Mode conversion in a randomly-stratified unmagnetized plasma

Kihong Kim[1], ***Dae Jung Yu***[2], and ***Dong-Hun Lee***[3]

[1] Division of Energy Systems Research, Ajou University, Suwon, Korea, khkim@ajou.ac.kr
[2] Division of Energy Systems Research, Ajou University, Suwon, Korea, yudau@ajou.ac.kr
[3] School of Space Research, Kyung Hee University, Yongin, Korea, dhlee@khu.ac.kr

Abstract

In real space plasmas, there exist random spatial variations of the plasma density as well as regular variations. The random density profile can affect the behaviors of resonance and wave propagation. In this paper, we investigate how the mode conversion from electromagnetic waves into electrostatic modes in a stratified unmagnetized plasma is affected by random density variations superimposed to the linear profile near the resonance. We obtain a surprising result that mode conversion is substantially enhanced in the presence of weak randomness. We interpret this result in terms of the enhancement of tunneling due to weak randomness.

1 Introduction

The simplest kind of mode conversion, which is the conversion of electromagnetic waves into electrostatic modes in cold, unmagnetized and stratified plasmas has been studied extensively for a long time [1-4]. In real space plasmas, it is reasonable to consider the plasma density with random spatial variations. In 1958, Anderson proposed for the first time that waves in random media cannot be extended over entire region but are localized when randomness is sufficiently strong [5,6]. This concept can also be applied to electromagnetic waves interacting with plasmas with a random distribution of the electron density. In the present work, we investigate the influence of randomness on mode conversion phenomena theoretically. We use the invariant imbedding method developed by two of us previously and calculate the mode conversion coefficient and the field distribution in a numerically exact manner [7,8]. We find a surprising result that mode conversion is substantially enhanced by weak randomness.

2 Theory

We assume that a plane wave of unit magnitude $\tilde{H}(x,z) = H(z)e^{iqx} = e^{ip(L-z)+iqx}$ is incident on a stratified plasma from the region where $z > L$ and transmitted to the region where $z < 0$. When the vacuum wave number is k_0 $(= \omega/c)$ and the incident angle is θ, the z and x components of the wave vector are expressed as $p = \sqrt{\epsilon_1}k_0 \cos\theta$ and $q = \sqrt{\epsilon_1}k_0 \cos\theta$ respectively, where ϵ_1 is the dielectric permittivity in the incident region. In cold, unmagnetized plasmas, the mode conversion occurs only for p waves. Then the complex amplitude of the magnetic field H satisfies

$$\frac{d^2H(z)}{dz^2} - \frac{1}{\epsilon(z)}\frac{d\epsilon(z)}{dz}\frac{dH(z)}{dz} + [k_0^2\epsilon(z) - q^2]H(z) = 0. \tag{1}$$

The field outside the inhomogeneous medium defines the reflection and transmission coefficients r and t as follows:

$$\tilde{H}(x,z) = \begin{cases} e^{ip(L-z)+iqx} + r(L)e^{ip(z-L)+iqx}, & \text{if } z \geq L \\ t(L)e^{-ip'z+iqx}, & \text{if } z \leq 0 \end{cases} . \tag{2}$$

The dielectric permittivity ϵ inside the plasma is given by

$$\epsilon(z) = 1 - \frac{{\omega_p}^2}{\omega(\omega + i\gamma)}, \quad {\omega_p}^2 = \frac{4\pi e^2}{m}n(z), \tag{3}$$

where ω_p is the electron plasma frequency and $n(z)$ is the electron density. γ is the phenomenological damping parameter and e and m are the electron charge and mass.

978-1-4244-6051-9/11/$26.00

We introduce dimensionless quantities $\zeta = k_0\Lambda$, $\tilde{z} = z/\Lambda$ and $\tilde{L} = L/\Lambda$, where Λ is a parameter defining the length scale. The averaged electron density increases linearly from $\tilde{z} = \tilde{L}$ to $\tilde{z} = 0$

$$n(z) = n_0\left[1 - \tilde{z} + \tilde{L}/2 - G(\tilde{z})\right], \tag{4}$$

where $G(\tilde{z})$ is a random function the value of which is uniformly distributed in the range $[-g, g]$. Then the dielectric permittivity is given by

$$\epsilon(\tilde{z}) = \begin{cases} \tilde{L}/2, & \text{if } \tilde{z} > \tilde{L} \\ \tilde{z} - \tilde{L}/2 + G(\tilde{z}) + i\eta, & \text{if } 0 \le \tilde{z} \le \tilde{L} \\ -\tilde{L}/2, & \text{if } \tilde{z} < 0 \end{cases}. \tag{5}$$

The frequency of the incident wave is fixed to $\omega_0 = \sqrt{4\pi n_0 e^2/m}$.

In order to calculate the magnetic field distribution and the wave reflectance, we use the invariant imbedding methods, the main idea of which is to transform the boundary value problem of the second-order differential equation into the initial value problem of coupled first-order ordinary differential equations. The field amplitude H and the reflection coefficient r satisfy the invariant imbedding equations [7,8]

$$\begin{aligned} \frac{\partial H(z,l)}{\partial l} &= i\sqrt{\epsilon_1}k_0\cos\theta\left\{\frac{\epsilon(l)}{\epsilon_1} - \frac{1}{2}\left[\frac{\epsilon(l)}{\epsilon_1} - 1\right]\left[1 - \frac{\epsilon_1}{\epsilon(l)}\tan^2\theta\right][1 + r(l)]\right\}H(z,l), \\ \frac{dr(l)}{dl} &= 2i\sqrt{\epsilon_1}k_0\cos\theta\frac{\epsilon(l)}{\epsilon_1}r(l) - \frac{i}{2}\sqrt{\epsilon_1}k_0\cos\theta\left[\frac{\epsilon(l)}{\epsilon_1} - 1\right]\left[1 - \frac{\epsilon_1}{\epsilon(l)}\tan^2\theta\right][1 + r(l)]^2. \end{aligned} \tag{6}$$

These are integrated using the initial conditions for r and H. The initial condition for r is obtained from the Fresnel's formula

$$r(0) = \frac{\epsilon_2\sqrt{\epsilon_1}\cos\theta - \epsilon_1\sqrt{\epsilon_2 - \epsilon_1\sin^2\theta}}{\epsilon_2\sqrt{\epsilon_1}\cos\theta + \epsilon_1\sqrt{\epsilon_2 - \epsilon_1\sin^2\theta}}, \tag{7}$$

where ϵ_2 is the dielectric permittivity in the transmitted region. The initial condition for H is $H(z,z) = 1 + r(z)$.

When integrating the invariant imbedding equations, we need the randomly of the electron density described by $G(\tilde{z})$. We generate this function using a random number generating function $R(s)$ in the IMSL library

$$G(\tilde{z}) = 2g'\left\{[R(s+1) - R(s)]\left[\frac{\tilde{z} - (s-1)\Delta\tilde{z}}{\Delta\tilde{z}}\right] + R(s)\right\} - g', \tag{8}$$

where $g' = g(\tilde{L}/2)$ and $s = \mathrm{Int}(\tilde{z}/\Delta\tilde{z}) + 1$. We divided the inhomogeneous region in equal intervals equal to $\Delta\tilde{z}$. We set $\Delta\tilde{z} = 0.01$ in all calculations. When g is 0.1, the fluctuation is from -0.1 to 0.1 with the average equal to zero. $R(s)$ ranges from 0 to 1 for each s.

3 Results

When we solve the invariant imbedding equations for a given configuration of G, we obtain one set of data. We have repeated this calculation for many different random configurations and averaged the results. If there is no randomness in the electron density, we recover the results obtained previously. On the other hand, it is demonstrated in Fig. 1 that the mode conversion coefficient $\mathcal{A}$ $(= 1 - |r|^2)$ increases as g increases from 0.01 to 0.06, but decreases after g is over about 0.06. In this calculation, we have used the values $\zeta = 20$, $\tilde{L} = 20$ and $\eta = 10^{-8}$. The initial increase is partly due to that the tunneling of electromagnetic waves from the cutoff to the resonance point is enhanced by weak randomness in the tunneling barrier and

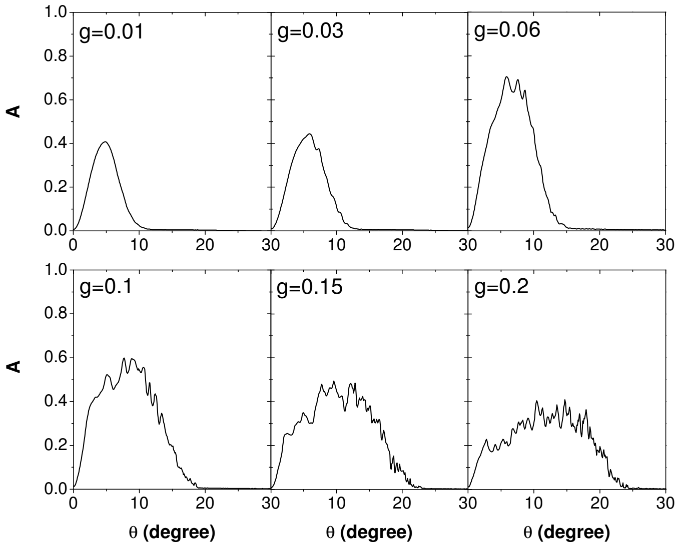

Fig. 1: Mode conversion coefficient $\mathcal{A}$ versus incident angle for disorder strength $g = 0.01$, 0.03, 0.06, 0.1, 0.15, 0.2, when $\zeta = 20$, $\tilde{L} = 20$ and $\eta = 10^{-8}$. The data are obtained by averaging over 40 random configurations for each g.

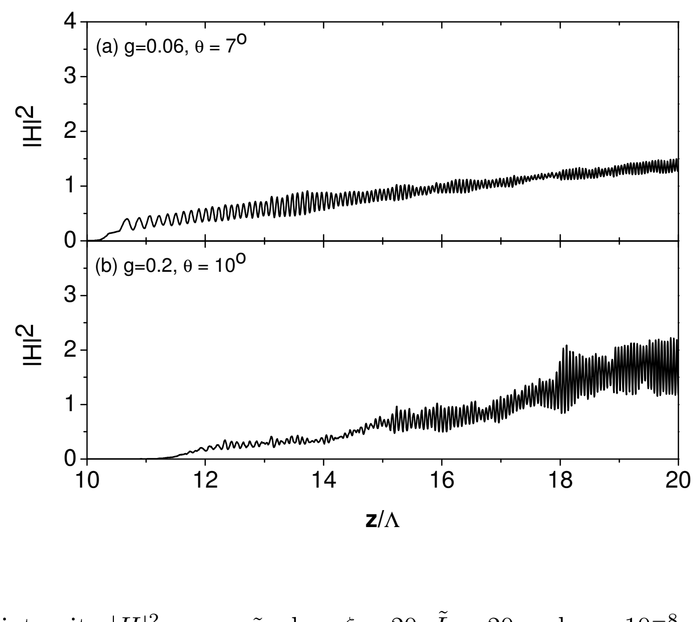

Fig. 2: Averaged field intensity $|H|^2$ versus $\tilde{z}$ when $\zeta = 20$, $\tilde{L} = 20$ and $\eta = 10^{-8}$. The figure (a) represents weak disorder and large mode conversion and the figure (b) represents strong disorder and small mode conversion. This data are obtained by averaging over 40 configurations.

partly due to the fluctuation of the rate of the density variation at the resonance point. The subsequent decrease is caused by Anderson localization effects.

When randomness is strong, a well-known theoretical result states that the field distribution has a non-monotonic shape and has a peak somewhere inside the disordered medium. In Fig. 2, we plot field distributions for two representative cases. When disorder is strong and mode conversion is weak ($g = 0.2$, $\theta = 10^\circ$), the field is enhanced and has a peak inside the plasma medium and the shape is non-monotonic. On the other hand, when disorder is weak and mode conversion is strong, the field profile is monotonic.

4 Conclusion

In this paper, we have studied the influence of random spatial variations of the plasma density on mode conversion phenomena in cold, unmagnetized plasmas using the invariant imbedding method. We have calculated the mode conversion coefficient averaged over a large number of random spatial configurations generated by a random number generator. We have also calculated the spatial distribution of the field intensity. We have obtained a surprising result that mode conversion is substantially enhanced in the presence of weak randomness. As the strength of randomness increases further, mode conversion has been found to be suppressed below the value in the absence of randomness due to Anderson localization effects.

5 Acknowledgments

This work has been supported by the Korea Science and Engineering Foundation grant (No. R0A-2007-000-20113-0) funded by the Korean Government.

6 References

1. T. H. Stix, *Waves in Plasmas*, AIP Press, New York, 1992.

2. D. G. Swanson, *Theory of Mode Conversion and Tunneling in Inhomogeneous Plasmas*, Wiley, New York, 1998.

3. K. G. Budden, *The Propagation of Radio Waves*, Cambridge University Press, Cambridge, 1985.

4. V. L. Ginzburg, *The Propagation of Electromagnetic Waves in Plasmas*, Pergamon, New York, 1970.

5. P. W. Anderson, "Absence of diffusion in certain random lattices," *Phys. Rev.*, 109, April 1958, pp. 1492-1505.

6. S. John, "Electromagnetic absorption in a disordered medium near a photon mobility edge," *Phy. Rev. Lett.*, 53, Nov. 1984, pp. 2169-2172.

7. K. Kim, H. Lim, and D.-H. Lee, "Invariant imbedding equations for electromagnetic waves in stratified magnetic media: Applications to one-dimensional photonic crystals," *J. Korean Phys. Soc.*, 39, Dec. 2001, pp. L956-L960.

8. K. Kim and D.-H. Lee, "Invariant imbedding theory of mode conversion in inhomogeneous plasmas. I. Exact calculation of the mode conversion coefficient in cold, unmagnetized plasmas," *Phys. Plasmas*, 12, June 2005, 062101.