IE-DDM-FFT for EM Scattering by Objects in Half-Space Structures

Lan-Wei Guo[1], Joshua Le-Wei Li[1,2], Yongpin P. Chen[1], and Jun Hu[1]

[1]School of Electronic Engineering and Institute of Electromagnetics,
University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China;
[2]Advanced Engineering Platform and School of Engineering, Monash University, Sunway, Selangor 46150, Malaysia
lanwei407@gmail.com;lwli@ieee.org;yongpinchen@gmail.com;hujun@uestc.edu.cn

Abstract

In this paper, an extension of the integral equation based on domain decomposition method (IE-DDM) is presented for dynamic electromagnetic scattering problems above a lossy half space. In the framework of the IE-DDM, the proposed method divides the composite object into several homogeneous sub-domains. Each sub-domain is properly described by a closed surface, and robin transmission condition is introduced to weakly enforce the continuity of tangential field across the touching-face between adjacent sub-domains. Multiple-grid precorrected fast Fourier transform (MG-pFFT) is adopted in each sub-domain independently to account for the self-interactions. Numerical experiments validate the accuracy and efficiency of this method.

1. Introduction

Surface integral equation (SIE) is one of the most powerful full wave method in the simulation of electromagnetic radiation and scattering problem. However, one of the major challenges in its application arises in the solution of the resulting dense and ill-conditioned matrix equation. One reason comes from the multi-scale nature of the geometry which may cause the “mixed-frequency” [1-2] problems and results in an extremely ill-conditioned matrix equation. The other reason is however relevant because of the inherently very large computational Scale [1]. The condition number problem is attacked by different algebraic preconditioners. Unfortunately, algebraic preconditioners are not always reliable or suffering from construction time.

To alleviate this difficulty, one popular strategy is to decompose original composite structure into several homogeneous and easily solvable sub-domains. This is the domain decomposition methods (DDMs), which have been employed as effective preconditioners to improve the ill-conditioning of the IE matrix. Recently, lots of domain decomposition methods based on integral equation formulations have been proposed. For example, the characteristic basis function (CBF) [3-4], tangential equivalence principle algorithm (T-EPA) [5], overlapped domain decomposition method (ODDM) [6], integral equation based domain decomposition method (IEDDM) [7-8], and so on. Instead of solving the original scattering problem as one large computational domain, we mesh and solve each small local sub-domain individually. Studies have shown that DDMs methods can reduce total unknowns and solving time [3-8]. The conditioning of the impedance matrix can also be improved [7-8].

In general, most literature about domain decomposition methods only considers electromagnetic phenomenon in an unbounded homogeneous background. However, there are a lot of electromagnetic phenomenon in real life need considering about scattering effects from the inhomogeneous background. One of the most useful models of inhomogeneous environment is planarly layered medium [9-11]. In this work, an extension of the IE-DDM is presented for analysis of EM scattering from multi-scale perfectly conducting structures above a lossy half space. Based on the dyadic Green's function, the scattering effects from the inhomogeneous of the lossy half space is considered. In the proposed DDM method, the conventional combined field integral equation (CFIE) solver is used in each sub-domain. Each sub-domain is described by a closed surface, and the continuity of tangential fields on the touching interfaces between sue-domains are weakly enforced by Robin transmission condition [7-8]. A novel multiple-grid pre-corrected fast Fourier transform (MG-pFFT) is adopted in each sub-domain independently to account for the self-interactions. Next, the conventional p-FFT is used for the mutual couplings of the sub-domains. Numerical results demonstrate that the proposed method yields rapid convergence in the outer Krylov iterative solution process [12].

2. Formulation

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For the sake of simplicity, a perfect electrically conducting (PEC) cylinder above a half space is being considered here. The whole computation domain Ω is first divided into two sub-domains Ω_1 and Ω_2 as shown in Fig. 1. Then, keep in mind that IEDDM is different from single scatter in free space, the incident fields of sub-domain Ω_1 or Ω_2 need to incorporate the scattered fields contributed from the other sub-domain. By applying the robin transmission condition, the continuity of tangential field across the touching-face between adjacent sub-domains is weakly enforced:

$$\mathbf{J}_{\Gamma_{21}} + \mathbf{J}_{\Gamma_{12}} = 0 \tag{1}$$

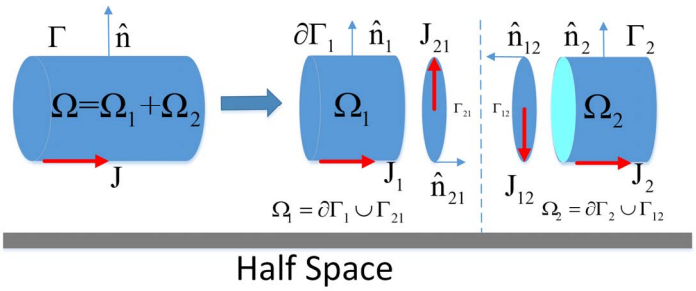

Fig. 1 The computational domain is divided into two sub-domains

Then the combined field integral equation (CFIE) is formulated in each sub-domains. By discretizing and testing of CFIE whit the RWG basis functions in the Galerkin method, a $N \times N$ dense matrix equation is formulated in this form

$$\mathbf{Z} \cdot \mathbf{I} = \mathbf{V} \tag{2}$$

A formulation similar to Michalski-Zheng C-formulation is chosen for dyadic Green`s functions (DGF) in the spatial domain [12]. According to the idea of IE-DDM [7], the discrete system Eq. (2) can then be cast into a matrix equation of the following form:

$$\mathbf{M}^{-1}(\mathbf{M} - \mathbf{N})\mathbf{I} = \mathbf{M}^{-1}\mathbf{V} \tag{3}$$

where $\mathbf{M}$ is the sub-domain solution for each of the sub-domain, and $\mathbf{N}$ is the matrix for couplings between sub-domains,

$$\mathbf{M} = \begin{bmatrix} A_{\partial\Gamma_1\partial\Gamma_1} & A_{\partial\Gamma_1\Gamma_{21}} & & \\ A_{\Gamma_{21}\partial\Gamma_1} & A_{\Gamma_{21}\Gamma_{21}} & & \\ & & A_{\Gamma_{12}\Gamma_{12}} & A_{\Gamma_{12}\partial\Gamma_2} \\ & & A_{\partial\Gamma_2\Gamma_{12}} & A_{\partial\Gamma_2\Gamma_{12}} \end{bmatrix}, [\mathbf{N}] = \begin{bmatrix} & & C_{\partial\Gamma_1\Gamma_{12}} & C_{\partial\Gamma_1\partial\Gamma_2} \\ B_{\Gamma_{21}\partial\Gamma_1} & B_{\Gamma_{21}\Gamma_{21}} & D_{\Gamma_{21}\Gamma_{12}} & \\ & D_{\Gamma_{12}\Gamma_{21}} & B_{\Gamma_{12}\Gamma_{12}} & B_{\Gamma_2\partial\Gamma_2} \\ C_{\partial\Gamma_2\partial\Gamma_1} & C_{\partial\Gamma_2\Gamma_{21}} & & \end{bmatrix} \tag{4}$$

Note that with a preconditioned Krylov subspace method, $\mathbf{M}^{-1}$ is a simple block-Jacobi preconditioner for the IE-DDM system. To speed up the computations of sub-domain solutions and $\mathbf{M}^{-1}$ in each inner iteration, the Krylov recycling algorithm (GCRO-DR) algorithm is employed [13].

In the proposed IE-DDM herein, we use a multiple-grid pre-corrected fast Fourier transform (MG-pFFT) to accelerate the matrix-vector multiplications. Here, the p-FFT is applied to each sub-domain independently, which employs multiple auxiliary grids that have different locations, grid spacing, and associated projection, propagation, and interpolation operators. Finally, a p-FFT solver is applied for the couplings matrix $\mathbf{N}$.

3. Results and Discussions

The numerical experiments are performed on an desktop computer configured by Intel I7 2600 quad core 3.4 GHz and 16 GB RAM. Numerical results are presented to validate the effectiveness of our proposed integral equation domain decomposition method. Four methods are investigated for this example: 1) conventional single domain p-FFT fast solver, named as p-FFT; 2) multiple-grid pre-corrected FFT with the IE-DDM frame, named as IEDDM-FFT; 3) conventional single domain p-FFT method with the largest grid space as 0.1 λ , named as FFTMAX; 4) conventional single domain pFFT method with the smallest grid space, 0.02 λ , named as FFTMIN. Both outer GMRES iteration and inner GCRO-DR process are terminated when the L^2 -norm of the residual vector is reduced to 10^{-3}.

To demonstrate the accuracy of the present IE-DDM frame, the biostatics RCS of one benchmark targets was calculated. The cylinder is above the half space with the bottom layer which has relative permittivity $\varepsilon_r = 6.38 - j0.663$. It is 3m long, has a radius of 0.5m, and is located 0.2m above the interface. A 600 MHz plane wave is incident from

$\theta_{inc} = 60^{\circ}$ and $\phi_{inc} = 0^{\circ}$. A transverse plane partitions the cylinder into three equal sized sub-domains, and then the surface of each sub-domain is discredited using triangles with $\lambda/10$ average edge length resulting in the RWG basis unknowns are 23,453. The RCS patterns calculated with the proposed IEDDM-FFT is compared to a reference AIM [14] as shown in Fig. 2-a. The patterns are visually identical. Fig. 2-b also give the current distribution of the cylinder, which shows that the electric currents calculated by IE-DDM is identical with one calculated by conventional single domain p-FFT.

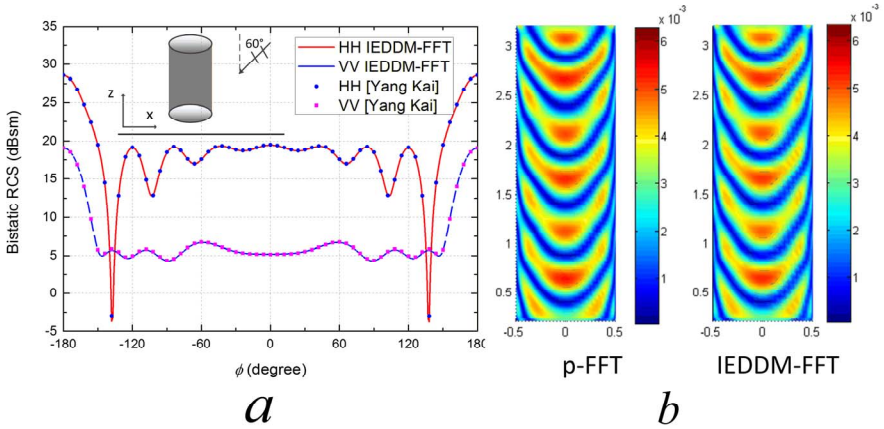

Fig. 2 Using MG-pFFT-IEDDM to solve a PEC cylinder scattering problem, (a) co-popularized biostatic RCS patterns of the cylinder above the half space in the θ-cut at 600 MHz; (b)current distribution.

Since the touching-face is introduced in our method, the unknowns of IEDDM-FFT may lager than the origin problem. Fortunately, DDM method enable mesh and solve each small local sub-domain individually, so it is very suitable for multi-scale problem. In order to examine the efficiency of our method, a multi-scale application is carried out on the multi-cylinders scattering problem.

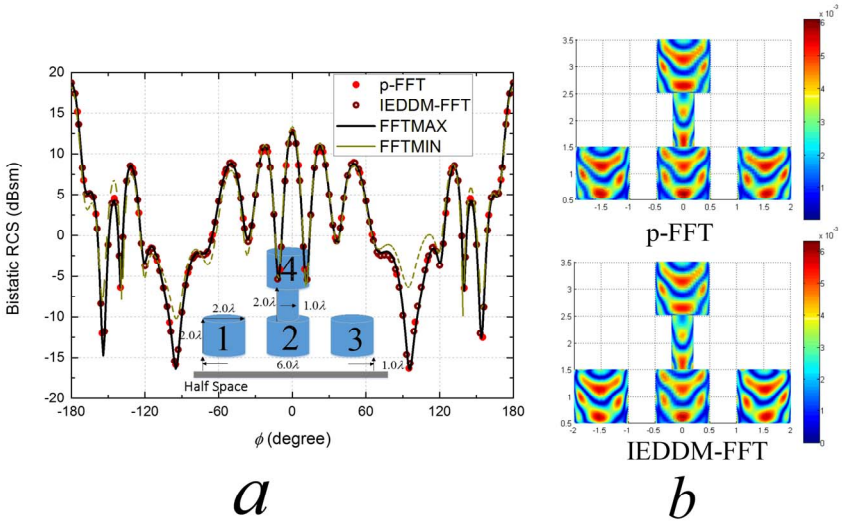

Fig. 3 Using MG-pFFT-IEDDM to solve a multi-scale problem, (a) VV-popularized biostatic RCS patterns of the multi-cylinders above the half space in the θ-cut at 600 MHz; (b)current distribution.

The dimension of the multi-cylinders is shown in Fig.3-a. The multi-cylinders are 0.5 m above the half space with the bottom layer which has relative permittivity $\varepsilon_r = 6.38 - j0.663$. The plane wave is set at f =600 MHz which is incident from $\theta_{inc} = 60^{\circ}$ and $\phi_{inc} = 0^{\circ}$. The overall geometry is divided into 4 closed objects, as shown in Fig.3-a. The four cylinders are meshed independent. The average size of edges for sub-domain 1 is 0.04 λ, while sub-domain 3 is 0.02 λ. For sub-domain 2 and 4 are 0.07 λ. The multi-cylinders result in 186,138 RWG unknowns. In the method of IEDDM-FFT, the four cylinders are solved independently by employing multiple auxiliary grids, and robin transmission condition is used for the touching-face between sub-domain 2 and 3. For comparison, FFTMAX and FFTMIN with the same mesh size are also given. The result from p-FFT is given as the reference. From table I and Fig.3, it can be seen

that the proposed IEDDM-FFT has a good convergence and accuracy. For FFTMAX, since the size of grid spacing is much bigger than RWG basis function, it need more memory and time for near-zone interactions matrix. What is worse, the larger the grid spacing, the higher order of grid and projection template is needed to insure the precision of interpolation and projection. For FFTMIN, the interpolation template for small grid size cannot cover the large domain of the RWG basis function. Hence, the precision of projection and interpolation in FFTMIN cannot be guaranteed. Therefore it is not able to obtain the correct result.

4. Conclusion

In this work, an extension of the IE-DDM is presented for analysis of EM scattering from perfectly conducting structures above a lossy half space. Based on domain decomposition method, multiple-grid precorrected fast Fourier transform is adopted in each sub-domain independently to account for the self-interactions. Numerical experiments validate the accuracy and efficiency of this method. It is also shown that the proposed IEDDM-FFT method is an efficient and robust preconditioner for the multi-scale EM problem.

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