An Efficient Multiregion Model for the EM Scattering of a Dielectric Rough Surface and a Dielectric Target

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Abstract

This paper is aimed at applying the multiregion model to the composite EM scattering from a dielectric target and a dielectric rough surface. In the multiregion model, the rough surface is divided into multiple regions, the method of moment is only adopted in the dominant region. Hence, this model can markedly reduce the number of unknowns. Firstly, we derived the single integral equation (SIE) in which the number of unknowns is half of those in the conventional MoM and the the equations will be easier to deal with. Then the multiregion model is extended by SIE. With the multiregion model, one can obtain the accurate equivalent currents on the dominant region and accurate bistatic scattering coefficient (BSC) in small and moderate scattering angles with much less time and memory requirement.

1. Introduction

Electromagnetic wave scattering from rough surface and targets is becoming a problem of long-term interest. This topic is important in many fields, such as electromagnetics, applied optics, communications, remote sensing, oceanography, material science, target recognition, electronic countermeasure (ECM), etc. For the rough surface-target composite scattering, many methods are developed. The multiregion model is one of them which focus on reducing the number of unknowns of MoM. So far, only a few studies on EM scattering from targets or rough surfaces based on the multiregion model have been published. In the beginning, the multiregion model was devoted to EM scattering by perfectly electric conducting (PEC) targets and the radiation of point sources or wire antennas near PEC targets[1]. Later on, this efficient model has been applied to the scattering from a PEC rough surface with or without a PEC target above it[2]. Recently, the efficient multiregion model is extended by the single integral equation (SIE) method for the first time to calculate the EM scattering from a dielectric rough surface with or without a perfectly electric conducting target above it[3]. In [3], two intermediate regions including their targets are chosen as the dominant region, and the electric currents in the dominant region are calculated by MoM. Induced electric currents in the other regions are then obtained by approximately considering the mutual coupling between different regions along the rough surface. This multiregion model markedly reduces the number of unknowns and saves computation time.

2. Formulation of the composite problem

2.1 SIE for Target Located above the Rough Surface

As depicted in Fig. 1, a dielectric target is located above the dielectric rough surface. The problem is assumed to be variant in the (x, z) plane. The region \( \Omega_a , \Omega_b \) and \( \Omega_0 \) denotes the free space above the rough surface(excluding the target), below the rough surface, and in the target, respectively. The subscripts a and b indicate above below the rough surface, ta indicate the target, they are the same for subsequent variables and operators.

The EM field scattered from a homogeneous dielectric rough surface and a dielectric target above it will be determined. Accordingly, we will use the equivalence principle[4] to derive the single integral equations. The electric field integral equations (EFIE) for HH polarization can be written as

\[
|Z_a \vec{L}_a(J_a) + Z_b \vec{L}_b(\vec{n} \times \vec{K}_a(J_a)) - \vec{K}_b[Z_a \vec{L}_a(J_a) \times \vec{n}_a] + Z_a \vec{L}_a(J_a) + Z_b \vec{L}_b[\vec{n} \times \vec{K}_a(J_a)]| = - E_{inc} \times \vec{n}_a - Z_a \vec{L}_a[\vec{n} \times H_{inc}^{\perp}] \times \vec{n}_a + \vec{K}_b[E_{inc} \times \vec{n}_a] \times \vec{n}_a
\]

(1)

\[
|Z_a \vec{L}_a(J_a) + Z_b \vec{L}_b(\vec{n} \times \vec{K}_a(J_a)) - \vec{K}_b[Z_a \vec{L}_a(J_a) \times \vec{n}_a] + Z_a \vec{L}_a(J_a) + Z_b \vec{L}_b[\vec{n} \times \vec{K}_a(J_a)]| = - E_{inc} \times \vec{n}_a - Z_a \vec{L}_a[\vec{n} \times H_{inc}^{\perp}] \times \vec{n}_a + \vec{K}_b[E_{inc} \times \vec{n}_a] \times \vec{n}_a
\]

(2)
where $E_s^{inc}$ and $H_s^{inc}$ are the electric and magnetic fields on the rough surface, respectively; and $E_o^{inc}$ and $H_o^{inc}$ are the electric and magnetic fields on the surface of the target. $Z$ is the wave impedance in the space. The two operators $\bar{L}$ and $\bar{K}$ are defined as

$$\bar{L} \cdot X = -j\kappa \int \left[ X + \frac{1}{\kappa^2} \nabla X \right] G(r, r') ds'$$

(3)

$$\bar{K} \cdot X = -\int X \times G(r, r') ds'$$

(4)

where $G(r, r') = -j/4 H_0^{(2)} (k |r-r'|)$ is the 2D Green’s function, $k$ is the wavenumber, and $H_0^{(2)}$ is the zeroth-order Hankel function of the second kind. Finally, the EFIE is written in terms of the surface equivalent electric current $J_s$, $J_o$ for the magnetic currents $M_s = 0$, $M_o = 0$.

$$E_s = E_s^{inc} + E_s^{sca}$$

$$H_s = H_s^{inc} + H_s^{sca}$$

2.2 Application of the Multiresolution Model

As depicted in Fig. 2. Because the amplitude of the incident tapered plane wave is Gaussian, the induced electric current in the center of the rough surface is very smaller than that in the center. Hence, the rough surface $S_o$ is split into multiple regions(Fig. 2). The two intermediate regions $S_{UL}$ and $S_{LR}$ are chosen as the dominant region and handled by the MoM. The minimum width of the dominant region should be at least equal to the length of $g$. Then, the amplitude of the incident tapered wave satisfies $|\psi^{inc}(r)|_{z=0} \leq e^{-1/4}$ [2,3] at the edge of the dominant region. If a target $S_o$ is located above the rough surface, the target will be also included in the dominant region.

For HH polarization, the scattered electric and magnetic fields can be expressed as

$$E_s^{sca} = Z_a \sum_{p=1}^{N} \hat{p}_a^{pl} \cdot J_p^{pl} + Z_s \sum_{p=1}^{N} \hat{p}_a^{pr} \cdot J_p^{pr}$$

(5)

$$H_s^{sca} = \sum_{p=1}^{N} \hat{k}_a^{pl} \cdot J_p^{pl} + \sum_{p=1}^{N} \hat{k}_a^{pr} \cdot J_p^{pr}$$

(6)

where $n = Ul, 1r, 2l, 2r, \ldots, Nl, Nr$ correspond to regions $S_{Ul}, S_{2l}, \ldots, S_{Nl}, S_{Nr}$, respectively.

In the multiresolution model, $J_{2l}$ and $J_{2r}$ are approximately obtained as follows

$$J_{2l}(r) = 2\hat{n}_x \times H_{2l}^{inc}(r) + 2\hat{n}_y \times \bar{K}_{2l}^{ul} \cdot J_{Ul}$$

(7)

$$J_{2r}(r) = 2\hat{n}_x \times H_{2r}^{inc}(r) + 2\hat{n}_y \times \bar{K}_{2r}^{ul} \cdot J_{Ur}$$

(8)

Applying the MoM to the dominant region and substituting (9) and (10) into (5) and (6) enable the matrix equation of EM scattering from a dielectric rough surface and a target above it to be written as

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} J_{Ul} \\ J_{Ur} \\ J_o \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$$

(9)

$$A_{11} = Z_a \bar{P}_{a}^{ul} + Z_a \bar{P}_{a}^{ur} + Z_b \bar{P}_{b}^{ul} + Z_b \bar{P}_{b}^{ur} + Z_a \bar{P}_{a}^{ul} + Z_a \bar{P}_{a}^{ur} + Z_b \bar{P}_{b}^{ul} + Z_b \bar{P}_{b}^{ur} + Z_a \bar{P}_{a}^{ul} + Z_a \bar{P}_{a}^{ur} + Z_b \bar{P}_{b}^{ul} + Z_b \bar{P}_{b}^{ur}$$

$$A_{12} = -\bar{K}_{b}^{ul} + Z_a \bar{P}_{a}^{ul} + Z_a \bar{P}_{a}^{ur} + Z_b \bar{P}_{b}^{ul} + Z_b \bar{P}_{b}^{ur} + Z_a \bar{P}_{a}^{ul} + Z_a \bar{P}_{a}^{ur} + Z_b \bar{P}_{b}^{ul} + Z_b \bar{P}_{b}^{ur}$$

$$A_{13} = -\bar{K}_{b}^{ur} + Z_a \bar{P}_{a}^{ul} + Z_a \bar{P}_{a}^{ur} + Z_b \bar{P}_{b}^{ul} + Z_b \bar{P}_{b}^{ur} + Z_a \bar{P}_{a}^{ul} + Z_a \bar{P}_{a}^{ur} + Z_b \bar{P}_{b}^{ul} + Z_b \bar{P}_{b}^{ur}$$
The simulation time for one dielectric rough surface obtained by full MoM and multi-MoM is also presented in the above dielectric cylinder are \( \lambda = 128 \). The relative permittivity of those in the full MoM and multi-MoM, respectively. The electric currents \( J_{\text{nl}} \) and \( J_{\text{pl}} \) on the dominant region are obtained, \( J_{\text{nl}} \) and \( J_{\text{pl}} \) can be obtained in the same way.

Once the induced electric currents \( J_{\text{nl}} \) and \( J_{\text{pl}} \) on the dominant region are obtained, \( J_{\text{nl}} \) and \( J_{\text{pl}} \) can be calculated through (9) and (10), respectively. Furthermore, the electric currents \( J_{\text{nl}} \) can be obtained by

\[
J_{\text{nl}}(r) = 2\hat{n}_s \times H_{\text{nl}}^{\text{inc}}(r) + \sum_{p=1}^{n-1} 2\hat{n}_s \times \vec{K}_{\text{nl}}^{p-1} \cdot J_{\text{pl}}
\]

where \( n = 3, \cdots, N \), \( J_{\text{nl}} \) can be obtained in the same way.

In the observational direction with \( k_s = k_0 \left( \hat{x} \sin \theta_i + \hat{z} \cos \theta_i \right) \), the bistatic scattering coefficient (BSC) for HH polarization is defined as the same as in [5]

\[
\sigma(\theta_i) = \frac{| - jk_0 Z_a \int J_s(r') \exp(jk_0 \hat{r} \cdot \hat{R}) ds - jk_0 Z_a \int J_o(r') \exp(jk_0 \hat{r} \cdot \hat{R}) dl |^2}{8 \pi k_0 g \sqrt{2} \cos \theta_i \left[ 1 - \frac{1 + 2 \tan^2 \theta_i}{2(k_0 g \cos \theta_i)^2} \right]}
\]

and \( r' = x \hat{x} + f(x) \hat{z}, \hat{R} = \sin \theta_i \hat{x} + \cos \theta_i \hat{z} \), and \( ds = \sqrt{1 + f'^2(x)} dx \).

3. Numerical Simulations and Discussion

In the simulation, the parameters of the incident tapered wave are given by \( f = 3.0 \text{GHz} \) ( \( \lambda = 0.1 \text{m} \)) and \( \theta_i = 30^\circ \). The radius and height of the above dielectric cylinder are \( r = 1.0 \lambda \), \( h = 10.0 \lambda \) respectively. The number of samples \( f \) or the target is \( \text{N}_f = 100 \), the relative permittivity of the target is \( \varepsilon_r = (3.9,3.3) \). The Gaussian rough surface is divided into six regions with \( N_y = 128, N_x = 128, N_{x,y} = 128, N_{x,y} = 128, N_{x,y} = 256, \) and \( N_{x,y} = 256 \), with simulated length \( L = 102.4 \lambda = 10.24 \text{m} \). The rms height and correlation length are \( \delta = 0.1 \lambda \) and \( l = \lambda \), respectively, and the tapered parameter is \( g = L / 4 \). The relative permittivity of the space below the surface is \( \varepsilon_r = (2.5,0.18) \).

The distribution of the induced current along the rough surface is depicted in Fig. 3 for one composite realization. The dash and solid lines are obtained through the full MoM and multi-MoM, respectively. The electric currents on the dominant region obtained by the two different methods fairly well agree, but an discrepancy exists in not-dominant regions because the induced current is only approximately obtained by considering the mutual coupling between different regions along the rough surface. Fig. 4 shows the induced currents along the cylinder. The results obtained by the two different methods are almost the same primarily because the cylinder is dealt with by the full MoM in this multiregion model. Fig. 5 compares the BSC obtained by full MoM and multi-MoM for 30 Monte Carlo realizations. The results obtained by the two methods are found to agree well for small and moderate scattering angles. However, an obvious discrepancy appears in the large scattering angles. We can conclude that the BSC in the large scattering angle region depends more on the currents at the edge of the rough surface.

The simulation application for one dielectric rough surface obtained by full MoM and multi-MoM is also presented in Fig. 6. After the application of the multiregion model, the number of unknowns is reduced to 25% of those in the full model.
MoM. The total times using the multi-SIE are about 9.54% of those using the full MoM. It is obvious that the multi-SIE can markedly reduce the number of unknowns and save computation time.

![Fig. 3. Comparison of induced electric currents along the Gaussian rough surface](image1)

![Fig. 4. Comparison of induced electric currents along the cylinder](image2)

<table>
<thead>
<tr>
<th>Method</th>
<th>Full MoM</th>
<th>Multi-MoM</th>
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<tbody>
<tr>
<td>Number of unknowns</td>
<td>1124</td>
<td>356</td>
</tr>
<tr>
<td>Filling time (s)</td>
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<td>17.799</td>
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<tr>
<td>Solving time (s)</td>
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<td>0.562</td>
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<tr>
<td>Total time (s)</td>
<td>199.712</td>
<td>19.048</td>
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</tbody>
</table>

![Fig. 5. Comparison of BSC](image3)

![Fig. 6. Comparison of simulation times](image4)

4. Acknowledgments

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5. References


