Numerical Simulation of Tomography-SAR Imaging and the Object Reconstruction using the Compressive Sensing Approach with $L_{1/2}$-Norm Regularization

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Abstract

Making use of multiple acquisitions of the SAR observations over the same area, the tomography-SAR (tomo-SAR) technology can achieve a three-dimensional (3-D) imagery of the interested scene. Given the sparseness of the elevation signals, the compressive sensing (CS) approach has become an effective and innovative method. Some regularization tools are also employed in CS approach to reconstruct the reflectivity profile of the objects. In this paper, we present a novel $L_{1/2}$-norm regularization to realize 3-D reconstruction. As a non-convex optimization problem, the $L_{1/2}$-norm regularization is solved by an iterative algorithm. Multiple acquisitions might cost expensive big data, e.g. extending one or more years. Based on our studies on polarimetric scattering and SAR image simulation, we first apply numerical simulation of polarimetric scattering for multi-pass SAR observations and produce the tomo-SAR image of the terrain objects. It can be of great help for parameterized analysis of tomo-SAR imagery. As an example, the tomo-SAR image and 3-D reconstruction of the Beijing National Stadium is presented.

1. Introduction

Conventional SAR achieves resolving ability in the range and azimuth directions. In spotlight mode SAR system, high resolution in azimuth can be achieved by processing the return signals from different angular views, whereas high resolution in range can be achieved by focusing wide-band linear frequency modulated pulse [1]. According to the side-looking geometry of the radar, the resulting image presents the projection of the real 3-D scene backscattering properties over the azimuth slant-range plane. Thus, some scatters might map into one azimuth-range resolution cell, especially in the urban areas full of high buildings, which causes inconvenience in image interpretation. In order to retrieve real 3-D objects, the tomography SAR (tomo-SAR) was presented in the 1990s [2]. It extends the synthetic aperture principle to the elevation direction (perpendicular to the incident plane) for the 3-D imaging. Resolution in elevation can be achieved by processing the return signals from different looking angles, which resembles the azimuth direction. With the launch of German TerraSAR-X and Italian COSMO-Skymed spaceborne SAR system, one-meter resolution may identify the structural details of building objects. Multi-pass very high resolution SAR images cost big data extending a long period. To fully understand and make parameterized analysis of complex terrain scene from tomo-SAR technology, numerical modeling and simulation of polarimetric scattering and tomo-SAR imaging are developed in this paper.

The limited passes result in small elevation aperture, and the corresponding Rayleigh resolution is much worse than the azimuth or range direction, usually less than one order. In recent years, compressive sensing (CS) based on regularization has been well studied for radar signal processing [3]. It is beyond the Shannon sampling theorem and is able to reconstruct sparse signals with a small quantity of irregular baselines. To carry out regularization for inversion, the $L_{1/2}$-norm was studied as a good approximation of L_0-norm [4]. In this paper, we apply the non-convex $L_{1/2}$-norm regularization in iterative algorithm to the inversion of tomo-SAR images. As an example, our simulation+ $L_{1/2}$-norm regularization is applied to 3-D reconstruction of the Beijing National Stadium.

2. Formulation of SAR tomography

As shown in Fig. 1, when the radar flies along the x-direction over the scene once, each obtained 2-D SAR image is the projection of the scene over the azimuth-slant range plane (incident plane). If the radar makes multi-passes over the scene, i.e. N flights, a new synthetic aperture is formed along the s-direction (referred to as the elevation), which is vertical to the referenced incident plane (x-r). In Fig. 1, b_n is the aperture position, the length between the adjacent aperture positions, Δb_n, is named as baseline, and the synthetic aperture length b is the overall baseline length. The n-th received signal $g_n(x_0, r_0)$, measured at the aperture position b_n of the pixel (x_0, r_0), is the integral of the complex reflectivity function along the elevation direction [5]:

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$$g_n = \int_{\Delta s} \gamma(s)\exp(-j2\pi\xi_n s)ds, \qquad n = 1,\ldots,N \tag{1}$$

where γ (s) is the reflectivity function along the s-direction and $\xi_n = 2\theta_n/\lambda$ represents the spatial frequency. θ_n is measured in radian. Substituting this integral on the s-direction for numerical summation of discrete points, the continuous system model can be approximated as:

$$g_n \approx \delta_s \cdot \sum_{l=1}^{L} \gamma(s_l)\exp(-j2\pi\xi_n s_l), \qquad n = 1,\ldots,N \tag{2}$$

where L is the number of discrete units on the s-direction and $\delta_s = \Delta s/(L-1)$ is the discrete interval. To ignore the constant, this model of the system is rewritten as a matrix equation:

$$\mathbf{g} = \overline{\mathbf{R}} \cdot \boldsymbol{\gamma} \tag{3}$$

where $\mathbf{g}$ is the measurement vector with N elements, $\overline{\mathbf{R}}$ is the $N\times L$ system matrix with $R_{nl}=\exp(-j2\pi\xi_n s_l)$, and $\boldsymbol{\gamma}$ is the discrete reflectivity vector with L elements $\gamma_l = \gamma(s_l)$.

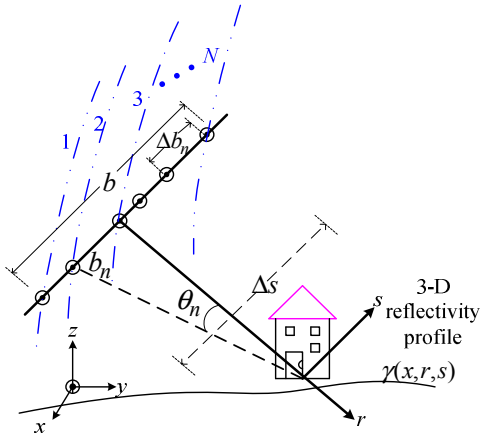

Fig. 1: Geometry of tomo-SAR imaging

3. Compressive sensing in SAR tomography

According to the Shannon sampling theorem for band-limited signals of spotlight SAR, the corresponding Rayleigh resolution of slant elevation is decided by the overall range of different viewing angles, i.e. $\rho_s=\lambda/2\theta_n$. The limited number of samples on elevation results in small distribution range of viewing angles, which makes the elevation resolution much lower than the azimuth and range directions. Such a low resolution is unbearable for high resolution 3-D imaging. New imaging algorithm against the Rayleigh limit should be studied.

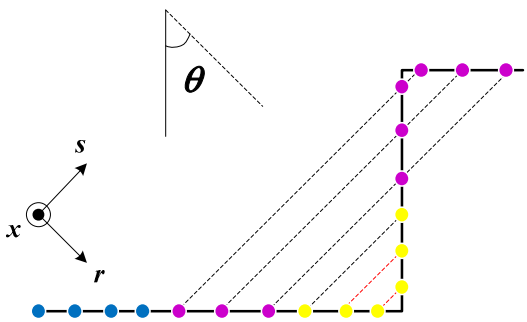

Fig. 2 Overlapped scatters on elevation *s*

In fact, the signals on elevation are usually sparse, unlike the continuous band-limited signal on azimuth or range directions, which usually consist of one to three typical scatters. As shown in Fig. 2, strong echoes from a building are those dominant signals on the elevation s contributed by the scatters, i.e. ground, wall and roof. In Fig. 2, it

indicates that the purple dots denote three scatters overlapping in one azimuth-range pixel, the yellow points are for two scatters, and the blue points are just for one scatter. These discrete scatters with unknown locations, amplitudes and phases should be separated correctly, even some scatters are with small elevation distances, e.g. as red dashed lines show.

Based on CS studies [6], if the unknown signal $\boldsymbol{\gamma}$ is sparse, and the system matrix $\overline{\mathbf{R}}$ is random Fourier samplings, the sparse signal can be exactly recovered from the limited measurements. Usually, the minimal number of the measurements $N > O(K \log(L / K))$ should be satisfied, where K is the sparsity of the unknown signal. The unique sparse solution can be found by L_0-norm minimization. In the presence of measurement noise, the question can be described as:

$$\min_{\boldsymbol{\gamma}} \left\{ \left\| \mathbf{g} - \overline{\mathbf{R}} \cdot \boldsymbol{\gamma} \right\|_2^2 + \mu \left\| \boldsymbol{\gamma} \right\|_0 \right\} \tag{4}$$

where the first item is to minimize the residual, and the second term as L_0-norm regularization is to obtain the sparse solution, and here μ is a tuning parameter between these two terms based on the noise level.

Eq. (4) is actually an optimization problem how to minimize the objective function. Theoretically, the L_0-regularization can have the exact sparse solution, but it is impossible to be solved. Since the L_1-norm is convex, it is often adopted to approximate L_0-norm for solving the sparse solution. In recent years, more general L_p-norm ($0 < p < 1$) regularization has been studied. It was found that the L_p-norm regularization is even sparser than L_1-norm, but is much easier to be solved. Especially, the $L_{1/2}$-norm regularization is the sparsest and robust when $1/2 \le p < 1$. Since as long as $0 < p \le 1/2$, all L_p-norm has similar properties, we consider the $L_{1/2}$-norm regularization:

$$\hat{\boldsymbol{\gamma}}_{L1/2} = \min_{\boldsymbol{\gamma}} \left\{ \left\| \mathbf{g} - \overline{\mathbf{R}} \cdot \boldsymbol{\gamma} \right\|_2^2 + \mu \sum_{i=1}^{L} \left| \gamma_i \right|^{1/2} \right\} \tag{5}$$

Eq. (5) is non-convex and is difficult to solve. The local minima of this concave penalty function, which more closely resembles the L_0-norm, can be found by an iterative algorithm. The algorithm can be divided into a series of weighted L_1-norm regularizations, which can be implemented using available software. It has been proved that such iterative algorithm for $L_{1/2}$-norm can be well and quickly converged to local minima [4, 7]. To compare L_1-norm and $L_{1/2}$-norm, we have tested a simple 3-D virtual surface modeled by many points for these two regularizations, and found that $L_{1/2}$-norm regularization has better super resolving ability and can reconstruct more scatters.

4. An example of 3-D object reconstruction

As an example of simulation of tomo-SAR imaging with 3-D reconstruction by $L_{1/2}$-norm regularization, we use a model of the Beijing National Stadium, which consists of irregular steel bars. The stadium occupies the area of about 120×120 m^2, and the height is about 30 m. Radar center frequency is f_c=15 GHz and the bandwidth B is 150MHz. The x-direction is the azimuth, and the incident angle θ is 45°. Total 200 samples are taken on both azimuth and slant-range directions. Only 11 acquisitions are acquired on slant-elevation direction, and the corresponding Rayleigh resolution is just 20 m.

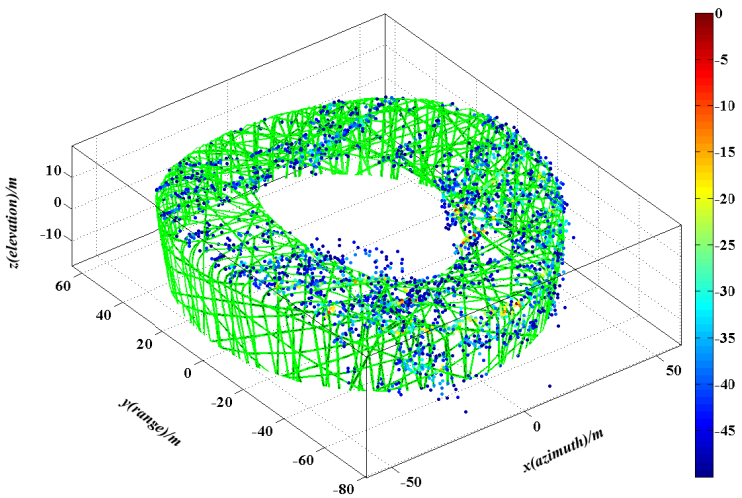

Fig. 3 A 3-D reconstruction of the Beijing National Stadium

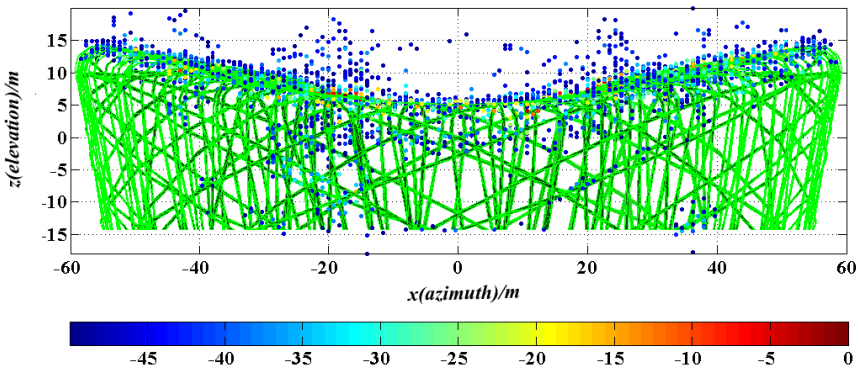

Fig. 4 front view of the 3-D reconstruction

We employ the software BART (Bidirectional Analytical Ray Tracing) developed in our laboratory [8] to calculate the HH co-polarized backscattering data. The stadium model is divided into 42185 triangular surfaces. Taking account of single scattering, the BART runs 37.2 hours to calculate scattering of this model on the computer with 4 cores, 3.2G CPU and 8G RAM. The final 3-D reconstruction is shown in Figs. 3, 4. The normalized scattering intensity above —50 dB is drawn in the figures. The results have been converted from the imaging space to the target space. As an intuitive comparison, the model of steel bars is also placed in the same figure.

It can be found from Figs. 3 and 4 that many scatters on the top and near-radar side are correctly reconstructed. In the front view of Fig. 4, main strong scattering comes from the top of the Stadium, and the heights of these scatters are well inverted.

5. Conclusion

This paper presents a novel numerical simulation of polarimetric scattering for tomo-SAR imaging and 3-D reconstruction with $L_{1/2}$-norm regularization for a complex object. Numerical simulation of tomo-SAR imaging may provide parameterized analysis for tomo-SAR observations over complex and variable scene with complex 3-D objects. A new $L_{1/2}$-norm regularization is well solved and applied to 3-D reconstruction of complex objects from tomo-SAR imaging. A reconstruction of the Beijing National Stadium model from simulated tomo-SAR image with $L_{1/2}$-norm is presented. Our approach of simulation of tomo-SAR with $L_{1/2}$-norm inversion can be extended to more natural scenes with complex objects.

6. References

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