

# Physics-based Scattering Model of Rainfall over Sea Surface

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## Abstract

Because of its critical role in tropical storm events, rainfall over sea surface has always been an important subject for remote sensing. Recently, space-borne high-resolution synthetic aperture radar (SAR) has been found to be a potential tool for rainfall observation. However, many interesting phenomenon of rain-wind over sea surface as revealed by SAR images are still not fully understood. This paper attempts to develop a physics-based radiative transfer model to capture the scattering behavior of rainfall over rough sea surface. Raindrop scattering and attenuation is modeled as Rayleigh spherical particle, while rain-induced rough surface is described by the Log-Gaussian ring-wave spectrum. The model is compared with empirical models and measurements.

## 1. Introduction

With SAR images, localized rainfall cells and tropical cyclone can now be observed in a never-before level of details. Many interesting and mysterious phenomenon have been reported and qualitatively analyzed [1,2]. Earlier studies focus on higher bands [e.g. 3], as raindrops are usually believed to be transparent to lower bands such as C/L-band. Now the general consensus is that rain-induced surface roughness has observable impact on the total scattering power even in lower bands.

Some empirical models have been developed from associated remote sensing and weather prediction data [e.g. 4-6]. Empirical models are dependent on the selection of fitting data points, susceptible to data error/noise and may have over-fitting issues. Derived from specific datasets, empirical models may not be generally applicable to different settings, e.g. incident angle, frequency bands, polarization, environment conditions. Thus, in order to better understand and interpret various phenomenon being observed by SAR, and then extract useful information from SAR data, a general physic-based scattering model has to be established.

Besides the volumetric scattering from raindrop itself and the attenuation of wave propagating through rainfall, sea surface alteration caused by raindrop and its associated effects are other major impacts to scattering. It is very difficult to model the complicated air-water interface when coupled with dynamic raindrop-induced splashes and turbulences. The major factors through which rainfall may affect sea surface are generally believed to be [e.g. 1,7]:

- a) Raindrops striking on water surface create ring-waves;
- b) Raindrops striking on water surface create stalks and crowns;
- c) Raindrops generate turbulence in the upper water layer which attenuate the short gravity wave spectrum;
- d) Downsplayed airflow associated with rain event may alter the original wind field and then further alter the sea surface spectrum.

The first factor has been studied via rain experiments and empirical log-Gaussian ring-wave spectra were derived [e.g. 7]. The rest factors have not been well studied and are not completely understood. However, previous literatures suggest that factors such as turbulence will become significant only in high incident angles (e.g. <30deg) [e.g. 1, 5].

This study attempts to establish a physics-based scattering model which consists of the radiative transfer (RT) model of rainfall layer and the rough sea surface model of linearly combined both wind-driven gravity-wave spectrum and the rain-induced ring-wave spectrum. Raindrop is modeled as Rayleigh spherical partible while the empirical D-V spectra and the empirical log-Gaussian spectra [7] are used to describe the wind/rain-impacted sea surface.

## 2. Radiative Transfer Model

Let's assume a locally uniform distribution of rain over rough sea surface. The height of cloud is denoted as  $H$ . Apparently, a ranging cell appeared in SAR image corresponds to the summation of the scattered powers from all

scatterers in the strip perpendicular to radar incidence direction. According to the mapping and projection principle [8], the scattering power of the corresponding cell in SAR image can be written as a sum of all volume scatters and the limited area of surface scatterer.

For an infinitesimal volume along the strip, its scattering contribution is written as [8]:

$$d\mathbf{M}_{vol} = n_0 dR dx dy \exp(-x \tan \theta \boldsymbol{\kappa}^+) \cdot \mathbf{P} \cdot \exp(-x \tan \theta \boldsymbol{\kappa}^-) \quad (1)$$

where  $dR dx dy$  denotes the volume;  $n_0$  denotes number of raindrop particles per unit volume;  $\mathbf{P}$  and  $\boldsymbol{\kappa}$  denotes the phase matrix and extinction matrix of an individual raindrop particle, respectively. The superscript  $\pm$  of  $\boldsymbol{\kappa}$  denotes upward and downward propagation direction, respectively. Note that Mueller matrix, phase matrix and extinction matrix are 4 by 4 real matrix (in bold) which describe the fully polarimetric scattering characteristics.

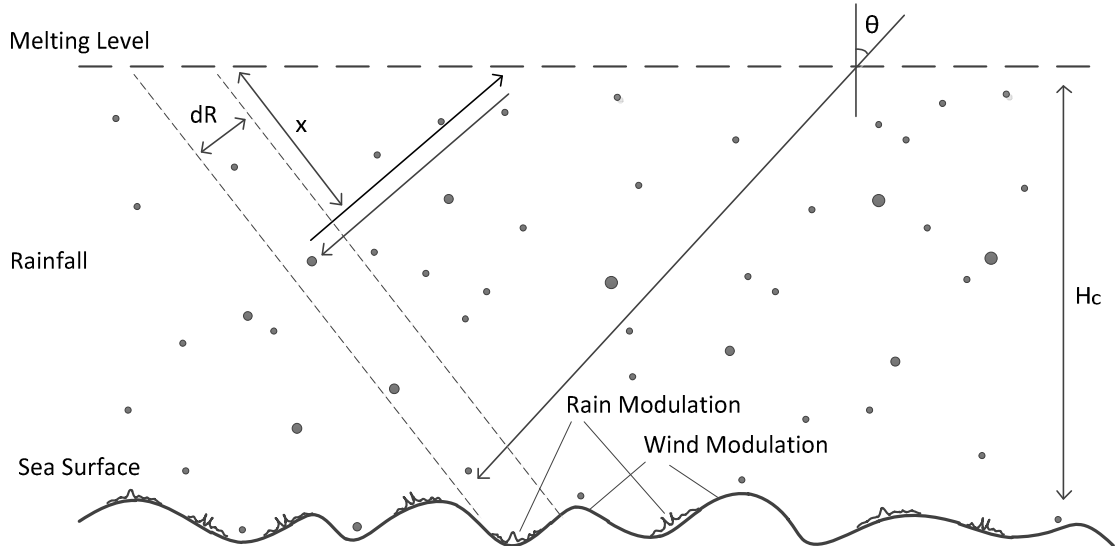
Evaluating the integral along the narrow stripe analytically, it yields

$$\mathbf{M}_{vol} = n_0 dR dy \mathbf{E}^+ \cdot \left\{ \frac{1 - \exp[-H \sec \theta (\beta_i^+ + \beta_j^-)]}{\tan \theta (\beta_i^+ + \beta_j^-)} P'_{ij} \right\}_{4 \times 4} \cdot \mathbf{E}^{-1} \quad (2)$$

According to the mapping and projection principle, the corresponding small area of the underlying sea surface contributes to the SAR image as:

$$\mathbf{M}_{surf} = dR dy \sec \theta \exp(-H \sec \theta \boldsymbol{\kappa}^+) \cdot \mathbf{R} \cdot \exp(-H \sec \theta \boldsymbol{\kappa}^-) \quad (3)$$

where  $\mathbf{R}$  is the rough surface Mueller matrix of an unit area.



**Figure 1 Mapping and projection model for rain over sea surface.**

Consider that the rough sea surface is modulated by a linear sum of two major parts of forces, i.e. wind and rain. Following from the integral equation method (IEM) [8] for randomly rough surface, the Mueller matrix is a linear function of the spectral density function,  $W$ , of rough surface, i.e.

$$\mathbf{R}(W_{wind} + W_{rain}) = \mathbf{R}(W_{wind}) + \mathbf{R}(W_{rain}) \quad (4)$$

where  $W_{wind}$ ,  $W_{rain}$  denotes the wind-driven spectral density function and the rain-modulated spectral density function, respectively.

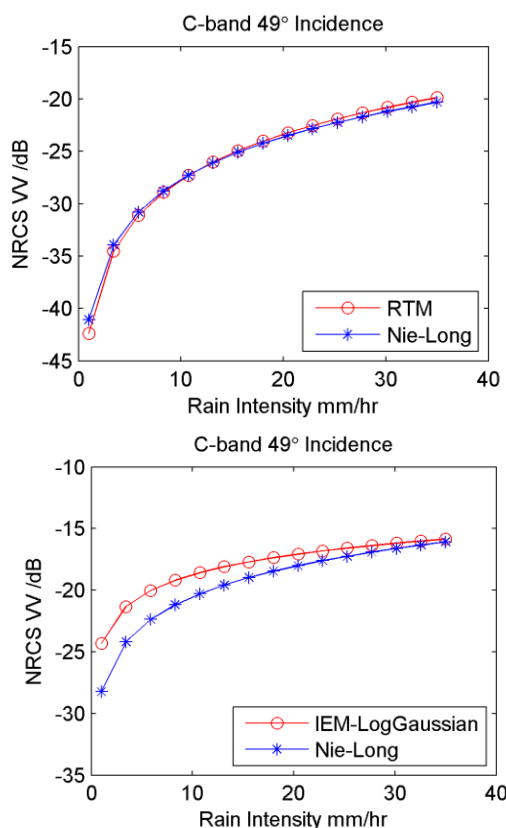
Ring wave is the dominant scatterer at about 30deg incident angle, while stalks become dominant at grazing angles. Since most SAR observations are taken at 30-40deg incidences, we focus on the scattering from ring waves for now. Bliven et al. (1997) and Lemaire et al. (2002) conducted experiments to measure the surface spectrum of water surface perturbed by simulated rain. Log-Gaussian type function is used to approximate the spectral density function of rain roughened surface.

The D-V spectrum is used with IEM model to model wind-drive sea surface scattering.

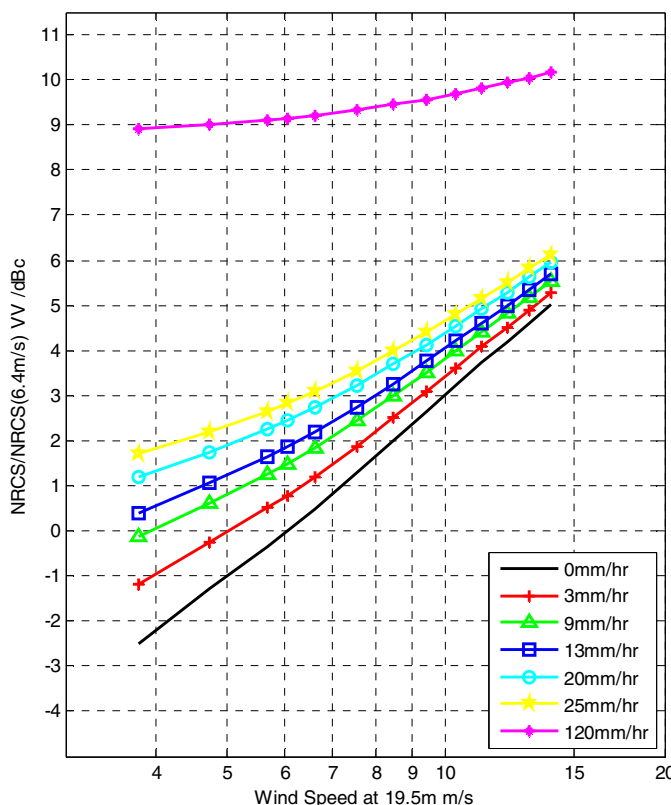
### 3. Preliminary Results

Some preliminary results are given below. In Figure 2, the volumetric scattering from raindrops are calculated and compared with respect to the empirical model given by Nie and Long [5]. The result matches well under different rain intensities. Note that the rainfall height is chosen as uniform 4km. According to [9], most rain types have the melting layer located at around 5km. Considering the effect of raindrop density decreases as altitude increases, we choose uniform 4km as an approximation of the real scenario. We compare our results of rain-induced surface scattering modeled by IEM and the Log-Gaussian ring-wave spectrum to Nie-Long empirical model. The comparison indicates a discrepancy up to 5dB in some cases. The Nie-Long prediction increases faster as rain gets stronger than our model, where the reason is still unknown.

Moore et al. (1979) gives measurements at Ku-band of simulated rain over wind-driven sea surface. Using IEM with incoherent wind-driven rough surface described by the D-V spectrum and rain-induced rough surface described by the Log-Gaussian ring-wave spectrum, we are able to plot the results at the same settings as Fig. 1 of [3]. It appears that the increment of backscattering due to rain-induced roughness is well captured by our model. But for extremely high rain rates (120mm/hr), our prediction is about 2-3dB higher. This might be attributable to the ignorance of the damping effect of strong rain applying to wind-driven roughness as well as other factors that are not included in this model.



**Figure 2 Top panel: Raindrop volumetric scattering: RTM vs. Nie-Long empirical model. Bottom panel: Rain-induced ring-wave backscattering: IEM vs. Nie-Long empirical model.**

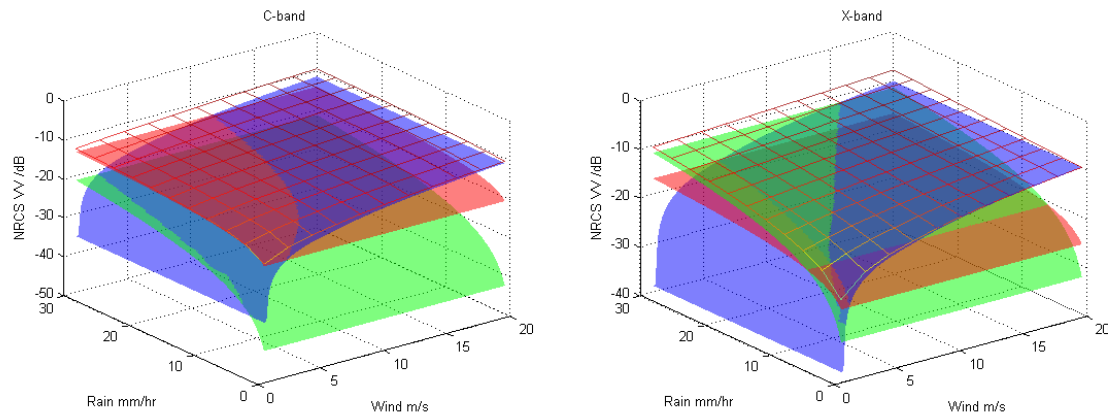


**Figure 3 Our model simulated hybrid wind-rain induced sea surface backscattering at Ku-band. The same settings were used in fig.1 of [3].**

Using the developed RTM model, we can now easily analyze the dependencies of SAR signal on various factors, namely, rain rate, wind speed, frequency etc. In Figure 4, we plot the dependencies of total scattering, wind-induced surface scattering (without rain attenuation), rain-induced surface scattering (with rain attenuation) and raindrop volumetric scattering as 3D curved surfaces on the plane of rain rate vs. wind speed, and for C and X frequency bands, respectively. The incident angle is selected as 40deg, while the uniform rainfall height is chosen as 4km. The following observations can be made:

At C-band, the total scattering is mainly consisted of wind-induced surface scattering and rain-induced surface scattering, while raindrop volumetric scattering is negligible. A clear boundary can be found along which the two types of scattering compete for dominance.

At X-band, raindrop volumetric scattering becomes stronger than rain-induced surface scattering for rain rate over  $\sim 6$  mm/hr. Hence, raindrop volumetric scattering (RVS) replaces rain-induced surface scattering's (RSS) role to compete with wind-induced surface scattering (WSS) for dominance. This is the case for Ku-band as well (not shown).



**Figure 4** Dependencies of different scattering contributions on wind speed (down-wind) and rain rate. Legend: Mesh surface: Total scattering; Blue surface: Wind-induced surface scattering; Red surface: Rain-induced surface scattering; Green surface: Raindrop volumetric scattering.

## 4. Conclusion

A physics-based scattering model for rainfall over sea surface is established taking into account raindrop volumetric scattering and attenuation, rain-induced ring-wave scattering and wind-driven sea surface scattering. It is preliminarily validated against empirical models and measurements at C and Ku bands. Numerical analyses reveal that rain-induced surface scattering plays important role in radar backscattering at C-band, while raindrop volumetric scattering becomes dominant at higher bands such as X and Ku bands.

## 5. References

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