An Improved Omni-directional Azimuth Angle Measurement Algorithm of VHF Radar

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Abstract

Attributing to the small array aperture of azimuth omni-directional VHF radar, the mutual coupling between antennas can not be neglected so that extra measurement error may be produced. To resolve this problem, an improved unambiguous algorithm is proposed. By simultaneously taking use of mode -1, mode 0 and mode 1 excited by phase mode in angle measurement, the proposed algorithm can eliminate the effect of mutual coupling and acquire the unambiguous value of the azimuth angle. Theoretical analyses and experimental results all demonstrate the effectiveness of the proposed algorithm.

1. Introduction

More and more attention has been attracted to new VHF radar systems for their inherent advantage in anti-stealthy and anti-ARM [1, 2]. Compared with the well known Melissa radar and synthetic impulse and aperture radar (SIAR), "Owl" radar, RLS radar and KB radar are much more agile and less expensive. Therefore, they are appropriate to be deployed in a large scale for air defense and early warning. Angle measurement algorithms of these omni-directional radars have been studied by many researchers. Omni-directional angle measurement algorithm for arrays with four elements were introduced in [3, 4] but there were no further analysis about factors attributing to the immanent error. Some simulation results were listed in [5] to demonstrate that the immanent error is related with some array parameters but analytical expressions were not derived. It is pointed out that mutual coupling might cause measurement error in [6] but how to decrease this error was not mentioned. A new algorithm was proposed in [7] to eliminate the effect of mutual coupling to angle measurement but neglected the occurrence of ambiguity. In this paper, an improved omni-directional angle measurement algorithm is proposed. With the proposed algorithm, unambiguous angle measurement can be accomplished no matter how serious the mutual coupling is.

2. Principle of omni-directional angle measurement

Uniform circular array with *N* elements is illustrated in Figure 1. The radius of the array is r. The azimuth angle and elevation angle of a unknown target are θ_A and θ_E respectively. Then the range difference between target-to-phase center and target-to-the *k*th element can be expressed as $r\cos\theta_E\cos(\theta_A - 2\pi k/N)$ approximately. If the transmitted signal is a narrow-band signal, the received signal of the *k*th element can be expressed as

$$s_k(t) \approx s_0(t)\exp\left[\frac{\mathrm{j}2\pi r}{\lambda}\cos\theta_E\cos\left(\theta_A - \frac{2\pi(k-1)}{N}\right)\right] \tag{1}$$

where $k = 1 \sim N$, λ is the wavelength, $s_0(t)$ is the received signal of the phase center.

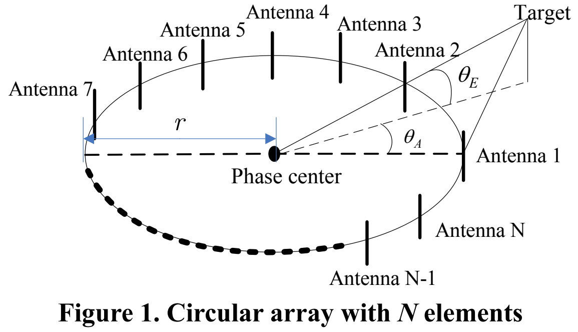

Figure 1. Circular array with *N* elements

The spatial fast Fourier transform (FFT) of the received signals of *N* elements is written as

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$$F_l=\sum_{k=0}^{N-1}s_{k+1}(t)\,\mathrm{e}^{-\mathrm{j}2\pi kl/N}\quad,\quad l=0\sim N-1 \tag{2}$$

Because $\exp(\mathrm{j}x\sin\theta)=\sum_{n=-\infty}^{+\infty}J_n(x)\exp(\mathrm{j}n\theta)$ where $J_n(x)$ is the nth order Bessel function of the first kind and $\sum_{k=0}^{N\text{-}1}\exp\left[\mathrm{j}\dfrac{2\pi k(n\text{-}l)}{N}\right]=\begin{cases}N, & n-l=QN\\ 0, & \text{else}\end{cases}$ where Q is an integer，it can be derived from (2) that

$$F_l=Ns_0(t)\sum_{Q=-\infty}^{+\infty}\mathrm{j}^{l+QN}J_{l+QN}\left(2\pi r\cos\theta_E/\lambda\right)\exp\left[-\mathrm{j}(l+QN)\theta_A\right] \tag{3}$$

Define $\alpha=2\pi r\cos\theta_E/\lambda$. According to the decay property of Bessel function, it can be obtained that $J_{l+QN}(\alpha)\to 0$ when $|l+QN|>\alpha$. Hence, if $2\pi r/\lambda<N/2$ is satisfied, (3) can be simplified as

$$F_l\approx Ns_0(t)J_l(\alpha)\,\mathrm{j}^l\exp\left(-\mathrm{j}l\theta_A\right) \tag{4}$$

In conventional omni-directional angle measurement algorithm, only F_{-1} or F_1 is used. However, the initial phase of $s_0(t)$ is actually unknown. Therefore, F_{-1} or F_1 is often utilized in associate with F_0, which is expressed as

$$F_1/F_0\approx\mathrm{j}\exp\left(-\mathrm{j}\theta_A\right)J_1(\alpha)/J_0(\alpha) \tag{7}$$

$$F_{-1}/F_0\approx-\mathrm{j}\exp\left(\mathrm{j}\theta_A\right)J_1(\alpha)/J_0(\alpha) \tag{8}$$

3. Unambiguous angle measurement with mutual coupling

In order to decrease the gain loss of the array and ensure the array pattern is omni-directional, the array aperture is generally small so that the mutual coupling is serious. Measurement error may be large if (7) or (8) is still adopted. To solve this problem, we may take use of F_{-1}, F_0 and F_1 simultaneously.

The mutual coupling matrix of a uniform circular array is a symmetrical Toeplitz matrix. When the element amount is even, the mutual coupling matrix $\boldsymbol{Z}$ can be given as

$$\boldsymbol{Z}=\left(z_{ij}\right)_{N\times N}=\begin{bmatrix} c_0 & c_1 & \cdots c_{N/2} & c_{N/2-1}\cdots & c_1\\ c_1 & c_0 & \cdots c_{N/2-1} & c_{N/2} & \cdots\ c_2\\ \vdots & \vdots & \cdots\ \vdots & \vdots & \vdots\ \vdots\\ c_{N/2} & c_{N/2-1} & \cdots c_0 & c_1 & \cdots c_{N/2-1}\\ c_{N/2-1} & c_{N/2-2} & \cdots c_1 & c_0 & \cdots c_{N/2-2}\\ \vdots & \vdots & \vdots\ \vdots & \vdots & \vdots\ \vdots\\ c_1 & c_2 & \cdots c_{N/2} & c_{N/2-1} & \cdots c_0 \end{bmatrix} \tag{9}$$

Then the received signal of the k th element can be rewritten as

$$sc_k(t)=s_0(t)\sum_{m=0}^{N-1}z_{k,m+1}\exp\left[\mathrm{j}\alpha\cos\left(\theta_A-2\pi m/N\right)\right] \tag{10}$$

Substituting (10) into (2), we have

$$F_l=s_0(t)\sum_{k=0}^{N-1}z_{k+1,m+1}\,\mathrm{e}^{-\mathrm{j}2\pi kl/N}\sum_{m=0}^{N-1}\exp\left[\mathrm{j}\alpha\cos\left(\theta_A-2\pi m/N\right)\right] \tag{11}$$

It is not hard to find that if taking the first column as a reference, the other N-1 columns may be obtained by ordinal cyclic shift of the first column. Meanwhile, considering the cyclic shift property of discrete Fourier transform, we can get

$$\sum_{k=0}^{N-1}z_{k+1,m+1}\,\mathrm{e}^{-\mathrm{j}2\pi kl/N}=\left(\sum_{k=0}^{N-1}z_{k+1,1}\,\mathrm{e}^{-\mathrm{j}2\pi kl/N}\right)\mathrm{e}^{-\mathrm{j}2\pi ml/N} \tag{12}$$

Substituting (12) into (11) and combining (7) and (8), we have

$$F_1/F_0\approx\mathrm{j}\exp\left(-\mathrm{j}\theta_A\right)J_1(\alpha)\sum_{k=0}^{N-1}z_{k+1,1}\,\mathrm{e}^{-\mathrm{j}2\pi k/N}/[J_0(\alpha)\sum_{k=0}^{N-1}z_{k+1,1}] \tag{13}$$

$$F_{-1}/F_0\approx-\mathrm{j}\exp\left(\mathrm{j}\theta_A\right)J_1(\alpha)\sum_{k=0}^{N-1}z_{k+1,1}\,\mathrm{e}^{\mathrm{j}2\pi k/N}/[J_0(\alpha)\sum_{k=0}^{N-1}z_{k+1,1}] \tag{14}$$

According to the first column of the mutual coupling matrix, we may get

$$\sum_{k=0}^{N-1}z_{k+1,1}\,\mathrm{e}^{-\mathrm{j}2\pi k/N}=\sum_{k=0}^{N-1}z_{k+1,1}\,\mathrm{e}^{\mathrm{j}2\pi k/N}=c_0-c_{N/2}+2\sum_{k=1}^{N/2-1}c_k\cos\left(2\pi k/N\right) \tag{15}$$

Define $\sum_{k=0}^{N-1} z_{k+1,1} \mathrm{e}^{-\mathrm{j}2\pi k/N} / \sum_{k=0}^{N-1} z_{k+1,1} = A\exp(\mathrm{j}\theta_C)$ and $B = \mathrm{j}AJ_1(\alpha)/J_0(\alpha)$. Then (13) and (14) can be respectively simplified as

$$F_1 / F_0 \approx B\exp\left[-\mathrm{j}\left(\theta_A - \theta_C\right)\right] \tag{16}$$

$$F_{-1} / F_0 \approx -B\exp\left[\mathrm{j}\left(\theta_A + \theta_C\right)\right] \tag{17}$$

From (16) and (17), we can obtain $\theta_1 = \theta_A - \theta_C$ and $\theta_2 = \theta_A + \theta_C$ respectively. If $|\theta_C| < \pi/2$, $\theta_A = (\theta_1 + \theta_2)/2$ is just the azimuth angle of the target. Otherwise there may be ambiguity of π as shown in Figure 2.

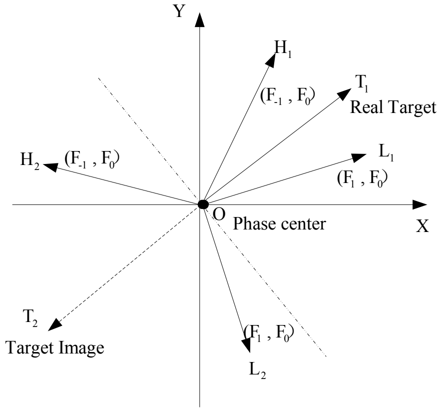

Figure 2. Ambiguity of angle measurement

It can be observed from Figure 2 that if $|\theta_C| < \pi/2$, the real angle $\angle T_1OX$ may be obtained by use of $\angle H_1OX$ and $\angle L_1OX$. If $|\theta_C| > \pi/2$, $\angle L_2OX$ and $\angle H_2OX$ are what we get from (16) and (17) respectively. As a result, we obtain $\angle T_2OX$ which is the angle of target's image. And the difference of $\angle T_1OX$ and $\angle T_2OX$ is exactly π. However, it is not that hard to satisfy the condition $|\theta_C| < \pi/2$ in engineering.

4. Experimental results

In order to demonstrate that the measurement error entailed by mutual coupling can be limited to be less than $\pi/2$ in practical engineering, we have done an experiment with a simplified radar system. In this radar system, there are four elements for simplicity. The polarization of each antenna is vertical, the transmitted signal is a continuous signal with carrier frequency 148MHz, and the radius-to-wavelength ratio is 0.25. The transmitting antenna is fixed and the receiving antennas can be rotated. During the experiment, we rotate the receiving antennas two circles. The total time of the rotation is 240 time frames and each time frame is 0.5 second.

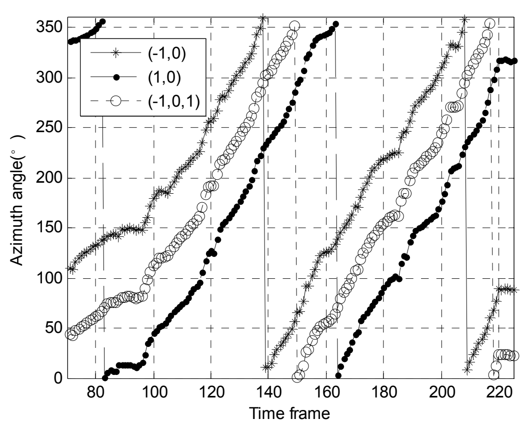

Figure 3. Comparison of three algorithms

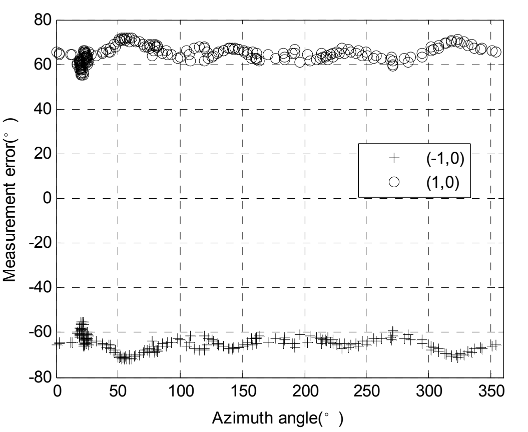

Figure 4. Error entailed by mutual coupling

Figure 3 illustrates the comparison of three algorithms. We can see that the measured angle by use of either F_{-1} and F_0 or F_0 and F_1 is quite different from that obtained via F_{-1}, F_0 and F_1. The difference is just the measurement error produced by mutual coupling. More specific difference is shown in Figure 4. It can be observed that the measurement error caused by mutual coupling is approximately 65° which is less than 90°. It means that the assumption in section 3 is reasonable and the proposed algorithm is feasible in engineering.

5. Conclusion

In order to decrease the gain loss of the array and ensure the array pattern is omni-directional, the array aperture is generally small so that the mutual coupling is serious. As a result, angle measurement error may be large. Accordingly, a new algorithm is proposed. Compared with conventional omni-directional angle measurement algorithm, mode -1, mode 0 and mode 1 excited by phase mode are used simultaneously for the elimination of mutual coupling effect and the solution of ambiguity. Theoretical derivation and measured data both demonstrate the correctness and feasibility of the presented algorithm.

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