On statistical distribution of characteristics of chorus element generation

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Abstract

A generation mechanism for VLF chorus was suggested by V. Yu. Trakhtengerts [1, 2] on the basis of the backward wave oscillator (BWO) regime of magnetospheric cyclotron maser. Up to now many predictions of the BWO regime were supported by observations. In this report we discuss a statistical distribution of the dimensionless parameter q, quantifying an excess of the electron flux over the absolute-instability threshold. Typically the q parameter values deduced from VLF chorus elements observed by WBD instrument onboard the CLUSTER spacecraft exhibit significant scatter, while their mean values were ~10. We suppose that so large excess of the instability threshold cannot be permanently supported in the magnetospheric plasma. On the basis of the discrete numerical model we demonstrate that if the noised "on-off" intermittency regime generation is realized, then the observed q values deduced from chorus elements should be extreme ones, but the average value over the entire event can be much smaller. We stress an importance of taking the noise-induced type of chorus generation into account.

1. Introduction

While the ELF-VLF chorus emissions are studied during more than 50 years, the only mechanism of their generation is well supported by theory. This the is the backward wave oscillator (BWO) regime of chorus generation which was proposed by V. Trakhtengerts [1, 2] and is based on the analogy with operation of the backward wave oscillator in laboratory electronic. The BWO regime of chorus generation occure when the conditions of absolute cyclotron instability are satisfied for radiation belt electrons near the equatorial region. Up to now many predictions of the BWO regime were supported by observations [3-5]. However to explain statistical features of succession of chorus elements within the BWO model it is necessary in addition to take into account fluctuations of the electron distribution function which determines the excess over generation threshold.

Kozelov et al. [6] presented a discrete numerical model which allowed them to reproduce statistical peculiarities of the ELF-VLF emission dynamics in ground-based observations. The model is based on the assumption that due to the occurrence of noise the magnetospheric BWO generator should operate in the so-called "on-off" intermittency regime. Further, the same model with parameters deduced from CLUSTER data allowed us to reproduce the dynamics of the ELF-VLF emissions observed by CLUSTER spacecraft in the magnetosphere near the generation region [3].

In this report we use the model adopted from [6] to explain recently obtained statistical distribution of the dimensionless parameter q, quantifying an excess of the electron flux over the absolute-instability threshold. According to [3]:

$$q = \gamma^2_{step}/\gamma^2_{thr} . \qquad (1)$$

Here γ_{step} is growth rate of waves in a homogeneous unbounded medium when a step-like deformation exists on the velocity distribution of energetic electrons, γ^2_{step} being proportional to the step height; $\gamma^2_{thr} = \pi/T_0$ is a threshold value of γ^2_{step} for the transition to the BWO regime; $T_0 = l_{BWO}(1/v_g + 1/v_{step})$ is the BWO characteristic time scale, l_{BWO} is the BWO length, v_g is the group velocity of the whistler-mode waves, and v_{step} is the absolute value of the parallel velocity of resonant electrons.

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2. Experimental distribution of the *q* parameter

Figure 1 presents a typical distribution of q-parameter values deduced from VLF chorus elements observed by WBD instrument onboard the Cluster spacecraft during the event on December 6, 2003. This distribution is non-symmetrical one, it has a maximum at q=10-15 and a longer wing to higher values. So large values obtained from the data agree well to results of numerical calculations [7], where it was shown that this is the values when continuous generation with slow frequency variation is changed by generation of discrete chorus-like elements. However, it would be difficult to assume that so large excess over the instability threshold can be permanently supported in the magnetospheric plasma in the generation region. The problem can be resolved if we take into account the noise-induced type of chorus generation. Then it is clear that the q values observed during chorus element generation should be extreme ones and, therefore, their distribution deduced from chorus elements should differ significantly from the distribution over the entire event. Let us demonstrate this deduction by numerical example.

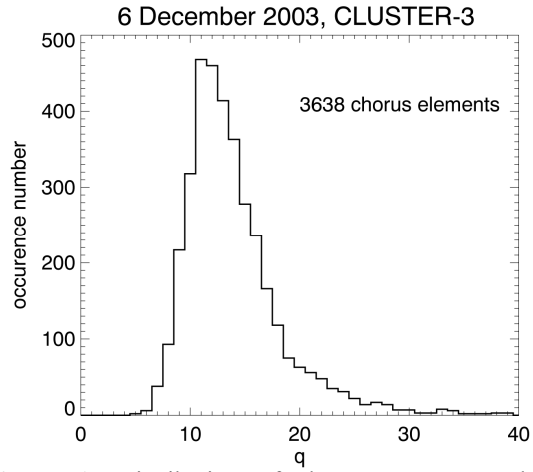

Figure 1. Distribution of the q-parameter values obtained from chorus elements observed by CLUSTER spacecraft during the event on December 6, 2003. The plasma density was N_e =10 см$^{-3}$.

3. Numerical model

As in papers [6, 8], we consider the discrete model of electronic generator in BWO regime:

$$A(t)=\lambda(t-T_0)\,[A(t-T_0)-A^3(t-T_0)]+\delta, \qquad (2)$$

where $A(t)$ is wave magnitude, T_0 is characteristic time delay of feedback (it is a time step for the discrete model), δ is a small constant which corresponds to the initial background wave, and λ is the growth rate. For the magnetospheric BWO $T_0 \sim 0.1$ s, and λ can be obtained as [1, 2]:

$$\lambda = 1 + \pi\,(\gamma_{step}\, l_{лов}/(v_g v_{step})^{1/2} - \pi/2) = 1 + \pi^2\,(q^{1/2}-1)/2$$

For example, $\lambda \approx 14$ for q =12. The relationships between the model parameters and physical parameters of the magnetospheric BWO were discussed in [3, 6]. Let us consider λ as a random variable:

$$\lambda(t) = C_0 + C_1\,\xi, \qquad (3)$$

where C_0 and C_1 are constants, and ξ is a random number. The simplest assumption that the random number $\xi(t)$ have the uniform distribution in the range from 0 to 1 was considered in paper [6]. Now we consider a case of normal distribution of ξ with a zero mean and unit variance. Then the constants C_0 and C_1 in equation (3) determine the mean value and variance of $\lambda(t)$. Additionally, we assume that $\lambda(t)$ is non-negative: $\lambda(t) = 0$ for $C_0 + C_1\,\xi(t) < 0$.

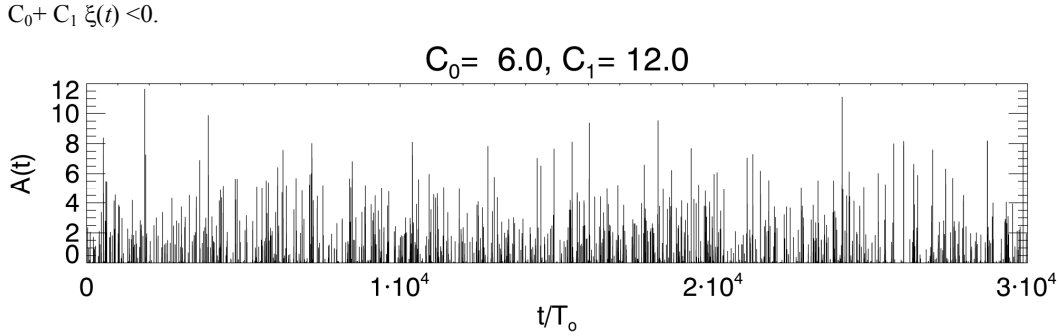

Figure 2. Dynamics of A(t) in the numerical model (2)-(3) with parameters C_0 = 6, C_1 = 12.0, $\delta = 10^{-8}$.

4. Results

The numerical model described by Eqs. (2)-(3) with parameters $C_0 = 6$, $C_1 = 12.0$, and $\delta = 10^{-8}$ demonstrates the generation of a sequence of peaks in $A(t)$, see Figure 2. At a level $A_{thr} = 3$ the peaks are well distinguished. Let us consider the probability density functions (PDF) of the growth rate λ for the entire simulation interval and separately for the moments just before exceeding the threshold value A_{thr}. The latter moments are actually the starting points of the chorus elements, so the corresponding parameter represent the BWO parameters deduced from the observed chorus elements. The PDFs are shown in Figure 3. One can see that the PDF for the λ values over the entire simulation time occupies much smaller values than the PDF for the moments when A_{thr} is exceeded. More detail about the evolution of the model parameters near the moments of the threshold excess have been obtained by superimposed epoch method, see Figure 4. One can see that at these moments the wave amplitude is on average at least one order of magnitude higher than at other times, but the average growth rate is by 50% higher at than just one time step before.

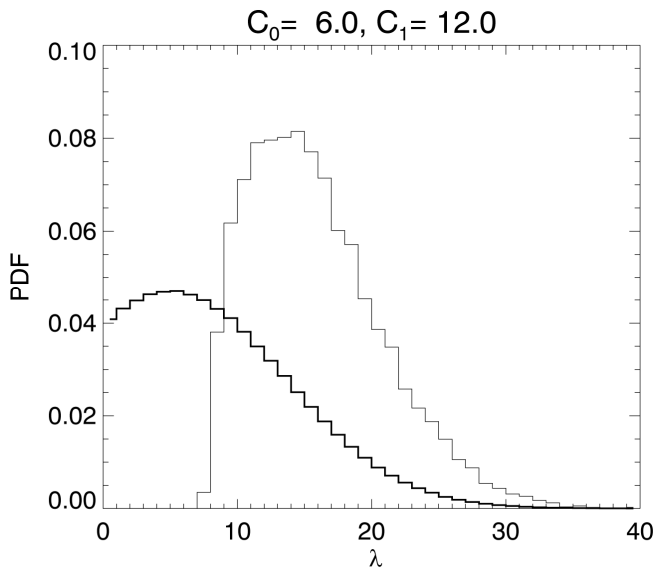

Figure 3. Probability density function of the growth rate λ: solid line - for the entire simulation time; thin line - separately for intervals when the threshold is exceeded.

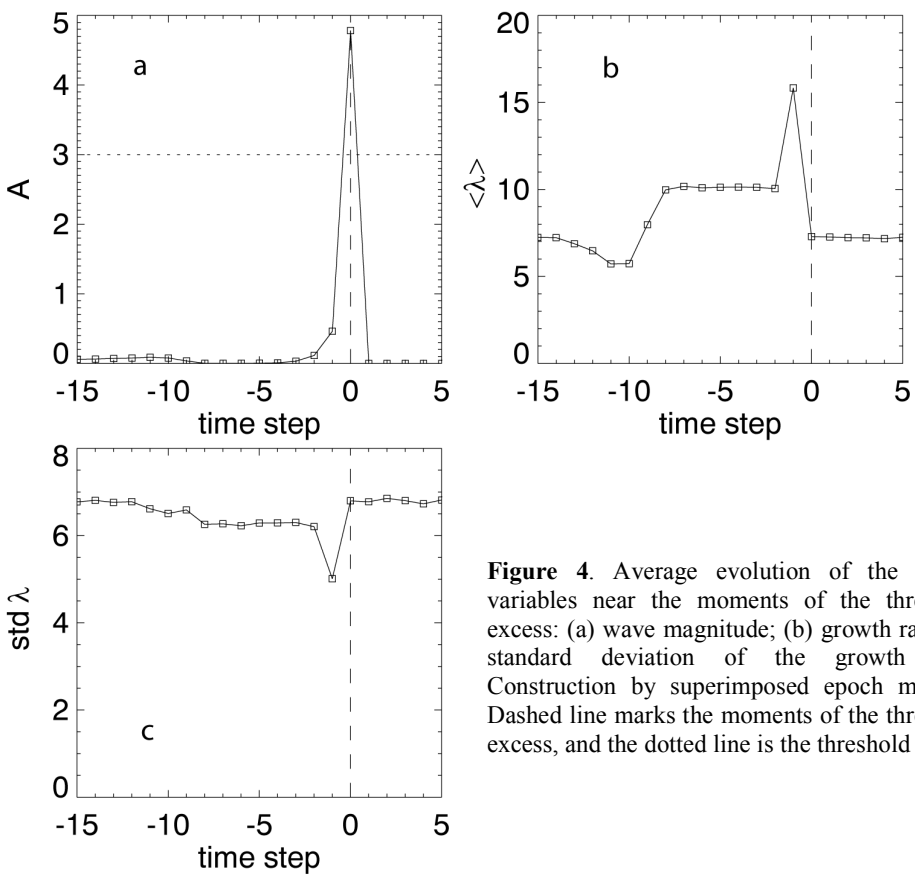

Figure 4. Average evolution of the model variables near the moments of the threshold excess: (a) wave magnitude; (b) growth rate; (c) standard deviation of the growth rate. Construction by superimposed epoch method. Dashed line marks the moments of the threshold excess, and the dotted line is the threshold level.

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