

# Joint Time-Frequency Channel Estimation Method for OFDM Systems Based on Compressive Sensing

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## Abstract

Time-domain synchronous orthogonal frequency division multiplexing (TDS-OFDM) outperforms the classical cyclic prefix OFDM (CP-OFDM) in higher spectral efficiency and faster synchronization. However, it has the difficulty to support high-order modulations like 256QAM and suffers from performance loss especially under severely fading channels. To solve this problem, a channel estimation method for OFDM system is proposed under the framework of compressive sensing (CS) in this paper. Firstly, by exploiting the signal structure, the auxiliary channel information is obtained. Secondly, we propose the auxiliary information based compressive sampling matching pursuit (A-CoSaMP) algorithm to utilize a very few frequency-domain pilots embedded in the OFDM block for the exact channel impulse response estimation. Simulation results demonstrate that the CS-based OFDM outperforms the conventional dual pseudo noise padded OFDM and CS-based TDS-OFDM schemes in both static and mobile environments.

## 1. Introduction

Time domain synchronous orthogonal frequency division multiplexing (TDS-OFDM) is the core technique of the widely deployed digital television/terrestrial multimedia broadcasting (DTMB) standard [1]. TDS-OFDM distinguishes the classical cyclic prefix OFDM (CP-OFDM) by replacing CP with the prior known pseudo noise (PN) sequence as the guard interval (GI). The PN sequence can also work as the training sequence (TS) for both synchronization and channel estimation (CE) and hence results higher spectral efficiency for TDS-OFDM.

However, there is one main drawback for TDS-OFDM, which is that the TS and the OFDM block will cause mutual inter-block interferences (IBI) to each other. Thus, iterative interference cancellation algorithm with high complexity has to be adopted for CE and equalization in TDS-OFDM systems [2]. One exciting solution is the dual-PN padded OFDM (DPN-OFDM) scheme, whereby the PN sequence is duplicated twice to make the second PN sequence immune from the IBI caused by the preceding OFDM block [3]. However, the spectral efficiency of the DPN-OFDM scheme is significantly decreased by the doubled PN sequence length. To prevent the spectral efficiency loss, the compressive sensing (CS) theory is exploited to solve the problem of TDS-OFDM, whereby the IBI-free region of small size within the received PN sequence is utilized to reconstruct the channel with high dimension [4]. However, such CS-based TDS-OFDM scheme cannot work once the maximum channel delay spread is close to the GI length due to the reduced size of the required observations.

To enhance the CE performance under severely fading channels with long delays, we propose the joint time-frequency CE method under the framework of CS. The work is inspired by the newly proposed time-frequency training OFDM (TFT-OFDM) scheme which is modified from TDS-OFDM [5]. The specific contributions of this paper can be summarized as follows: 1) The proposed joint time-frequency CE method uses the PN sequence to acquire the coarse channel path delay estimation, while the exact channel impulse response (CIR) estimation depends on a very few frequency-domain pilots embedded in the OFDM block based on CS. 2) Unlike the IBI caused by the preceding OFDM block to the current TS has to be cancelled completely to achieve good performance of TDS-OFDM, it is not necessary to remove such interference in TFT-OFDM since the received PN sequence is only used for the coarse channel path delay estimation. 3) With the use of CS and sparse channel nature, the number of pilots embedded in the OFDM block in TFT-OFDM could be significantly saved.

*Notation:* We use boldface letters to denote matrices and column vectors;  $\mathbf{F}_{N \times N}$  denotes the  $N$ -point fast Fourier transform (FFT) matrix with the  $(n+1, k+1)$ th entry being  $\exp(-j2\pi nk / N) / \sqrt{N}$ ;  $\mathbf{0}$  denotes the zero vector;  $\otimes$  represents the circular correlation;  $(\cdot)^T$ ,  $(\cdot)^H$ ,  $(\cdot)^{-1}$ , and  $\|\cdot\|_p$  denote the transpose, conjugate transpose, matrix inversion and  $l_p$  norm operations, respectively;  $\mathbf{x}|_\Gamma$  denotes the entries of the vector  $\mathbf{x}$  on the set of  $\Gamma$ .

## 2. TFT-OFDM System Model

Unlike the conventional TDS-OFDM or CP-OFDM where the training information only exists in either the time or frequency domain [4], Fig. 1 shows that TFT-OFDM has training information in both the time and frequency domains for every signal symbol, i.e., the time-domain TS and the frequency-domain pilots scattered over the sub-carriers are jointly used in TFT-OFDM. The  $i$ th TFT-OFDM signal symbol  $\mathbf{s}^i = [s_0^i, s_1^i, \dots, s_{M+N-1}^i]^T$  is composed of the known PN sequence  $\mathbf{c} = [c_0, c_1, \dots, c_{M-1}]^T$  of length  $M$  and the OFDM block  $\mathbf{x}^i = [x_0^i, x_1^i, \dots, x_{N-1}^i]^T$  of length  $N$ . In contrast to the conventional TDS-OFDM, the OFDM block in TFT-OFDM contains not only the traffic data, but also a small number  $J$  of pilots denoted as  $\tilde{\mathbf{x}}^i|_{\Gamma}$ , where  $\Gamma = \{P_0, P_1, \dots, P_{J-1}\}$  is the pilot location set. By the utilization of CS theory and sparse channel nature, the pilot number  $J$  could be reduced significantly (far less than the  $L$ , around 1% of the  $N$ ). The power of the pilots could be boosted for better recovery performance, which is similar to that in the traditional pilot-aided CE method [5].

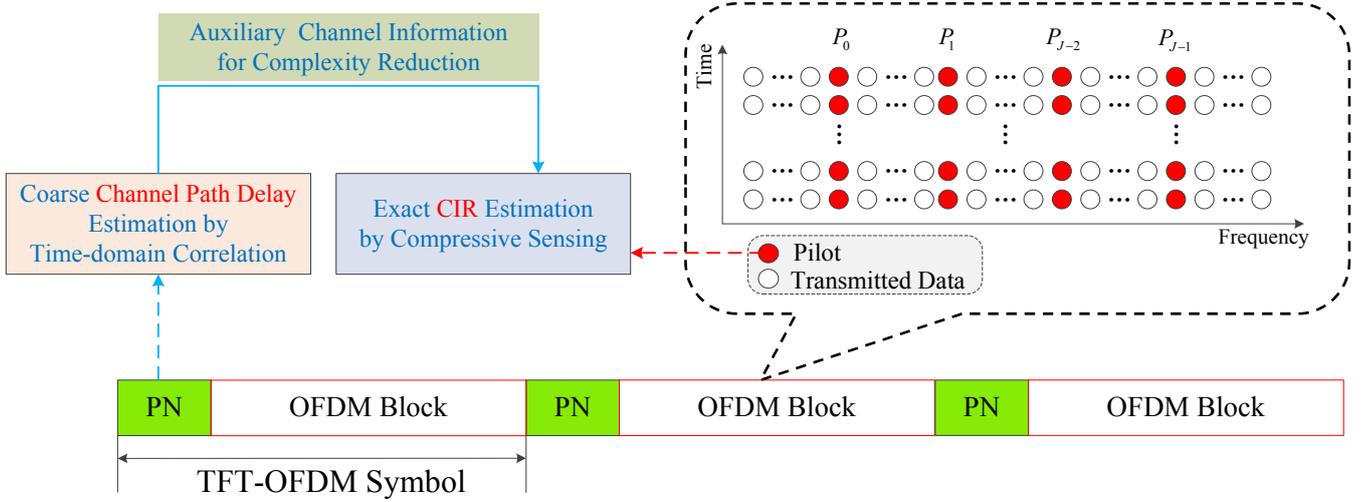


Fig. 1. Proposed frame structure and the corresponding CS-based CE for the TFT-OFDM scheme.

In the wireless propagation scenarios, the discrete-time CIR  $\mathbf{h}^i = [h_0^i, h_1^i, \dots, h_{L-1}^i]^T$  of length  $L$  comprising  $S$  resolvable propagation paths. Then, the time-domain OFDM block after cyclic reconstruction  $\mathbf{y}^i = [y_0^i, y_1^i, \dots, y_{N-1}^i]^T$  can be represented as

$$\mathbf{y}^i = \begin{bmatrix} x_0^i & x_{N-1}^i & \cdots & x_1^i \\ x_1^i & x_0^i & \cdots & x_2^i \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-1}^i & x_{N-2}^i & \cdots & x_0^i \end{bmatrix}_{N \times N} \begin{bmatrix} \mathbf{h}^i \\ \mathbf{0}_{N-L} \end{bmatrix} + \mathbf{w}^i = \Psi \mathbf{h}_N^i + \mathbf{w}^i, \quad (1)$$

where  $\Psi$  is the Toeplitz matrix of size  $N \times N$  determined by the transmitted OFDM block  $\mathbf{x}^i$ ,  $\mathbf{h}_N^i$  is the  $N$ -length CIR vector extended from the original  $L$ -length CIR vector  $\mathbf{h}^i$  with  $N-L$  zeros, and  $\mathbf{w}^i = [\omega_0^i, \omega_1^i, \dots, \omega_{N-1}^i]^T$  denotes the additive white Gaussian noise (AWGN) with zero mean and the variance of  $\sigma^2$ .

If the channel is exactly known at the receiver, the IBI of the PN sequence on the OFDM block can be completely removed. Then, the classical overlap and add (OLA) algorithm [2] can be utilized to add the ‘‘tail’’ of the OFDM block to its head so that the OFDM block is cyclically reconstructed and the effect of CP can be restored [5]. The frequency-domain OFDM block after cyclic reconstruction is  $\tilde{\mathbf{y}}^i = \mathbf{F}_{N \times N} \mathbf{y}^i$ , then the received frequency-domain pilots  $\tilde{\mathbf{y}}^i|_{\Gamma}$  can be represented by

$$\tilde{\mathbf{y}}^i|_{\Gamma} = \text{diag}(\tilde{\mathbf{x}}^i|_{\Gamma}) \mathbf{F}_{(L)}^{\Gamma} \mathbf{h}^i + \tilde{\mathbf{w}}^i|_{\Gamma}, \quad (2)$$

where  $\mathbf{F}_{(L)}^{\Gamma}$  represents the row sub matrix comprising the rows of the  $\mathbf{F}_{(L)}$  on the set of  $\Gamma$ .

### 3. Compressive Sensing Based Channel Estimation

The CS theory has shown that  $\mathbf{h}^i$  can be exactly recovered by a very small number of observations  $J$  which is far less than its length  $L$  if the target signal to be recovered is sparse [6]. Fortunately, numerous theoretical analysis and experimental results have verified that wireless channels are sparse in nature, i.e., the number of the active paths  $S$  is usually smaller than the dimension of the CIR  $L$  ( $S \ll L$ ) [7]. The CoSaMP is a widely used CS algorithm due to its robustness to noise, where the most significant  $S$  components of the original sparse signal are identified in an iterative manner [8].

In this section, by fully exploiting the joint time-frequency processing feature of TFT-OFDM, we propose the auxiliary information based CoSaMP (A-CoSaMP) for the CE. The proposed CS-based joint time-frequency CE firstly utilizes the PN-based correlation in the time domain to acquire the auxiliary channel information, and then the frequency-domain pilots are used for the final exact CIR estimation based on CS. The proposed CE based on A-CoSaMP is composed of three steps:

#### *Step 1: PN-Based Coarse Path Delay Estimation*

Based on the good auto-correlation property of the PN sequence [2], the received PN sequence  $\mathbf{d}^i$  is directly correlated with the locally known PN sequence to acquire the coarse CE  $\bar{\mathbf{h}}^i$  as

$$\bar{\mathbf{h}}^i = \frac{1}{M} \mathbf{c} \otimes \mathbf{d}^i = \mathbf{h}^i + \mathbf{v}, \quad (3)$$

where  $\mathbf{v}$  denotes the channel's AWGN as well as the effect of interference caused by the preceding OFDM block. Although the coarse CE is not accurate due to the IBI, the good auto-correlation property of the PN sequence ensures that the auxiliary channel information necessary for the following A-CoSaMP algorithm could be partially preserved. Such information includes the locations of the most significant taps, together with the approximate channel sparsity level.

#### *Step 2: Cyclic Reconstruction of OFDM Block*

The cyclic reconstruction of the OFDM block can be implemented by the simple OLA operation, which is a relatively mature technique widely used in the conventional TDS-OFDM systems [2].

#### *Step 3: Exact CIR Estimation Using A-CoSaMP*

The pilots are then extracted from the cyclically reconstructed OFDM block and processed for the accurate CE. Based on the basic idea of classical CoSaMP algorithm [8], the A-CoSaMP algorithm is proposed in this paper, whereby the auxiliary channel information obtained above is exploited to improve the CE performance and lower the computational complexity. First, the approximate channel sparsity level and SNR obtained in *Step 1* are used to configure the maximum iteration number in A-CoSaMP, which is usually less than that in CoSaMP. Second, the locations of the most significant taps obtained in *Step 1* are utilized to further reduce the number of iterations.

## 4. Simulation Results

This section investigates the performance of the CS-based joint time-frequency CE. The signal bandwidth is 7.56MHz locating at a central frequency of 760MHz. The OFDM block length  $N$  is 4096, and the PN sequence length  $M$  is 256. The low-density parity-check (LDPC) code with code rate of 0.6 and code length of 7488 in DTMB is adopted [1]. The modulation schemes 256QAM for the static channel and 16QAM with a receiver velocity of 60 km/h are both considered to evaluate the support for UHDTV and mobile services, respectively. The China digital television test 8th (CDT-8) channel model which has a very strong echo path ( $S=6, L=241$ ) close to the GI length [1] is adopted to evaluate the system performance. In the CS-based TDS-OFDM scheme, the last  $G=36$  samples of the IBI-free region are used for CE, while the number of frequency-domain pilots  $J=36$  with boosted power 3 dB is utilized in the proposed scheme.

Fig. 2(left) describes the MSE performance comparison of the proposed scheme with the conventional DPN-OFDM and CS-based TDS-OFDM schemes under CDT-8 channel. When the channel length  $L$  is fairly close to the GI length  $M$ , the MSE performance of the proposed scheme is 7.5 dB better than that of DPN-OFDM, while the recent CS-based TDS-OFDM cannot work due to the reduced size of the IBI-free region. Fig. 2(right) compares the LDPC-coded bit error rate (BER) performance of the proposed scheme with the conventional DPN-OFDM and CS-based TDS-OFDM schemes. The BER performance with the ideal CE is also presented as the benchmark for comparison. It can be observed from Fig. 2(right) that under the CDT-8 channel,

the BER performance of the proposed scheme is about 2 dB better than that of the DPN-OFDM, while the CS-based TDS-OFDM scheme cannot work due to the IBI-free region is contaminated by the long channel.

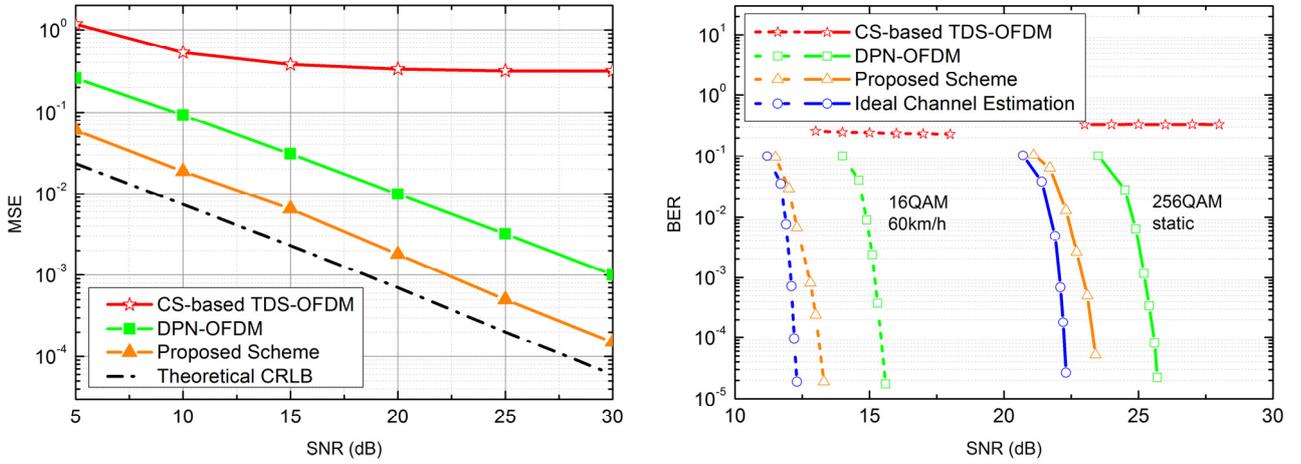


Fig. 2. The MSE(left) and BER(right) performances comparison under CDT-8 channel.

## 5. Conclusion

The MSE performance of the proposed CE method based on A-CoSaMP outperforms the conventional schemes and is close to the CRLB by simultaneously exploiting the time-domain PN sequence and frequency-domain pilots. Simulation results show that the proposed scheme has good BER performance in both static and mobile scenarios and can well support the 256QAM, especially when the maximum channel delay spread is fairly close to the GI length.

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