

# Energy Analysis of a Class of Nonlinear Detection Systems with Chaotic Oscillators

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**Abstract:** In areas of weak signal detection, considering the algorithm complexity and huge computation in phase transition identification of Duffing oscillator, a novel method based on energy distribution was proposed in this paper. Firstly, chaotic detection theory was analysed, then Teager Energy Operator was associated with nonlinear dynamic theory, which makes symbol identification achievable. Finally, the robustness and real-time property of the algorithm were analyzed. Simulation results show that, compared with the traditional methods, the computation and real-time property of this method has significantly improved to meet the needs of the weak signal detection under the strong noise background.

**Index Terms:** chaotic detection; energy operator, algorithm complexity; real-time

## I Introduction

With facing more complex space electromagnetic environment, the transmission channel susceptible to environmental factors, the input signal to noise ratio is too low for demodulation at the receiving end. And hence the information transferring is blocked, the difficulty of demodulation becomes larger, the error rate increase consumedly. In fact, enhancing the ability to detect weak digital signals effectively, improving the reliability of digital demodulation in strong noise background has been a hot issue of theory and engineering research. However, the 'linear' demodulation technology based on matched filtering, and the best correlation receiving has get to the extreme performance, closing to the theoretical limit. If wanting a further "low point" walk, there must have a new theoretical breakthrough, just as the research of nonlinear demodulation techniques.

In recent years, chaos theory has been widely applied in the field of signal detection, using the chaotic oscillator to detect weak signals is a complex mathematical process, whose implementation core is chaotic fractal identification method. At present, domestic

and foreign scholars have made a lot of work, but there are still some problems: the phase plane trajectory method proposed in the literature [1,2] was viewed as the criterion phase transition, but its vulnerability is in simulation time and other factors, the error is large and inefficient, the dual oscillators difference method in literature [3] improved the trajectory method, but still not trip out of image observation scope, Lyapunov exponent method in literature [4] can accurately identify the system in phase space trajectories and states, but Jacobian matrix of system equations is very difficult to resolve. The Melnikov method in literature [5] is relatively simple, but the identification accuracy is worse than others.

Considering these problems, a novel method based on chaotic energy was proposed in this paper. Firstly, chaotic detection theory based on Duffing oscillator was analysed in-depth. Then Teager Energy Operator was associated with nonlinear dynamic theory, which makes symbol identification achievable. Finally, the robustness and real-time property of the algorithm were analyzed. Simulation results show that the non-linear demodulation algorithm based on chaos theory has a good anti-noise performance for demodulating digital signals at low SNR background.

## II Chaotic energy detection principle

### 2.1 Duffing chaotic oscillators

The main chaotic dynamic modes are: Duffing, Lorenz, Vanderpol and etc, thereinto, research and application of Duffing equation has been in emphases, and was widely used in weak signal detection. Considering the detection model based on Duffing equation:

$$x''(t) + \delta x'(t) + u = \gamma + a + \eta(t) \quad (1)$$

Where, we defined the system driving force:  $a = \cos(\omega t)$ , damping ratio:  $\delta$ , restoring force:  $u = -x + x^3$ , circumstance

noise:  $\eta(t)$ , signal to-be-measured:  $\gamma_d \cos(\omega t)$ . Besides, the frequencies of system and signal-to-be-measured are equal.

The chaotic system based on Duffing equation have a very high sensibility to periodic driving force, when the damping ratio is fixed, the system will represent ample nonlinear fractal states orderly: chummage, bifurcation, chaos, critical and periodic trace [6]. However, the system trace in chaos is some anomalistic approaches, and correspondingly one simple closed curve in periodic state. By the differences between the two states, the weak signal could be detected.

## 2.2 Energy tracking model

In the 1990s, Kaiser firstly preferred the conception of the energy operator<sup>[7]</sup>, and approved it could track the nonlinear signal accurately, particularly envelope of amplitude modulation signals and instantaneous feature of frequency modulation signals<sup>[8]</sup>.

To the continue signal  $x(t)$ , we defined its operator  $\psi_c$ :

$$\psi_c[x(t)] = [x'(t)]^2 - x(t)x''(t) \quad (2)$$

Where,  $x'(t)$ ,  $x''(t)$  are individually the first derivative and second derivative of the signal  $x(t)$ , the physics sense represents energy is proportional to square of amplitude and frequency.

Energy detection algorithm is defined on small sample signals, which makes the approach of nonlinear system's instantaneous frequency, instantaneous amplitude, envelope be one real-time processing.

## 2.3 Fractal states identification

Chaotic system is a high-order nonlinear stochastic differential equation, which doesn't have accurate analytic solution. And hence we could only resolve the numerical solution by discretize- tion approach. By the approach of Euler-Maruyama<sup>[9]</sup> method, recurrence form is as follows:

$$\begin{cases} x(k+1) = hy(k) \\ y(k+1) = (-\delta y(k) + \mu + \gamma)h + \sqrt{2D}\sqrt{h}\epsilon_k \end{cases} \quad (3)$$

Where,  $h$  is integration step,  $\epsilon_k$  is standard Gauss random series,  $D$  is the intension of white noise.

For the system energy in stable state, continuously sample  $n$  times as the collected assemble, each integration step is considered

as one point  $x(k)$ , and continuous-time  $t$  is displaced with discrete-time variable  $nT$ :

$$x'(t) \leftrightarrow [x(k+1) - x(k-1)]/2T \quad (4)$$

So we can get the discrete signal energy operator:

$$\psi_d[x(k)] = x^2(k) - x(k-1)x(k+1) \quad (5)$$

In different background noise circumstances, the system energy operator calculated by the algorithm varies correspondingly. In order to better distinguish chaotic phase change, we introduce statistics knowledge in quantitative analysis of system energy.

## III Simulation experiment

### 3.1 Feature analysis and Detection method

Build chaotic system model consisting of Holmes type Duffing equation, resolve system by Teager energy algorithm to get the energy distribution characteristics, the results are shown in Figure 1, Figure 2. As we can see from the figures, the energy distribution characteristics of large-scale periodic state, after the initial fluctuations, the envelope soon goes into convergence and becomes stability. By contrary, in chaotic state, there are a lot of negative pulses present in energy fluctuations. From the above analysis, the difference between the the energy distribution characteristics under two states is conspicuous, which will be used as one fractal states identification method.

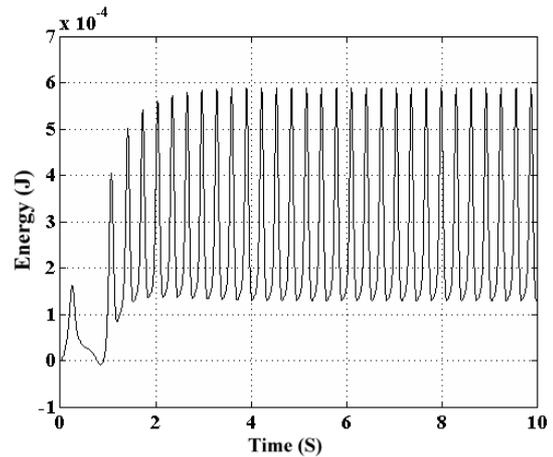


Fig.1 Chaotic energy distribution characteristic

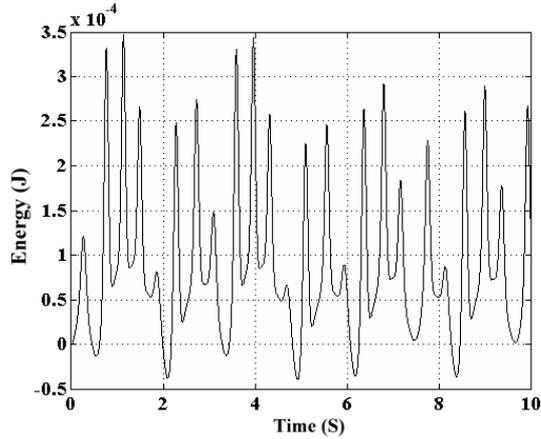


Fig.2 Periodic energy distribution characteristic

Based on the above analysis, the chaotic energy algorithm could be described as follows:

- (1) Adjust the intensity of the cycle driving force to make the system be in a critical state,
- (2) Input the to-be-measured signal with noise, and record the system output,
- (3) Use the energy operator to track the system output, obtain the energy distribution of the system characteristics, and calculate phase change threshold in accordance with Equation (5),
- (4) Comparing the phase transition threshold average and the minimum energy value, if  $E > 0$ , the system has a phase transition; Instead, the system phase transition doesn't occur.

### 3.2 Real-time analysis

In this chapter, weak electrical carrier signals are used in simulation. Simulation parameters: the electrical carrier signal's intensity 0.02V, frequency 1 rad/s, calculation step  $h = 0.01$  s. When the phase of to-be-measured signal transfers from "0" and " $\pi$ ", the Duffing system switches between from chaos and large-scale periodic state. The detection equation is derived to obtain the phase detecting range<sup>[11]</sup>:

$$\arccos(a/2\gamma) - \pi \leq \tau \leq \arccos(a/2\gamma) + \pi \quad (6)$$

When the phase  $\tau$  is located in phase detection range, the Duffing system can be used for signal detection.

Currently, Lyapunov exponent method is one of the most widely used classical phase transition algorithms. Therefore compared with

the chaotic energy method, the real-time feature will be analysed as follows.

The simulation time is 0 ~ 200s, within 100 Monte Carlo experiments, the single simulation computation time results with different data sizes are shown in Figure 3.

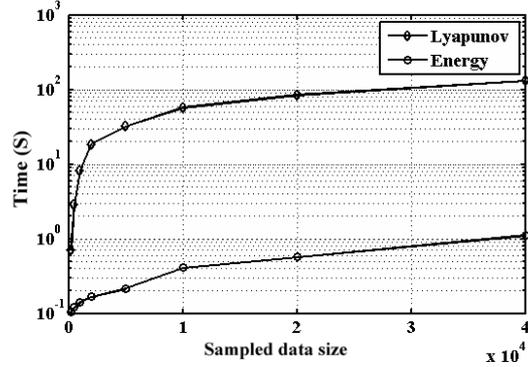


Fig.3 Fractal states identification methods'

simulation-time with different data sizes

In the simulation results, the average simulation computation time of single energy detection method is 0.0146s, while Lyapunov exponent calculation method is 6.1621s, the method proposed in paper reduced than the latter by approximately two orders of magnitude. Compared with the method in literature [1,2], the method proposed in this paper has a higher detection efficiency. Furthermore, it's more conducive to the computer automatically detects and quick judgment to avoid the influence of human factors.

### 3.3 Reliability analysis

In order to quantitatively analyze the algorithm's anti-noise performance, one group of weak electric-power line carrier signal (1000 random symbols) under strong background noise is detected in experiments.

The detection model is Duffing system, the main parameters of simulation is the same to the former chapter. By the approach of Energy algorithm, we obtain the energy distribution characteristics of system for each symbol, which is shown in Figure 4. To reduce the algorithm complexity furtherly, the sample ratio  $k$  is introduced.

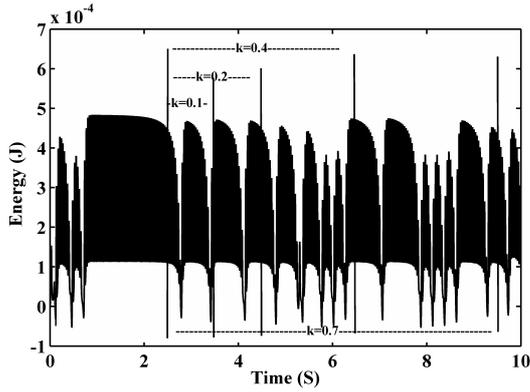


Fig.4 Different ratios of sampled data for detection

However, as shown in Figure 5, with the noise intensity increasing, the accurate identification probability is decreasing. In addition, the identification probability is in direct proportion to the sample ratio  $k$ . The simulation data are 10 groups of weak power line carrier signals, traditional detection methods are indaptable, iteration method is Euler-Maruyama approach and the mian simulation parameters is the same to the former.

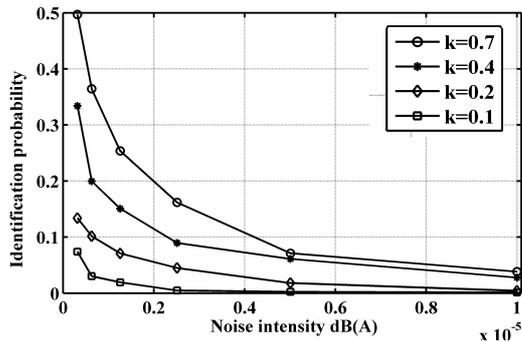


Fig.5 Chaotic Energy identification probability with different Noise-intensity

#### IV Conclusion

For detecting weak signals in strong noise accurately, chaotic method is combined with Teager energy operator in paper, named as chaotic energy algorithm. With extremely low computational complexity and excellent noise immunity, this method can achieve the reliable detection requirements of 0dB~30dB signals. Results show that the algorithm is a very

effective detection method for signals identification in strong noise background.

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