A Numerical Analysis of the Time Evolution of Plasma Wake Structure around a Simplified 2D Cube Satellite Model

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Abstract

A two-dimensional numerical model was developed to study the time evolution of wake structure around a cube satellite moving in a plasma with transonic speed. The cube satellite was simplified to be a 2D metal cylinder with a square cross section. An electric potential is then applied to the metal cylinder. The numerical scheme was compared to the analytical solution for a 1D problem before being applied to the 2D model. The study provided an initial understanding of plasma behavior around the 2D cube satellite model when it is in its orbit.

1. Introduction

As cube satellites have become very low-cost space exploration vehicle, it is important to study the physics of the environment sounding the satellite. This is because the behavior of a range of instruments, from surface mount antennas [1] to impedance probes [2] are affect by the media where the satellite orbits. This paper treats a cube satellite as a metal box and studies the plasma particle profiles around the box. The assumption of the metal box is valid because most cube satellites have surface mount solar cells, which have a metal backing and there are some surface charges and associated electric potential on the metal. To achieve the initial understanding of the plasma behavior around the cube satellite, at the current stage, we ignore effect of complex solar cell surface on plasma properties.

2. Fluid Transport Equations and Numerical Approach

Plasma properties can be described by fluid transport equations obtained by taking moments of the Boltzmann's equation [3]. A separate fluid transport equation needs to be written for each type of plasma particles (i.e. electrons, ions, and neutral particles). The first two equations are continuity equation and momentum equation as follows.

$$\frac{\partial n_k}{\partial t} + \nabla.\left(n_k \vec{u}_k\right) = 0, \tag{1}$$

and

$$\frac{\partial\left(n_k \vec{u}_k\right)}{\partial t} + \nabla.\left(n_k \vec{u}_k \vec{u}_k\right) = -\frac{\nabla P_k}{m_k} + \frac{n_k}{m_k}\vec{F}. \tag{2}$$

Equation (1) describes describing the conservation of mass and (2) is the momentum equation where n^k is the density, $\vec{u}_k$ is the velocity, m^k is the mass, and p^k is the pressure of the k^{th} species. Other symbols in equation (2) are as follows. $\vec{F}$ is the Lorentz force given by $e(\vec{E} + \vec{u}_k \times \vec{B})$, where e is the charge of the ion or electron, $\vec{E}$ represents the electric field, and $\vec{B}$ is the magnetic field applied to the plasma.

These two equations need to be written for all three types of plasma particles (i.e. electrons, ions and neutral particles). The electrostatic potential ϕ in the plasma sheath region is governed by Poisson's equation

$$\nabla^2 \phi = \frac{e}{\varepsilon_0}(n_i - n_e), \tag{3}$$

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where n_i is the density of ions and n_e is the density of electrons. Equations (1), (2) and (3) can describe the evolution of plasma sheath in a space with respect to time.

The numerical solution is formulated by defining a uniform spatial and temporal grid. The Euler scheme is used to the march the finite difference equation forward in time. The time step Δt varies according to the CFL condition given by

$$\Delta t = C \times min(\frac{a^x_{j+1/2,k}}{\Delta x}, \quad \frac{a^y_{j,k+1/2}}{\Delta y}), \tag{4}$$

where C is the Courant number and is set to 1/8 in our simulations. $a^x_{j+1/2,k}$ and $a^y_{j,k+1/2}$ are the local speed in each cell [4].

The Poisson's equation (3) has a non-linear term due to the presence of Boltzmann's relation for electron density. This equation can be written as

$$h(\phi) = \frac{(\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j})}{(\Delta x)^2} + \frac{(\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1})}{(\Delta y)^2} + \frac{e}{\varepsilon_0}[n_i - n_0 \exp(\frac{\phi_i - \phi_0}{kT_e})] = 0, \tag{5}$$

and can be solved using iteration method outlined as follows.

When ϕ^k is the electric potential from k^{th} iteration, the next iteration ϕ^{k+1} can be found from the Taylor series expansion by neglecting the higher order terms, where

$$h(\phi^{k+1}) = h(\phi^k) + h'(\phi^k)(\phi^{k+1} - \phi^k) = 0. \tag{6}$$

If we define the error term as $\delta^k = \phi^{k+1} - \phi^k$, then we have

$$h'(\varphi^k)\delta^k = h(\phi^k). \tag{7}$$

Equation (7) is in form of AX=B, and A can be determined from Jacobian matrix

$$A_{mn} = \begin{cases} 1/(\Delta x)^2 & m = i+1 \text{ or } i-1, n = j \\ -\frac{2}{(\Delta x)^2} - \frac{2}{(\Delta x)^2} - \frac{n_0 e}{kT_e \varepsilon} \exp(\frac{\phi - \phi_0}{kT_e}) & m = i, n = j \\ 1/(\Delta y)^2 & m = j, n = j+1 \text{ or } j-1 \end{cases} \tag{8}$$

In each iteration, a linear system of equations is to be solved to obtain the value for the correction term. This value is then added to ϕ^k to obtain the new value electric potential at each spatial node, which is then used in next iteration. Since the A matrix is sparse, the bi-conjugate gradient scheme is used to solve the linear system of equations [5]. For each iteration, the tolerance is decided for the RMS error. In our case, we took the tolerance to be 10^{-7}. The method is observed to converge fast for an appropriate initial guess.

3. Simulation Parameters and Validation of the Numerical Scheme with 1D Problem

In our study, the length, density, velocity and time were normalized for electron Debye length, average ion density, acoustic speed of the ions and plasma period (calculated from plasma frequency) respectively [3]. The simulation parameters for other quantities are as follows. It should be noted that the model of the system was made to evolve with the time.

Density n_0: 1

Electron temperature: 0.1eV
Ion temperature: 0.1eV
Plasma potential: 0V
Potential applied at the metal surface: -0.2V

Before using the numerical scheme to compute a 2D problem, we tested the scheme by comparing the solution for the 1D problem and the analytical solutions. To perform the comparison, an infinite metal wall was placed at the left boundary and the inflow of the particles was placed at the right boundary. The distance between the two boundaries is chosen to be 50 Debye length. The value for Δx is set to be 0.2. The initial conditions were set to be n_i (x,0)=1 and u_{ix}(x,0)=0. The numerical simulation was performed with MATLAB™ on Windows environment using Dual Core AMD Opteron Processor 165 (1.81GHz). The RAM memory required for the simulation domain was 50MB. The time taken for one plasma period to complete was 5 seconds.

The results of the one-dimensional problem obtained after running the simulation for 3257 plasma periods together with the analytical solutions are summarized in Fig. 1 to 3. From Fig. 1 and Fig. 2, it is seen that the numerical solution agrees well with the analytical results, and validates the computational scheme [6]. Fig. 3 shows the initial velocity of ions reaches the ion acoustic speed after achieving the steady state.

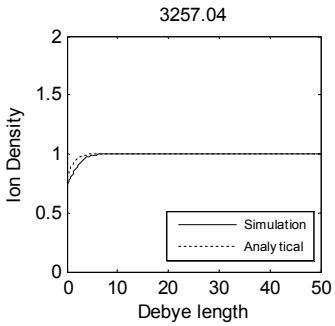

Fig. 1 Ion Density

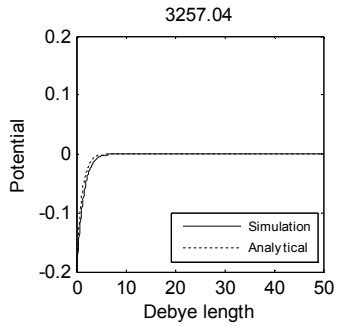

Fig. 2 Electric Potential

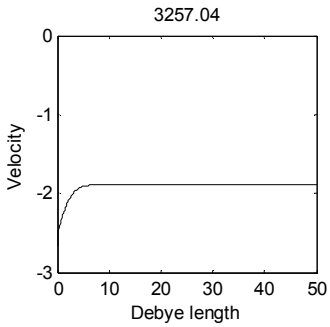

Fig. 3 Ion Velocity

4. 2D Problem and Results

The purpose of solving a 2D problem is to study the plasma particles around a simplified Cube Satellite model. When a Cube Satellite is in its orbit, the ion flux can be approximated as coming towards the satellite surface (i.e. ion flux coming along y direction and perpendicular to the satellite surface xz) (Fig. 4). The variation of plasma particle along the z axis is simplified as invariant and accordingly the satellite is approximated as a 2D cylinder (Fig. 4).

In simulation, the x and y boundaries were set at 70 and 150 Debye length respectively. The metal cylinder is placed at (25, 75) and the x, y boundaries are set sufficiently large compared to the size of the cylinder. Since Boltzmann's relationship for the electron density was used, the mass of the ions was that of the realistic model. The ion to electron temperature ratio was assumed to be 1. The density and velocity at inflow boundaries were fixed on the initial values, whereas at outflow boundaries of the metal surface and the computational domain were calculated from the first order extrapolation using the interior cells. The electric potential was specified on the metal surface and was extrapolated on the boundaries of computational domain using the linear extrapolation. The initial conditions were set to be $n_i(x, 0)=1$, $u_{ix}(x, 0)=2.5$, and $u_{iy}(x, 0)=0$.

The RAM memory required for the simulation domain 450MB. The time taken for one plasma period to complete was 3.5 hours. The density profile starts with single tail in the rarefaction region which divides into two. The profile shows symmetric behavior around the ion flux direction. The wake structure expands both in the direction perpendicular and parallel to the ion flux direction. After running the simulation for 100.51 plasma periods, we obtained the ion density profile (Fig. 5) and electric potential profile (Fig. 6), where we can see the rarefaction. There is a small region behind the metal cube which shows a density buildup. This is because the negative voltage applied to the metal cube forces the ions near the surface towards the cube and causes the velocity of ions in the flux direction to reverse the direction.

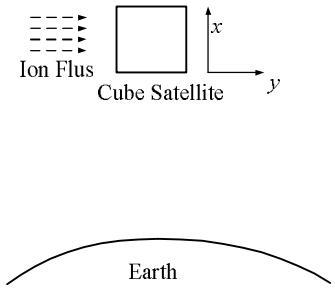

Fig. 4 Cube Satellite Model

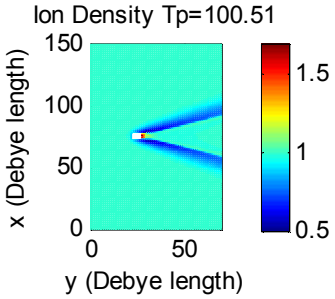

Fig. 5

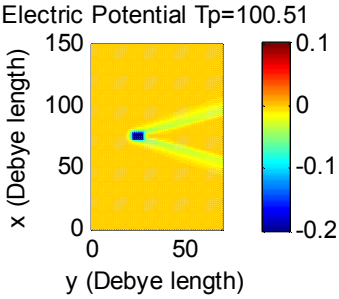

Fig. 6

5. Conclusion

Wake structure around a simplified Cube Satellite model in 2D is studied. The numerical method is validated by comparing it with the analytical solutions for the 1D problem. The numerical simulation clearly shows the wave refraction around the satellite and we were able to obtain properties of plasma surrounding the cube satellite. As future studies, a more complex 3D model needs to be considered to provide more detailed observation of plasma particle profiles around the satellite.

6. References

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