-H. Chung, and D.-S. Shiu, "Syntehsizing low autocorrelation and low PAPR OFDM sequences under spectral constraints through convex optimization and GS algorithm," *IEEE Trans. Signal Process.*, vol. 59, pp. 2234-2243, May 2011.

[12] P. Stoica, H. He, and J. Li, "New algorithms for designing unimodular sequences with good correlation properties," *IEEE Trans. on Signal Process.*, vol. 57, no. 4, pp. 1415-1425, Apr. 2009.

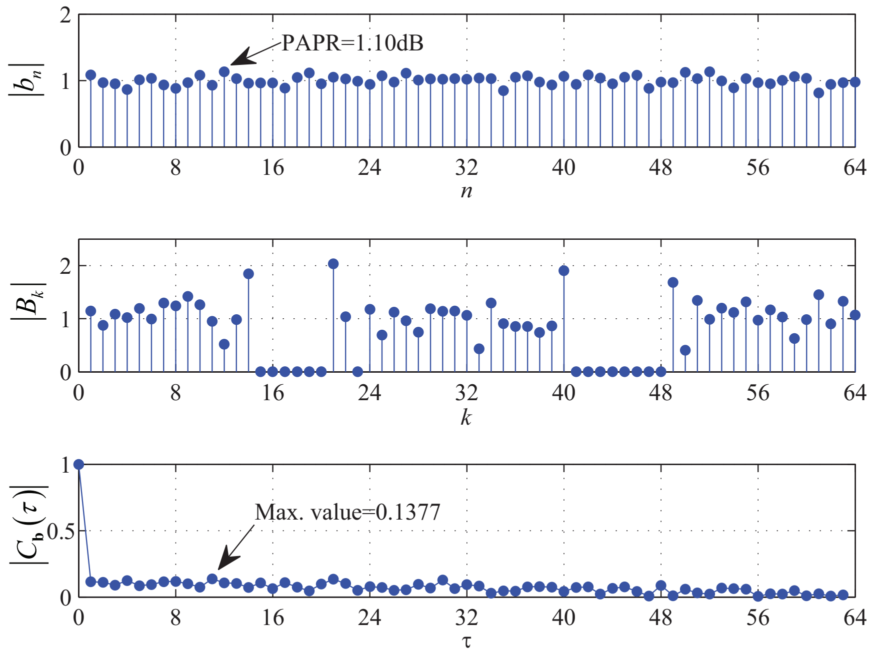

Fig. 1: CR sequence obtained from the proposed algorithm