Whistler Wave Excitation by a Pulsed Loop Antenna Located in a Cylindrical Duct with Enhanced Plasma Density

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Abstract

Pulsed radiation from a loop antenna located in a cylindrical duct with enhanced plasma density is studied. An expression for the radiated energy is derived and its distribution over the spatial and frequency spectra of the excited waves as a function of the source and duct parameters is analyzed. Numerical results referring to the case where the frequency spectrum of the current is concentrated in the whistler range are reported. It is shown that under ionospheric conditions, the presence of an artificial duct with enhanced density can lead to a significant increase in the energy radiated from a pulsed loop antenna compared with the case where the same source is immersed in the surrounding uniform magnetoplasma.

1. Introduction

Electromagnetic radiation from monochromatic sources immersed in homogeneous and inhomogeneous magnetized plasmas has received much careful study and there are many accounts of it (see, e.g., [1, 2] and references therein). Over the past decade, there has been shown a substantial degree of interest in the excitation and propagation of nonmonochromatic signals in a magnetoplasma [3–5]. This interest has been motivated by the importance of transient phenomena for propagation of electromagnetic waves through the magnetosphere and the ionosphere, as well as for plasma diagnostics using pulsed signals launched from antennas on spacecraft. Much previous theoretical work on the subject is focused on studying the fields and radiation characteristics of pulsed sources emitting electromagnetic waves to a homogeneous magnetoplasma [5]. However, there exists a very little theory of the radiation from nonmonochromatic sources located in cylindrical magnetic-field-aligned density irregularities, commonly known as density ducts. Note that such ducts can arise due to various nonlinear effects near electromagnetic sources in a magnetoplasma and are capable of guiding whistler-range waves, which play an important role in many promising applications.

It is the purpose of the present paper to discuss energy characteristics of the radiation from a pulsed loop antenna located in an enhanced-density duct that is surrounded by a uniform cold magnetoplasma such as exists in the Earth's ionosphere. The emphasis will be placed on the case where the frequency spectrum of the antenna current is concentrated in the whistler range.

2. Formulation of the Problem and Basic Equations

We consider a circular loop antenna placed coaxially in a cylindrical duct of radius a. The duct is aligned with the z axis of a cylindrical coordinate system (ρ, ϕ, z). Parallel to this axis is a uniform dc magnetic field $\boldsymbol{B}_0 = B_0\hat{\boldsymbol{z}}_0$. The plasma density is equal to $\tilde{N}$ inside the duct, and to N_a in the ambient uniform plasma surrounding the duct. The electric current density of the antenna is specified as

$$\boldsymbol{J}(\boldsymbol{r}, t) = \hat{\boldsymbol{\phi}}_0 I_0 \delta(\rho - b)\delta(z)\chi(t), \tag{1}$$

where I_0 is the amplitude of total current, b is the antenna radius ($b < a$), δ is a Dirac function, and $\chi(t)$ is a dimensionless function describing the current behavior in time. The function $\chi(t)$ has the maximum value equal to

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unity and is assumed nonzero in the time interval $0 < t < \tau$, where τ is the current pulse duration, which in principle can be infinitely long. The plasma is described by the permittivity tensor $\boldsymbol{\varepsilon} = \varepsilon\hat{\boldsymbol{\rho}}_0\hat{\boldsymbol{\rho}}_0 + ig\hat{\boldsymbol{\rho}}_0\hat{\boldsymbol{\phi}}_0 - ig\hat{\boldsymbol{\phi}}_0\hat{\boldsymbol{\rho}}_0 + \varepsilon\hat{\boldsymbol{\phi}}_0\hat{\boldsymbol{\phi}}_0 + \eta\hat{\boldsymbol{z}}_0\hat{\boldsymbol{z}}_0$. Expressions for the elements of the tensor $\boldsymbol{\varepsilon}$ can be found elsewhere [2].

The total energy W radiated from a current $\boldsymbol{J}(\boldsymbol{r},t)$ with duration τ is obtained as

$$W = -\int_0^\tau dt \int_V \boldsymbol{J}(\boldsymbol{r},t)\cdot\boldsymbol{E}(\boldsymbol{r},t)d\boldsymbol{r}, \tag{2}$$

where integration with respect to the spatial coordinates is performed over the volume V occupied by the source current, and $\boldsymbol{E}(\boldsymbol{r},t)$ is the electric field excited by the source. To evaluate W, we need to express $\boldsymbol{E}(\boldsymbol{r},t)$ in terms of the source current. To do this, it is convenient to use the Laplace transforms of these quantities as functions of time. Throughout this paper, we use the following convention for the definition of Laplace and inverse Laplace transforms:

$$f(\omega) = \int_0^\infty f(t)\exp(-i\omega t)dt,\ \ \mathrm{Im}\,\omega = -\sigma < 0,\ \ \sigma = \mathrm{const};\quad f(t) = \frac{1}{2\pi}\int_{-i\sigma-\infty}^{-i\sigma+\infty} f(\omega)\exp(i\omega t)d\omega. \tag{3}$$

For a source with current (1), only the azimuthal electric-field component $E_\phi(\boldsymbol{r},t)$ is required to evaluate W. It can be shown that the Laplace-transformed quantity $E_\phi(\boldsymbol{r},\omega)$ is given by the formula [2]

$$E_\phi(\boldsymbol{r},\omega) = \sum_n a_{s,n}(\omega)E_{\phi;s,n}(\rho,\omega)\exp(-ih_{s,n}z) + \sum_\alpha\int_0^\infty a_{s,\alpha}(k_\perp,\omega)E_{\phi;s,\alpha}(\rho,k_\perp,\omega)\exp(-ih_{s,\alpha}z)dk_\perp, \tag{4}$$

where $E_{\phi;s,n}(\rho,\omega)$ and $E_{\phi;s,\alpha}(\rho,k_\perp,\omega)$ are the azimuthal components of the vector wave functions describing the radial distribution of the electric fields of eigenmodes (discrete-spectrum waves) and continuous-spectrum waves of the duct, respectively, at a fixed frequency ω; $a_{s,n}(\omega)$ and $a_{s,\alpha}(k_\perp,\omega)$ are the excitation coefficients of the corresponding waves; n is the eigenmode radial index ($n = 0, 1, \ldots$); $h_{s,n}$ is the longitudinal wave number of an eigenmode with the index n; $k_\perp$ is the transverse (with respect to $\boldsymbol{B}_0$) wave number in the ambient uniform magnetoplasma; the functions $h_{s,\alpha}(k_\perp,\omega)$ stand for the longitudinal wave numbers of the ordinary ($\alpha = \mathrm{o}$) and extraordinary ($\alpha = \mathrm{e}$) waves of the ambient magnetoplasma; and the subscript s designates the wave propagation direction ($s = +$ and $s = -$ correspond to waves propagating in the positive and negative directions of the z axis, respectively). Expressions for $h_{s,\alpha}(k_\perp,\omega)$, as well as for the vector functions describing the fields of the discrete- and continuous-spectrum waves are given in [2].

Using the well-known technique developed for finding the excitation coefficients of open guiding systems in a magnetoplasma [2], we can write

$$a_{\pm,n}(\omega) = 2\pi bI_0\chi(\omega)N_n^{-1}(\omega)E^{(\mathrm{T})}_{\phi;\mp,n}(b,\omega),\quad a_{\pm,\alpha}(k_\perp,\omega) = 2\pi bI_0\chi(\omega)N_\alpha^{-1}(k_\perp,\omega)E^{(\mathrm{T})}_{\phi;\mp,\alpha}(b,k_\perp,\omega). \tag{5}$$

Here, $\chi(\omega)$ is the Laplace transform of the source function $\chi(t)$, the superscript (T) denotes fields taken in a medium described by the transposed dielectric tensor $\boldsymbol{\varepsilon}^{\mathrm{T}}$, and $N_n(\omega)$ and $N_\alpha(k_\perp,\omega)$ are the normalization quantities that are deduced from the orthogonality relations for the discrete- and continuous-spectrum waves (see [2] for details).

Substituting the inverse Laplace transform of $E_\phi(\boldsymbol{r},\omega)$ into formula (2) and performing integration with respect to the spatial coordinates and time, we obtain

$$W = -I_0 b\int_{-i\sigma-\infty}^{-i\sigma+\infty} d\omega\,\chi(-\omega)E_\phi(\boldsymbol{r},\omega)|_{\rho=b,\,z=0}\,. \tag{6}$$

The integration path in (6) is symmetric about the imaginary ω axis. Passing to integration over the right-hand part of this path, for which $\mathrm{Re}\,\omega > 0$, and then making the limiting transition $\sigma \to 0$, we get the resulting expression

$$\begin{aligned} W \;=\; & \int_0^\infty d\omega\,(-I_0^2)4\pi b^2\chi(-\omega)\chi(\omega)\mathrm{Re}\left[\sum_n N_n^{-1}(\omega)E^{(\mathrm{T})}_{\phi;-s,n}(b,\omega)E_{\phi;s,n}(b,\omega)\right. \\ & \left. + \sum_\alpha\int_0^\infty N_\alpha^{-1}(k_\perp,\omega)E^{(\mathrm{T})}_{\phi;-s,\alpha}(b,k_\perp,\omega)E_{\phi;s,\alpha}(b,k_\perp,\omega)dk_\perp\right]. \end{aligned} \tag{7}$$

Note that only the regions of integration over positive $k_\perp$ values for which the functions $h_{s,\alpha}$ are purely real make nonzero contributions, along with the propagated eigenmodes, to the radiated energy W given by formula (7).

We first examine the case where the temporal behavior of the source current is taken as a pulse whose filling comprises a few half-periods of a monochromatic oscillation:

$$\chi(t) = \sin(\omega_0 t)[H(t) - H(t-\tau)]. \tag{8}$$

Here, $H(t)$ is a Heaviside step function, $\tau = kT/2 = \pi k/\omega_0$ is the signal duration ($k = 1, 2, \ldots$), and ω_0 is the frequency corresponding to a given period $T = 2\pi/\omega_0$. In addition, we will also discuss the case of a single current pulse without modulation:

$$\chi(t) = (t/t_0)\exp[-(t-t_0)/t_0]. \tag{9}$$

Since the current pulse described by (9) for $t_0 = T/4 \equiv \pi/2\omega_0$ and that described by (8) for $k = 1$ are similar in shape, it is instructive to compare the radiation characteristics of the antenna for both signals.

3. Numerical Results

The quantity W was evaluated numerically for plasma parameters chosen to be typical of the Earth's ionosphere: the ambient plasma density $N_a = 10^6$ cm^{-3} and the external static magnetic field $B_0 = 0.5$ G. With these values, the ambient plasma had the electron plasma frequency $\omega_{\rm p} = 5.6 \times 10^7$ s^{-1}, the electron gyrofrequency $\omega_{\rm H} = 8.8 \times 10^6$ s^{-1}, and the effective ion gyrofrequency $\Omega_{\rm H} = 200$ s^{-1}. It was assumed that the source radius $b = 2.5$ m, the duct radius $a = 5$ m, and $\tilde{N} > N_a$. In the case where the source function $\chi(t)$ is described by formula (8), we choose the parameter ω_0 such as to satisfy the condition $\omega_{\rm LH} < \omega_0 \ll \omega_{\rm H}$, which corresponds to the resonant part of the whistler range. Here, $\omega_{\rm LH} = (\omega_{\rm H}\Omega_{\rm H})^{1/2}$ is the lower hybrid frequency. Calculations show that in this case, the dominant contribution to the radiated energy is ensured by slightly leaky modes, which can be separated from the $\alpha = {\rm e}$ term in (7), rather than by other terms, including a single axisymmetric eigenmode of the surface type with the index $n = 0$, which is supported by such a duct. The leaky modes have complex longitudinal wave numbers $h = k_0(p' - ip'')$, where k_0 is the wave number in free space, and are separated by appropriately deforming the path of integration over $k_\perp$ in (7) [2]. Their contributions to the expression under the sign of the integral over ω in (7) will be denoted as $w_\nu(\omega)$, where ν is the leaky-mode radial index ($\nu = 1, 2, \ldots$).

As an example, Fig. 1(a) shows the normalized (to I_0^2) contributions $w_\nu(\omega)$ of the leaky modes to the frequency spectrum of the energy radiated from the source with time dependence (8) for $\omega_0 = 1.9 \times 10^5$ s^{-1}, $k = 5$, and $\tilde{N}/N_a = 30$. Note that the projections of the diagrams in Fig. 1(a) onto the horizontal plane represent the dependences of the leaky-mode normalized propagation constants p' on the frequency ω. Note that the dependences $p'(\omega)$ for the dominant modes lie between the lower boundary $p = 2\tilde{\omega}_{\rm p}/\omega_{\rm H}$ and the upper boundary $p = \tilde{\omega}_{\rm p}/[\omega(\omega_{\rm H} - \omega)]^{1/2}$, which are shown by the red lines in Fig. 1(a). Here, $\tilde{\omega}_{\rm p}$ is the electron plasma frequency corresponding to the plasma density $\tilde{N}$ inside the duct. Results of numerical calculations of the total energy radiated from the loop antenna are shown in Fig. 1(b) for the previously chosen values of $\omega_{\rm p}$, $\omega_{\rm H}$, $\Omega_{\rm H}$, a, and b. The closed circles in the figure indicate the total energy radiated from the source with time dependence (8) for $\omega_0 = 1.9 \times 10^5$ s^{-1} and various values of $\tilde{N}/N_a$ and $k = \tau\omega_0/\pi$. For comparison, the values of the energy radiated from the source having time dependence (9) with $t_0 = \pi/2\omega_0$ are shown by the closed red squares in the figure. The open circles and the open square in Fig. 1(b) show the radiated energy when the loop antenna with the corresponding time dependences of the current is immersed in the surrounding uniform magnetoplasma.

It follows from Fig. 1(b) that the presence of a duct with enhanced density can lead to a significant increase in the energy radiated from a pulsed loop antenna compared with the case where the same source is immersed in the surrounding uniform magnetoplasma. Another important implication of the numerical results is that the radiated energy in the case where the current pulse is described by (8) obeys the relation $W = \overline{P}_{\rm rad}\tau$ with a fairly good accuracy. Here, $\overline{P}_{\rm rad}$ is the time-averaged power radiated from the source possessing a time-harmonic current with the frequency ω_0. It is important that such behavior is observed for the current containing even a few half-periods of a monochromatic oscillation, when the parameter k is moderately small, and is related to the features of excitation of whistler-mode waves by the loop antenna in a magnetoplasma. As the characteristic frequency ω_0 is increased, while remaining in the resonant part of the whistler range, the energy spectrum becomes wider and the marks representing the radiated energy approach the dependence $W = \overline{P}_{\rm rad}\tau$ for higher values of k. Nevertheless, the resonant part of the whistler range continues to give the predominant contribution to the radiation until the source spectrum is located in the frequency region below the electron gyrofrequency.

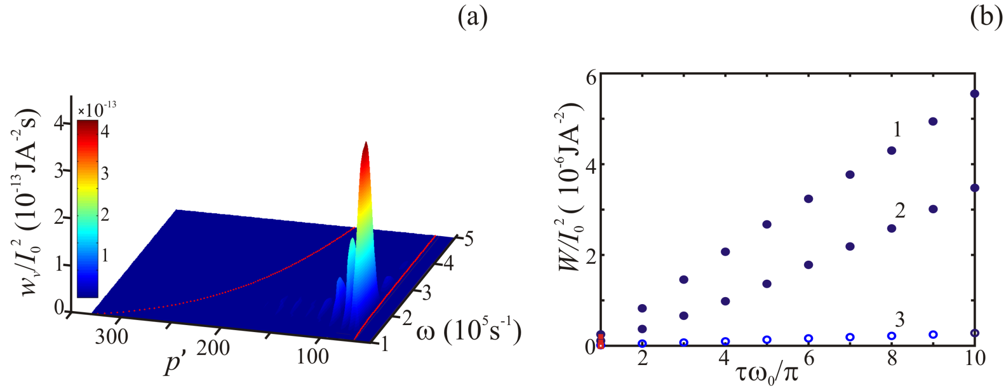

Figure 1. (a) Leaky-mode contributions to the frequency spectrum of the energy radiated from source (8) for $\tilde{N}/N_a = 30$ and $k = 5$, and (b) the total radiated energy as a function of the signal duration for $\tilde{N}/N_a = 30$, $\tilde{N}/N_a = 10$, and $\tilde{N}/N_a = 1$ (families of symbols labeled 1, 2, and 3, respectively; see text for discussion).

4. Conclusion

In this paper, we have studied the radiation characteristics of a pulsed loop antenna located in an enhanced-density duct in a magnetoplasma modeled upon the Earth's ionosphere. A notable increase in the energy radiated from such an antenna has been found to occur in the whistler range due to the presence of the duct. It has been shown that the radiated energy of a loop antenna whose current pulse contains only a few half-periods of a monochromatic oscillation with the frequency lying in the resonant part of the whistler range is very close to the product of the current duration by the time-averaged radiated power of the corresponding monochromatic source. In addition, conditions have been determined under which the radiation characteristics of the loop antenna with one half-period of a monochromatic current and those of the same source with a realistic-shape single current pulse of comparable duration are very close. The results obtained can be useful for explanations of the data of space and laboratory experiments on whistler wave excitation by pulsed sources in a magnetoplasma containing artificial field-aligned density irregularities.

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