Perturbation Analysis of the TE Plane Wave Scattering from the End-Face of a Waveguide System

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Abstract

The scattering of TE plane wave from the end-face of a two-dimensional waveguide system is treated by solving straightforwardly a wave equation for the electric field. The unknown scattered field is expanded into a perturbation series with respect to the difference of refractive indices of core and cladding. The first and second order terms of the series are analytically derived and it is shown that the second order term is sufficiently small compared with the first order term and can be ignored.

1. Introduction

An image fiber is composed of a large number of cores embedded in a single cladding, which is used to transmit directly an optical image. By illuminating the end-face of an image fiber with a laser beam a diffraction pattern reflecting the arrangement of cores can be simply observed. We can see experimentally that the diffraction pattern does not almost depend on the polarization of the laser beam. We are interested in the problem from a theoretical point of view.

The reflection and transmission of a guided mode by the cut-end of a dielectric slab waveguide and the coupling of a beam wave to a dielectric slab waveguide have been treated by the boundary integral equation[1,2]. In their papers the approximate form of the Green's function in a waveguide region has been used and the integral equation has been solved numerically. The analytical treatment of the boundary integral equation is very difficult.

The scattering of an electromagnetic wave from a dielectric body has been treated by the volume integral equation for the electric field[3]. The scattered field is the field radiated from the electric polarization induced in a dielectric body by the incident field and the physical image is very clear. The solution of the volume integral equation can be easily expanded into a perturbation series and each term of the series can be derived analytically. The analytical representation gives a deep understanding of the scattering properties of a dielectric body.

The scattering of a plane wave from the end-face of a waveguide system composed of a large number of cores has been treated by the volume integral equation for the electric field and it has been shown that the far scattered field does not almost depend on the polarization of an incident wave[4].

In this paper the scattering of a TE plane wave from the end-face of a two-dimensional waveguide system is treated by the perturbation method. The first and second order terms of the perturbation series are derived and the contribution of the second order term is clarified.

2. Formulation of the problem

We consider the scattering of a plane wave from the end-face of a two-dimensional waveguide system composed of a large number of cores and a single cladding as shown in Fig.1. The waveguide system is a model of an image fiber. For the total electric field $\boldsymbol{E}$ we can write

$$\nabla \times \nabla \times \boldsymbol{E} - k_0^2 n^2(\boldsymbol{r})\boldsymbol{E} = 0 \tag{1}$$

where $\boldsymbol{r}$ is a position vector in the xz-plane, k_0 is the wave number in the air region and $n(\boldsymbol{r})$ is the refractive index distribution for the entire region. The total field $\boldsymbol{E}$ is divided as follows:

$$\boldsymbol{E} = \begin{array}{ll} \boldsymbol{E}^i + \boldsymbol{E}^r + \boldsymbol{E}^{s1}, & z > 0 \\ \boldsymbol{E}^t + \boldsymbol{E}^{s2}, & z < 0 \end{array} \tag{2}$$

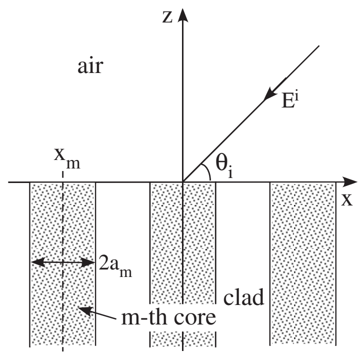

Fig.1 Geometry of the problem.

where $\boldsymbol{E}^i$ is the incident field and $\boldsymbol{E}^r$ and $\boldsymbol{E}^t$ are the reflected and transmitted fields by a plane surface between the air and the cladding, respectively. $\boldsymbol{E}^{s1}$ and $\boldsymbol{E}^{s2}$ are the scattered fields, which satisfy

$$\nabla \times \nabla \times \boldsymbol{E}^{s1} - k_0^2 \boldsymbol{E}^{s1} = 0 \tag{3}$$

$$\nabla \times \nabla \times \boldsymbol{E}^{s2} - k_0^2 n_2^2 \boldsymbol{E}^{s2} = k_0^2 \delta n^2 \sum_m \phi_{a_m}(x)\boldsymbol{E}^t + k_0^2 \delta n^2 \sum_m \phi_{a_m}(x)\boldsymbol{E}^{s2} \tag{4}$$

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where n_1 and n_2 are the refractive indices of core and cladding, respectively and $\delta n^2 = n_1^2 - n_2^2$. ϕ_{a_m} is defined by

$$\phi_{a_m}(x) = \begin{array}{ll} 1, & |x - x_m| < a_m \\ 0, & |x - x_m| > a_m \end{array} \tag{5}$$

a_m and x_m are the radius and the center position of the m-th core. The scattered fields satisfy the boundary condition at $z = 0$,

$$\overline{z} \times \boldsymbol{E}^{s1} = \overline{z} \times \boldsymbol{E}^{s2}, \qquad \overline{z} \times (\nabla \times \boldsymbol{E}^{s1}) = \overline{z} \times (\nabla \times \boldsymbol{E}^{s2}) \tag{6}$$

The scattered field is represented by

$$\boldsymbol{E}^{si}(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{\boldsymbol{E}}^{si}(p,z) e^{-jpx} dp, \qquad i = 1,2 \tag{7}$$

The scattered field is expanded into the perturbation series with respect to δn^2,

$$\tilde{\boldsymbol{E}}^{s1} = \delta n^2 \boldsymbol{A}_1(p) e^{-jq_0 z} + \delta n^4 \boldsymbol{A}_2(p) e^{-jq_0 z} + \cdots \tag{8}$$

$$\tilde{\boldsymbol{E}}^{s2} = \delta n^2 \boldsymbol{B}_1(p,z) + \delta n^4 \boldsymbol{B}_2(p,z) + \cdots \tag{9}$$

where $q_0 = \sqrt{k_0^2 - p^2}$. We can obtain the following ordinaly differential equation for $\boldsymbol{B}_n$,

$$\nabla_p \times \nabla_p \times \boldsymbol{B}_1 - k_0^2 n_2^2 \boldsymbol{B}_1 = \frac{k_0^2}{2\pi}\sum_m \int_{-\infty}^{\infty} \phi_{a_m}(x) \boldsymbol{E}^t(x,z) dx \tag{10}$$

$$\nabla_p \times \nabla_p \times \boldsymbol{B}_n - k_0^2 n_2^2 \boldsymbol{B}_n = \frac{k_0^2}{2\pi}\sum_m \int_{-\infty}^{\infty} \tilde{\phi}_{a_m}(p - p_1) \boldsymbol{B}_{n-1}(p_1,z) dp_1, \quad n \geq 2$$

where $\tilde{\phi}_{a_m}(p)$ is the Fourier transform of $\phi_{a_m}(x)$ and $\nabla_p = -jp\overline{x} + \overline{z}\frac{d}{dz}$.

3. Far scattered field

The incident electric field is expressed as

$$\boldsymbol{E}^i = \overline{y} e^{jk_0(x\cos\vartheta_i + z\sin\vartheta_i)} \tag{11}$$

Then the transmitted field is

$$\boldsymbol{E}^t = \overline{y} T_\perp e^{jk_0 n_2(x\cos\vartheta_t + z\sin\vartheta_t)} \tag{12}$$

ϑ_i and ϑ_t are the angles of incidence and transmission, respectively. $T_\perp$ is the transmission coefficient,

$$T_\perp = 2\sin\vartheta_i / (\sin\vartheta_i + n_2 \sin\vartheta_t) \tag{13}$$

From Eq.(10) we have

$$\frac{d^2 B_{1y}}{dz^2} + q_2 B_{1y} = -k_0^2 T_\perp \sum_m \tilde{\phi}_{a_m}(p + k_0 n_2 \cos\vartheta_t) e^{jk_0 n_2 \sin\vartheta_t z} \tag{14}$$

where $\boldsymbol{B}_1 = \overline{y} B_{1y}$ and $q_2 = \sqrt{k_0^2 n_2^2 - p^2}$. The solution is given by

$$B_{1y}(p,z) = b_1(p) e^{jq_2 z} - \frac{k_0^2 T_\perp}{q_2^2 - (k_0 n_2 \sin\vartheta_t)^2} \sum_m \tilde{\phi}_{a_m}(p + k_0 n_2 \cos\vartheta_t) e^{jk_0 n_2 \sin\vartheta_t z} \tag{15}$$

$\boldsymbol{A}_1 = \overline{y} A_{1y}$ and b_1 are determined from the boundary condition (6), which are given by

$$b_1(p) = k_0^2 T_\perp \frac{q_0 + k_0 n_2 \sin\vartheta_t}{(q_0 + q_2)\left(q_2^2 - (k_0 n_2 \sin\vartheta_t)^2\right)} \sum_m \tilde{\phi}_{a_m}(p + k_0 n_2 \cos\vartheta_t) \tag{16}$$

and

$$A_{1y}(p) = \sum_m A_{1y}^m(p) \tag{17}$$

where

$$A_{1y}^m(p) = -\frac{k_0^2 T_\perp}{(q_0 + q_2)(q_2 + k_0 n_2 \sin\vartheta_t)} \tilde{\phi}_{a_m}(p + k_0 n_2 \cos\vartheta_t) \tag{18}$$

A suffix m means the single scattering from the m-th core. We can obtain the second order term in the same manner as the first order term, which is given by

$$A_{2y}(p) = \sum_m \sum_l A_{2y}^{ml}(p) \tag{19}$$

where

$$A_{2y}^{ml}(p) = \frac{k_0^4 T_\perp}{2\pi(q_0(p)+q_2(p))(q_2(p)+k_0 n_2 \sin\vartheta_t)} \times \int_{-\infty}^{\infty} \frac{q_0(p_1)+q_2(p)+q_2(p_1)+k_0 n_2 \sin\vartheta_t}{(q_2(p)+q_2(p_1))(q_0(p_1)+q_2(p_1))(q_2(p_1)+k_0 n_2 \sin\vartheta_t)} \tilde{\phi}_{a_m}(p-p_1)\tilde{\phi}_{a_l}(p_1 + k_0 n_2 \cos\vartheta_t) dp_1 \tag{20}$$

Suffixes m and l mean the double scattering from the m-th and l-th cores.

The scattered field (7) is

$$\boldsymbol{E}^{s1}(x,z) = \overline{y} \sum_n \frac{\delta n^{2n}}{2\pi} \int_{-\infty}^{\infty} A_{ny}(p) e^{-jpx - jq_0 z} dp \tag{21}$$

By using the saddle point method the far scattered field is

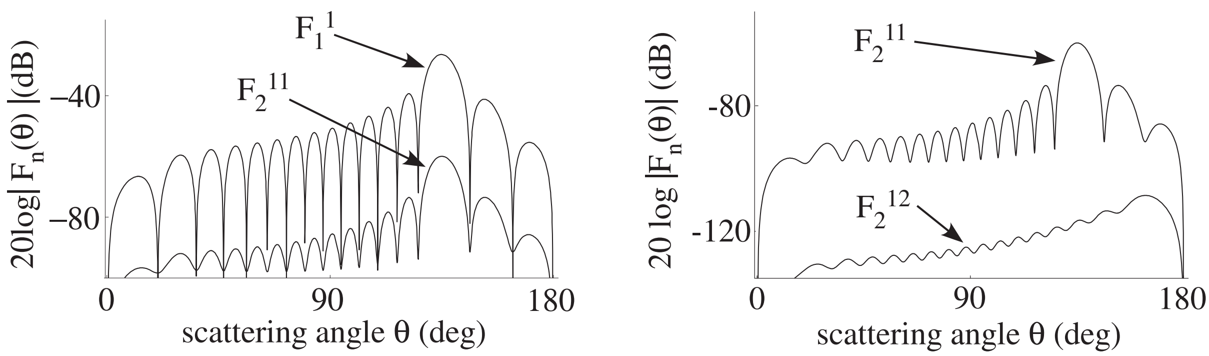
$$\boldsymbol{E}^{s1}(x,z) \sim \overline{y} \sqrt{\frac{2\pi}{k_0 r}} e^{-jk_0 r + j\frac{\pi}{4}} \sum_n F_n(\vartheta), \quad r \to \infty \tag{22}$$

$F_n(\vartheta)$ is the scattering amplitude of order n,

$$\begin{aligned} F_1(\vartheta) &= \textstyle\sum_m F_1^m(\vartheta) \\ F_1^m(\vartheta) &= \frac{\delta n^2}{2\pi} k_0 \sin\vartheta A_{1y}^m(k_0 \cos\vartheta) \\ F_2(\vartheta) &= \textstyle\sum_m \sum_l F_2^{ml}(\vartheta) \\ F_2^{ml}(\vartheta) &= \frac{\delta n^4}{2\pi} k_0 \sin\vartheta A_{2y}^{ml}(k_0 \cos\vartheta) \end{aligned} \tag{23}$$

4. Numerical examples

The scattering patterns for TE wave incidence are shown in Fig.2. For numerical calculation the wavelength λ, the core radius a, the core spacing h, the number of cores N, the refractive indices of core and cladding n_1 and n_2 and the angle of incidence ϑ_i are chosen as $\lambda = 0.633\mu m$, $a = 2.5\mu m$, $h = 8\mu m$, $N = 2$, $n_1 = 1.457$, $n_2 = 1.437$ and $\vartheta_i = 45(deg)$. F_1^m and F_2^{ml} represent the single scattering by the core m and the double scattering by the cores m and l, respectively. Since radii of two cores are equal $|F_1^1| = |F_1^2|$ and $|F_2^{11}| = |F_2^{22}|$. The infinite integral in the second order term is replaced by the integral over a finite interval and it is evaluated numerically. The second order term is sufficiently small compared with the first order term and can be ignored.



(a) F_1^1 and F_2^{11}. (b) F_2^{11} and F_2^{12}.

Fig2. Scattering patterns for TE wave incidence.

5. Conclusions

The scattering of a TE plane wave from the end-face of a waveguide system has been treated by solving straightforwardly a wave equation for the electric field. The scattered field has been expanded into the perturbation series and the first and second order terms of the series have been analytically derived. It has been shown that the second order term is sufficiently small compared with the first order term and can be ignored.

6. References

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