Analysis of the influence of the position of the mobile on SAR induced using polynomial chaos decomposition

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Abstract

The numerical analysis of the uncertainty of the Specific Absorption Rate (SAR) induced by a mobile phone requests large number of simulations. Today the FDTD is still time consuming even if large efforts have been to improve it. In order to overcome these difficulties, statistical process has been introduced to develop a metamodel technique which parameterizes the SAR 10g. The appropriate stochastic metamodel and the PC coefficients are estimated by ordinary least-square regression. This recent metamodeling technique appears to be a relevant approach in order to reduce the number of model’s evaluations.

1. Introduction

As reported by several studies [1] the Specific Absorption Rate (SAR) distribution in brain tissues and the maximum of 10 grams averaged SAR induced by a mobile phone RF depend on the position of the mobile phone relatively to the head. Numerical simulations using the Finite-Difference Time-Domain (FDTD) method [2] are increasingly used to assess the SAR. In spite of advances in High Performance Calculation the model complexity, the large number of parameters, it is still impossible to characterize the SAR distribution using method such as Monte Carlo. A preferable strategy is to rely on approximation models which are referred to as metamodels. These metamodels replace the expensive simulation. Metamodeling techniques can be used for design evaluation and optimization in the dosimetry and so specifically in the SAR’s computation. The aim of this study is to conceive a SAR’s statistical model induced in the heterogeneous head model with the different positions of the mobile phone.

2. The experimental evaluations

In this paper, the specific absorption rate (SAR) in scaled human head model is analysed to study possible differences between SAR in the head while using a mobile phone in different positions. A series of FDTD simulations are carried out to evaluate the field penetration into the heterogeneous head and to calculate the SAR at 900 MHz. The cell phone is modelled using patch antennas. The generic mobile phone is described by [3].
2.1 Description for testing position

The first step consists on maintaining the phone in the “cheek” position (EN 62209-1) where the phone is touching the ear and contacting the cheek of the phantom, then pivoting the mobile against the pinna and moving it from the mouth by an angle \( \theta \) between 0 and 30 degrees and vertically by a second angle \( \phi \). In second step, we translate the phone horizontally in the plane ear mouth with \( \Delta x \) and vertically with \( \Delta y \) in the plane orthogonal to this plane. The Table 2 shows the intervals of the four parameters used for positioning the phone inside the head.

### Table 1: Intervals of the four input random

<table>
<thead>
<tr>
<th>( x_1 = \theta )</th>
<th>( x_2 = \phi )</th>
<th>( x_3 = \Delta x )</th>
<th>( x_4 = \Delta y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0°  30°]</td>
<td>[-15° 15°]</td>
<td>[5    30]mm</td>
<td>[-10  10]mm</td>
</tr>
</tbody>
</table>

The SAR evaluation is conducted at 900 MHz under test at 80 positions of the phone on right side of the head. In this study we are interested in calculating the maximum of 10-g averaged SAR.


Various well-known methods have been studied for uncertainty propagation and sensitivity analysis. They are mainly based on intensive Monte Carlo simulation, which may lead to intractable calculations in case of a numerical SAR assessment. To overcome this difficulty, it is possible to substitute the model response by an analytical approximation, called metamodel. Metamodelling techniques consist in building an analytical approximation \( \hat{M} \) of the deterministic model function \( Y = M(X) \). Such an approach takes into account a statistical distribution of the input random variables, which is considered though in the framework of uncertainty propagation. Indeed, it is recalled that the input parameters are modelled by random variables. The purpose of the metamodelling approach described in the present work, aims at obtaining an accurate approximation of the model function \( M \) at any point of the domain of variation of the input parameters \( X \).

To select a suitable set of \( X \), the Latin Hypercube Sampling (LHS) is used. The LHS technique aims at generating pseudo-random numbers that are more representative of the joint distribution of the input random vector \( X \) than those generated by Monte Carlo simulation. This approach method is a stratified sampling technique where the random variable distributions are divided into equal probability intervals. The LHS method generates a sample size \( N \) from the \( n \) variables. A \( 1/N \) probability is randomly selected from within each interval that is partitioned into \( N \) non-overlapping ranges for each basic event [4]. In this study LHS is used to select randomized value for the input parameters.

To build the metamodel, let us consider now a physical model \( M \) which can be referred to as a black box function, represented by a deterministic mapping \( Y = MX \) by uncertainty propagation through the model \( M \), as illustrated in Figure 2.

![Figure 2: Meta model process for uncertainty propagation](image)
In our case the model function depends on a set of M input parameters denoted by \( X = \{ x_1, x_2, x_3, x_4 \} \) which present the two angles \( (x_1, x_2) \) and two translations \( (x_3, x_4) \), and returns a set of output quantities \( Y = \{ y_1, ..., y_{80} \} \) referred to the vector of the SAR10g calculated. The vector \( Y \) called the random model response in the sequel. The polynomial chaos (PC) expansion is well suited to this purpose[5]. The PC provides an explicit representation in terms of the input random variables \( (x_1, ..., x_4) \) and describes the approximation of the unknown random response of a model in a suitable finite-dimensional basis \( (\psi(X))_{k \in \mathbb{N}^p-1} \) as follows:

\[
Y \approx M(X) \equiv \sum_{k=0}^{p-1} \beta_k \psi_k(X) 
\]

The \( \beta_k \)'s are unknown deterministic coefficients, and the \( \psi(X) \)'s are multivariate polynomials.

In the present work, we are interesting in spectral expansion on to bases made of orthogonal polynomials, commonly referred to as polynomial chaos (PC) expansion. In this setup, characterizing the model response is equivalent to computing the deterministic coefficients \( \beta_k \).

Various methods have been used for building up an approximation of the model response on to a polynomial chaos basis. Since the polynomial are part of a base a projection method can be used to estimate the coefficient but such approach request large number of simulation (even if well below the monte carlo approach). In the present work, the regression-based PC approach has been retained among the various non intrusive methods since it is minimizing the computational [6].

4. Estimation of the coefficient of the model

The regression approach aims at computing the PC coefficients that minimize the mean-square error of approximation of the model response \( Y = M(X) \) by the PC metamodel. In the following we use the vector notation :

\[
\beta_k = \{ \beta_{k0}, ..., \beta_{kp-1} \}^T 
\]

\[
\psi(X) = \{ \psi_{k0}(X), ..., \psi_{kp-1}(X) \}^T 
\]

The regression problem may be cast as follows: Find \( \hat{\beta} \) that minimizes

\[
\Delta y \equiv 1E \left[ \left( \beta_k^T \psi(X) - M(X) \right)^2 \right] 
\]

Regression appears to be a relevant approach in order to reduce the number of model evaluations. Indeed the number of terms strongly increases with both the number of input random variables \( M \) and the PC degree \( p \). For numerical application, the number of input variable \( M \) is set equal to 4 and we select \( X = \{ x_1, x_2, x_3, x_4 \} \), in regression context, while 80 experimental associated with for dimensions \( (M=4) \), the maximum degree \( p \) will be set equal to for.

The number \( P \) of the coefficients to estimate may be cast as:

\[
P = \sum_{k=0}^{p} \binom{M}{M+k-1} = \frac{(M+p)!}{M!p!} 
\]

<table>
<thead>
<tr>
<th>( M=4 )</th>
<th>( p=1 )</th>
<th>( p=2 )</th>
<th>( p=3 )</th>
<th>( p=4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15</td>
<td>35</td>
<td>70</td>
<td></td>
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</table>

The convergence of the various PC approximations is observed that for \( p=4 \), as shown in Figure 4.
Degree (p=4) is sufficient to properly represent the model response allowing a computation of a 70 coefficients at a low computational coast compared to the usual PC.

4. Conclusion

After a careful review of the convergence estimates and computational cost associated with the regression method, it appears that the regression-based estimates should be the most accurate for a given number of performed model evaluations. This techniques provides a quite interpretable metamodel, allowing after a straightforward post-processing such as the computation of sensitivity indices which allows to deduce the most important parameter in the evaluation of SAR in the head [7].

5. References

1. Dimbylow P J 1993 FDTD calculations of the SAR for a dipole closely coupled to the head at 900 MHz and 1.9 GHz Phys. Med. Biol. 38 361