

# FDTD Analysis of Lightning Electromagnetic Pulses Considering Topography and Presence of Grounded Strike Object

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## Abstract

Azimuthal magnetic fields at a relatively far distance (50 km), generated by a lightning strike to the top of mountain and to the top of an 80-m-high grounded object located on the top of mountain, have been analyzed using the 2D finite-difference time-domain (FDTD) method in the cylindrical coordinate system. Also, the influence of the presence of a mountain range located between the lightning strike point and field observation point on the azimuthal magnetic field has been studied. Azimuthal magnetic fields associated with lightning strike to the top of mountain are higher than those associated with the corresponding strike to a flat ground for both fast- and slow-front lightning currents. The presence of an 80-m-high strike object further increases magnetic field peaks for higher ground conductivity ( $\geq 1000$  mS/m) and shorter current risetime ( $\leq 1$   $\mu$ s). The field enhancement for the case of lightning strike to a mountain decreases with decreasing the ground conductivity. The shielding effect due to a 1-km-high mountain range located between the lightning strike and observation points is insignificant for relatively-slow-front currents.

## 1. Introduction

Modern lightning locating systems provide lightning return-stroke peak currents estimated from measured electric or magnetic field peaks. Direct measurements of lightning currents on tall grounded objects are used for testing the validity of field-to-current conversion equations. Baba and Rakov [1] have derived far-field-to-current conversion factors for lightning strikes to tall grounded objects for (a) the initial peak current at the object top, (b) the largest peak current at the object top (due to the reflection at the object bottom), and (c) the peak current at the object bottom. The correction factor, which is defined as the ratio of a field-to-current conversion factor for lightning strike to a tall object to that for the corresponding lightning strike to flat ground (given as  $2\pi cd/v$  for magnetic field, where  $d$  is the distance between lightning strike and observation points), derived for the 553-m CN Tower for case (b) is given as  $f_{\text{tall\_top}} = [1 + \rho_{\text{bot}}(1 + \rho_{\text{top}})] v/(v+c)$ , where  $\rho_{\text{bot}}$  is the current reflection coefficient at the bottom of the tall object,  $\rho_{\text{top}}$  is the current reflection coefficient at the top of the tall object for upward-propagating waves,  $v$  is the return-stroke wavefront speed, and  $c$  is the speed of light. For  $\rho_{\text{bot}} = 0.9$ ,  $\rho_{\text{top}} = -0.67$  (these correspond to the lightning-channel equivalent impedance of  $1000 \Omega$  and the tall-object characteristic impedance of  $200 \Omega$ ), and  $v=c/3$ , the correction factor is  $f_{\text{tall\_top}} = 0.32$ , which is close to the observed ratio (0.38 on average) of the peak current measured directly near the tower top and corresponding NALDN-reported peak current [2].

In mountainous countries or areas, lightning are likely to frequently strike a tree or a grounded object located on the top of mountain. For a lightning strike to the top of mountain or the top of a grounded object on a mountain, far fields may be enhanced, similar to the case of lightning strike to a tall grounded object on flat ground. On the other hand, the presence of a mountain range between the lightning strike point and field observation point may cause reduction of the field magnitude owing to shielding effect.

In this paper, azimuthal magnetic fields, generated by lightning strikes to the top of mountain and to the top of a grounded object located on a mountain, are analyzed using the 2D finite-difference time-domain (FDTD) method [3] in the cylindrical coordinate system. Also, the influence of the presence of a mountain range, located between the lightning strike point and field observation point, on the magnitude of azimuthal magnetic field is studied.

## 2. Models

Fig. 1 shows the representation of a lightning strike to flat ground, to be analyzed using the 2D FDTD method in the cylindrical coordinate system. The working space of  $70 \text{ km} \times 36 \text{ km}$  is divided uniformly into  $5 \text{ m} \times 10 \text{ m}$  rectangular cells. The time increment is set to  $14.8 \text{ ns}$ , which fulfills the Courant stability condition. The ground thickness (between the ground surface and the bottom absorbing boundary) is set to  $1 \text{ km}$ . The upper, bottom, and right-side boundaries are set by specifying Liao's second-order absorbing boundary condition [4]. Lightning channel is represented by a vertical conductor with  $0.5\text{-}\Omega/\text{m}$  distributed series resistance and additional  $7.5\text{-}\mu\text{H}/\text{m}$  distributed inductance that is excited at its bottom by a lumped voltage source [5]. This representation of lightning return stroke is referred to as the RL model in this paper. The output voltage is set so that the resultant current peak is  $11 \text{ kA}$ . Fig. 2 shows waveforms of current at different heights for the risetime of the channel-base current being set to  $1 \mu\text{s}$ . The current wave propagates upward at speed about  $150 \text{ m}/\mu\text{s}$  and experiences attenuation and distortion. The transmission line (TL) model [6, 7] was also considered.

### 3. Analysis and Results

Fig. 3 shows waveforms of azimuthal magnetic field at a distance of  $50 \text{ km}$  for a lightning strike to flat, perfectly-conducting ground when the risetime of the channel-base current is  $1 \mu\text{s}$ . Note that the other plot shown in Fig. 3 is computed with the transmission-line (TL) model of lightning return stroke [6, 7], in which a current wave propagates upward at speed  $150 \text{ m}/\mu\text{s}$  without any attenuation or distortion. The peak value of the FDTD-computed magnetic field ( $15 \text{ mA}/\text{m}$ ) for the RL model is lower than that for the TL model ( $18 \text{ mA}/\text{m}$ ). Also, the magnetic field computed using the RL model decays faster than that computed using the TL model. The differences are due to the attenuation of upward-propagating current in the RL model. Table 1 summarizes peak values of magnetic field at  $50 \text{ km}$  computed for different values of ground conductivity and current risetime. Note that peak values in the parentheses in Table 1 are computed with the TL model (also in other tables). It appears from Table 1 that the peak value of magnetic field decreases with decreasing ground conductivity.

Fig. 4 shows waveforms of azimuthal magnetic field at  $50 \text{ km}$  for a lightning strike to the top of an  $80\text{-m}$  high grounded object located on flat, perfectly-conducting ground. The grounded object is represented by a vertical perfectly-conducting cylinder of radius  $0.675 \text{ m}$ . Corresponding waveform computed using the TL model extended to include a tall grounded object [8] are also shown in Fig. 4. High-frequency oscillations due to the successive current reflections within the object are observed in the magnetic-field waveforms. Peak values of magnetic field are enhanced by the presence of tall object. Table 2 summarizes peak values of magnetic field at  $50 \text{ km}$  computed for different values of ground conductivity and current risetime. It appears from comparison of Table 2 with Table 1 that the field peak value is enhanced by the presence of  $80\text{-m}$  grounded strike object for relatively fast-front lightning currents ( $0.5$  and  $1 \mu\text{s}$ ) and higher ground conductivity ( $\infty$ ,  $1000 \text{ mS}/\text{m}$ ). The magnetic-field enhancement for the case of the RL model is less significant than that for the TL model, which is probably due to the current attenuation in the RL model.

Table 3 summarizes peak values of magnetic field at  $50 \text{ km}$  computed for a lightning strike to the top of a mountain. The mountain has a shape of truncated cone of height  $1000 \text{ m}$ , upper-face radius  $1000 \text{ m}$ , and lower-base radius  $2000 \text{ m}$ . It appears from Table 3 that the magnetic-field peak is enhanced by the presence of mountain even for relatively slow-front lightning currents ( $5 \mu\text{s}$ ). Table 4 summarizes peak values of azimuthal magnetic field for a lightning strike to the top of a mountain of height  $500$ ,  $1000$ ,  $1500$  and  $2000 \text{ m}$ . Both mountain and ground are set to be perfectly conducting. It appears from Table 4 that the enhancement effect of magnetic field increases slightly with increasing the mountain height.

Table 5 summarizes peak values of azimuthal magnetic field at  $50 \text{ km}$  due to a lightning strike to the top of an  $80\text{-m}$  high grounded object located on the top of  $1000\text{-m}$  high mountain, as a function of ground conductivity and current risetime. It appears from comparison of Table 5 with Table 3 that the presence of an  $80\text{-m}$ -grounded object at the mountain top is significant for higher ground conductivity ( $\infty$ ,  $1000 \text{ mS}/\text{m}$ ) and shorter lightning current risetime ( $0.5$ ,  $1 \mu\text{s}$ ).

Table 6 summarizes peak values of azimuthal magnetic field at  $50 \text{ km}$  due to a lightning strike to flat ground, in the presence of  $1000\text{-m}$  high circular mountain range at a distance of  $25 \text{ km}$  from the strike (coaxial with the vertical lightning channel because of the rotational symmetry of the 2D-cylindrical representation), as a function of ground conductivity and current risetime. It appears from comparison of Table 6 with Table 1 that the field magnitude is slightly reduced in the presence of  $1000\text{-m}$  high mountain range for fast-front lightning currents ( $0.5 \mu\text{s}$ ) owing to the mountain shielding effect, while it is not influenced at all for a relatively slow-front current ( $5 \mu\text{s}$ ).

Table 7 summarizes peak values of azimuthal magnetic field at a distance of  $50 \text{ km}$  for a lightning strike to flat ground, in the presence of a mountain range of height  $500$ ,  $1000$ ,  $1500$ , and  $2000 \text{ m}$  at a distance of  $25 \text{ km}$ . The conductivity of ground and mountain is set to  $1000 \text{ mS}/\text{m}$ . It appears from Table 7 that the field magnitude decreases with increasing the mountain height.

Table 8 summarizes peak values of azimuthal magnetic field at 50 km due to a lightning strike to an 80-m tall object on flat ground, in the presence of a 1-km high mountain range at a distance of 25 km, as a function of ground conductivity and current risetime. It appears from comparison of Table 8 with Table 2 that the magnetic-field enhancement due to the presence of the 80-m grounded strike object decreases owing to the shielding effect of the mountain range between the source and observation points.

### 4. Conclusion

In this paper, azimuthal magnetic fields at a relatively far distance (50 km), generated by lightning strikes to the top of a mountain and to the top of an 80-m-high grounded object located on the top of mountain, have been analyzed using the 2D FDTD method. Also, the influence of the presence of a mountain range located between the lightning strike point and field observation point on the remote azimuthal magnetic field has been studied. The lightning return stroke channel is represented by a vertical conductor with 0.5-Ω/m distributed series resistance and additional 7.5-μH/m distributed inductance that is excited at its bottom by a lumped voltage source. A current wave propagates along this simulated channel at speed of about 150 m/μs with attenuation and distortion. Azimuthal magnetic fields associated with a lightning strike to the top of mountain are higher than those associated with the corresponding strike to flat ground for both fast- and slow-front lightning currents. The presence of an 80-m-high strike object further increases magnetic field peaks for higher ground conductivity ( $\geq 1000$  mS/m) and shorter current risetime ( $\leq 1$  μs). The field enhancement for the case of lightning strike to a mountain decreases with decreasing the ground conductivity. The shielding effect due to a 1000-m high mountain range located between the lightning strike and observation points is insignificant for relatively slow-front currents.

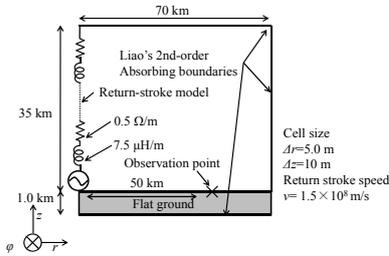


Fig. 1 Lightning return-stroke channel, represented by a vertical conductor with distributed series resistance and additional distributed inductance excited at its bottom by a lumped voltage source.

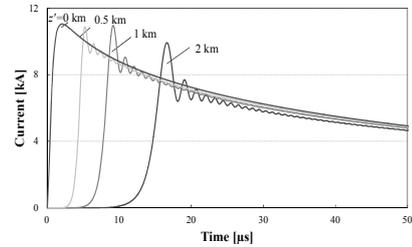


Fig. 2 Waveforms of current at different heights along the simulated vertical channel.

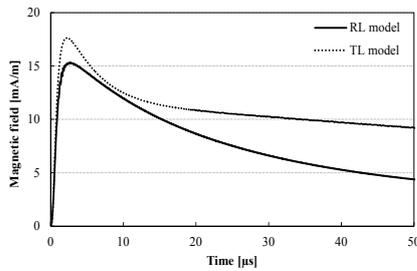


Fig. 3 FDTD-computed waveforms of azimuthal magnetic field at 50 km for a lightning strike to flat, perfectly-conducting ground.

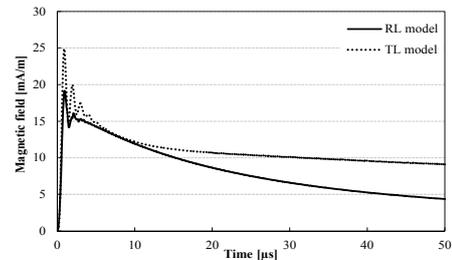


Fig. 4 FDTD-computed waveforms of azimuthal magnetic field at 50 km for a lightning strike to an 80-m tall object on flat perfectly-conducting ground.

Table 1 Peak values of the azimuthal magnetic field at 50 km due to a lightning strike to flat, perfectly-conducting ground, as a function of ground conductivity and current risetime.

|                    |          | Risetime [μs] |         |         |
|--------------------|----------|---------------|---------|---------|
|                    |          | 0.5           | 1.0     | 5.0     |
| $\sigma$<br>[mS/m] | $\infty$ | 16 (18)       | 15 (18) | 15 (18) |
|                    | 1000     | 16 (18)       | 15 (18) | 15 (18) |
|                    | 1.0      | 13 (16)       | 15 (16) | 15 (18) |
|                    | 0.1      | 11 (13)       | 13 (14) | 14 (17) |

Table 2 Peak values of the azimuthal magnetic field at 50 km due to a lightning strike to an 80-m grounded object on flat ground, as a function of the ground conductivity and current risetime.

|                    |          | Risetime [μs] |         |         |
|--------------------|----------|---------------|---------|---------|
|                    |          | 0.5           | 1.0     | 5.0     |
| $\sigma$<br>[mS/m] | $\infty$ | 26 (39)       | 19 (26) | 16 (18) |
|                    | 1000     | 26 (38)       | 19 (25) | 16 (18) |
|                    | 1.0      | 14 (16)       | 15 (17) | 15 (18) |
|                    | 0.1      | 11 (13)       | 13 (14) | 15 (17) |

Table 3 Peak values of azimuthal magnetic field at 50 km due to a lightning strike to the top of 1000-m-high mountain, as a function of ground conductivity and current risetime.

|                    |          | Risetime [ $\mu$ s] |         |         |
|--------------------|----------|---------------------|---------|---------|
|                    |          | 0.5                 | 1.0     | 5.0     |
| $\sigma$<br>[mS/m] | $\infty$ | 20 (24)             | 21 (24) | 22 (25) |
|                    | 1000     | 20 (24)             | 21 (24) | 22 (25) |
|                    | 1.0      | 18 (21)             | 19 (21) | 21 (25) |
|                    | 0.1      | 15 (18)             | 17 (19) | 20 (23) |

Table 5 Peak values of the azimuthal magnetic field at 50 km due to a lightning strike to an 80-m tall object on the top of 1000-m high mountain, as function of ground conductivity and current risetime.

|                    |          | Risetime [ $\mu$ s] |         |         |
|--------------------|----------|---------------------|---------|---------|
|                    |          | 0.5                 | 1.0     | 5.0     |
| $\sigma$<br>[mS/m] | $\infty$ | 30 (43)             | 22 (29) | 22 (25) |
|                    | 1000     | 30 (43)             | 22 (29) | 22 (25) |
|                    | 1.0      | 18 (21)             | 19 (22) | 22 (25) |
|                    | 0.1      | 15 (18)             | 17 (19) | 20 (22) |

Table 7 Peak values of azimuthal magnetic field at 50 km for a lightning strike to flat ground in the presence of a mountain range of height 500, 1000, 1500, and 2000 m at a distance of 25 km. The conductivity of ground and mountain range is set to 1000 mS/m.

|                     |      | Risetime [ $\mu$ s] |         |         |
|---------------------|------|---------------------|---------|---------|
|                     |      | 0.5                 | 1.0     | 5       |
| Mountain height [m] | 500  | 15 (18)             | 15 (18) | 15 (18) |
|                     | 1000 | 14 (17)             | 15 (17) | 15 (18) |
|                     | 1500 | 14 (16)             | 15 (16) | 15 (18) |
|                     | 2000 | 13 (15)             | 14 (16) | 15 (17) |

Table 4 Peak values of azimuthal magnetic field at 50 km for a lightning strike to the top of a mountain of height 500, 1000, 1500, and 2000 m. The ground and the mountain are set to be perfectly conducting.

|                     |      | Risetime [ $\mu$ s] |         |         |
|---------------------|------|---------------------|---------|---------|
|                     |      | 0.5                 | 1.0     | 5.0     |
| Mountain height [m] | 500  | 19 (22)             | 18 (21) | 19 (22) |
|                     | 1000 | 20 (24)             | 21 (24) | 22 (25) |
|                     | 1500 | 21 (24)             | 22 (25) | 24 (28) |
|                     | 2000 | 21 (24)             | 23 (25) | 26 (30) |

Table 6 Peak values of azimuthal magnetic field at 50 km due to a lightning strike to flat ground, in the presence of a 1000-m-high mountain range at a distance of 25 km from the strike, as a function of ground conductivity and current risetime.

|                    |          | Risetime [ $\mu$ s] |         |         |
|--------------------|----------|---------------------|---------|---------|
|                    |          | 0.5                 | 1.0     | 5.0     |
| $\sigma$<br>[mS/m] | $\infty$ | 14 (17)             | 15 (17) | 15 (18) |
|                    | 1000     | 14 (17)             | 15 (17) | 15 (18) |
|                    | 1.0      | 13 (15)             | 14 (16) | 15 (18) |
|                    | 0.1      | 11 (13)             | 12 (14) | 14 (17) |

Table 8 Peak values of azimuthal magnetic field at 50 km due to a lightning strike to an 80-m tall object on flat ground, in the presence of a 1-km high mountain range at a distance of 25 km, as a function of ground conductivity and current risetime.

|                    |          | Risetime [ $\mu$ s] |         |         |
|--------------------|----------|---------------------|---------|---------|
|                    |          | 0.5                 | 1.0     | 5.0     |
| $\sigma$<br>[mS/m] | $\infty$ | 19 (27)             | 16 (19) | 16 (18) |
|                    | 1000     | 19 (27)             | 16 (19) | 15 (18) |
|                    | 1.0      | 13 (15)             | 14 (16) | 15 (18) |
|                    | 0.1      | 11 (13)             | 12 (14) | 14 (16) |

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