One-dimensional Particle-In-Cell simulation of electron beam plasma interaction

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Abstract

The theory of the two-stream instability is used to validate the initial phase of a developing code which ultimately aims to simulate the generation of VLF chorus. A one dimensional Particle-in-Cell simulation shows that waves are generated when a warm electron beam is injected into a cold background plasma with components of the electric field being excited. After a few time steps the phase space reveals a distortion of the beam and the creation of potential holes, causing electrons to oscillate.

1 Introduction

Electron beams propagating away from the Sun play an important role in generating solar radio emissions and are a basic phenomenon observed in solar activity. When a solar flare occurs, electron beams are ejected from the Sun into the interplanetary medium. These beams propagate along the interplanetary magnetic field lines with parallel velocities that are much larger than the electron thermal speed of the solar wind [1].

2 Background: PIC Simulations

Particle-in-Cell (PIC) simulations use superparticles to represent different populations of plasmas and are used here to study the two-stream instability. The results can be compared to theory to validate the code and can be used for non-linear analysis of the system. PIC method exploits the force-at-a-point formulation and a field equation for the electric potential. Field quantities, which in the physical system pervade all space, are approximately represented by values on a one dimensional grid. Differential operators, such as the Laplacian $\bigtriangledown^2$, are replaced by finite-difference approximations on the grid. Potentials and forces at particle positions are obtained by interpolating between grid points. Densities are obtained by the opposite process of assigning the particle attributes to nearby grid points. Periodic boundary conditions are used and the time and length scales are normalized to the plasma frequency, ω_{pe}, and Debye length, λ_D [2].

3 Theory

In the initial stage of the simulation the electron beam and the background plasma are given the same density. The result is that the two-stream instability analysis can be applied to the system which requires

$$\frac{\omega_{pe} L}{v_0} > \frac{2\pi}{\sqrt{2}}$$

for instability to occur using $L = \dfrac{2 \cdot \pi}{k_0}$ where k_0 is the smallest wavenumber present and v_0 the beam's drift velocity [3].

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4 Simulation and Results

The simulation contains 10^6 superparticles (evenly split between the cold background plasma and the beam) and uses a grid of 10000 points where the grid spacing is $\triangle x = 0.01$ ($L = 100$). The background plasma has a thermal velocity equal to the electron thermal velocity, v_{th}, and a zero drift velocity, while the electron beam has a thermal velocity of $2v_{th}$ and a drift velocity of $5v_{th}$.

Figure 1 shows how the initial background plasma gets distorted by the electron beam after 25 time steps. Electron potential holes are seen after 100 time steps and electrons starts to oscillate in phase space. After many time steps the electron beam is destroyed and the electron velocity for the beam flattens out while the background density remains Maxwellian.

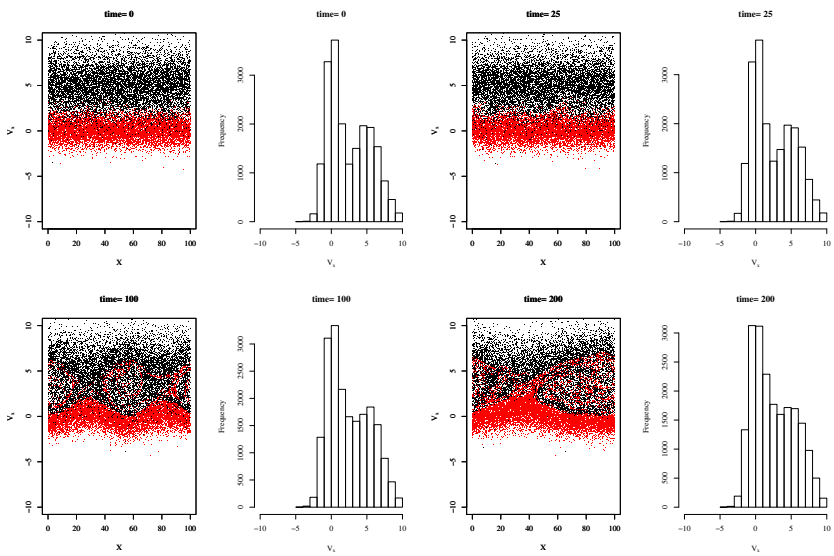

Figure 1: Phase space and velocity distribution development of the electrons. Red dots represent the cold electron population while the black dots represent the warm electron beam.

Figure 2 shows the spatial component of the electric field. Initially the field appears to be uniform but after about 25 time steps oscillations in the electric field become evident. The phase speed for the first mode can be observed from the diagonal curves which form in the electric field.

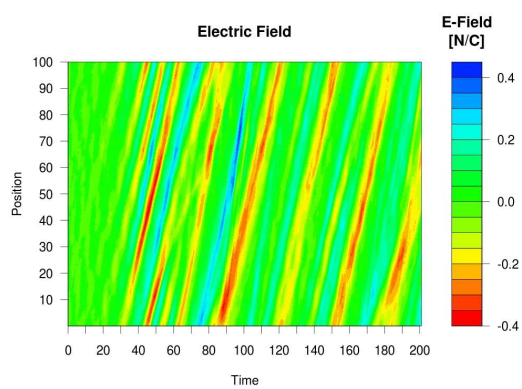

Figure 2: Spatial electric field versus time.

Initially there is a sudden drop in kinetic and increase in potential energy as particles are clumped together to form potential holes in phase space (Figure 3). Later there is a gradual increase in the kinetic energy as the system tends to thermal equilibrium. The total energy for the system remains constant, obeying the law of conservation, indicating that there are no non-physical parameters present in system or energy mechanisms which are not accounted for.

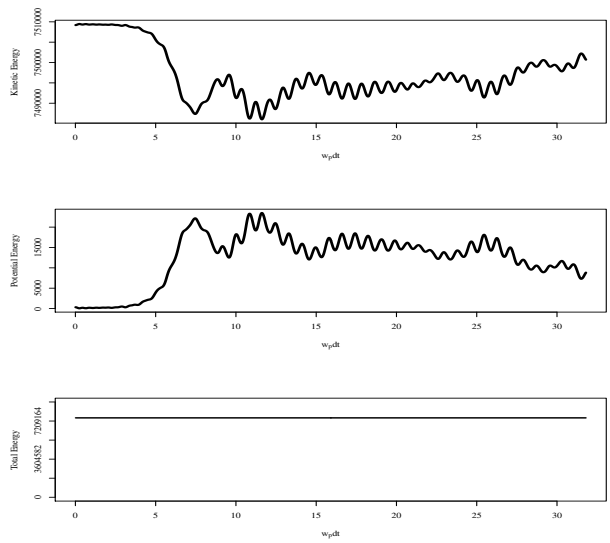

Figure 3: Kinetic, potential and total energy of the system.

The dispersion relationship is obtained by applying Fourier transforms to the spatial component of the electric field and then the time component of every wavenumber component. The dominant component of the system is seen to be for $k = 1$ and $k = 2$ oscillating less than the electron plasma frequency (Figure 4). There is also a strong frequency component for $k = 1$ near the plasma frequency.

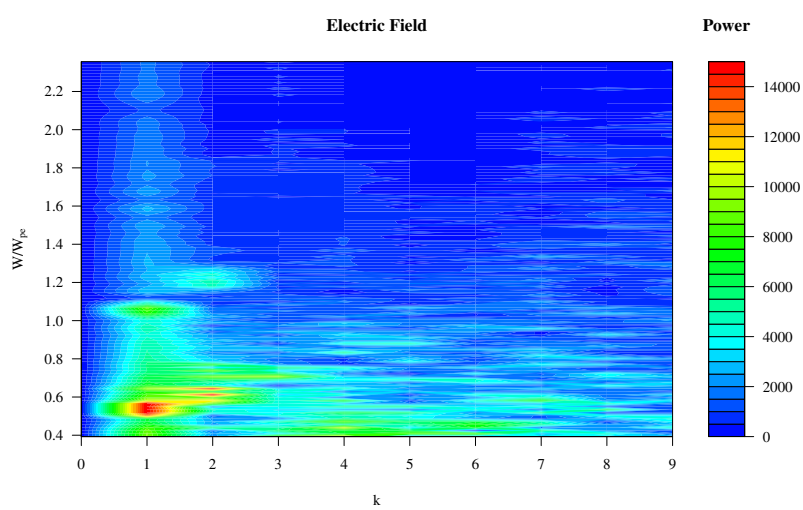

Figure 4: Dispersion relation of the system with frequencies normalised to the plasma frequency and k normalised to $2\pi/L$.

A portion of the electric field between $L = 50$ and 60 was analysed (Figure 5a) by taking the average of the electric field and then applying the Fourier transform at different time steps with a fixed sampling period to create a spectrogram (Figure 5b). The spectrogram confirms the growth of waves at frequencies below the plasma frequency. Initially there are little or no waves in the system but as the beam moves through the background plasma, both the beam and background plasma are distorted and generates waves near 40 time steps which are amplified near 50 time steps.

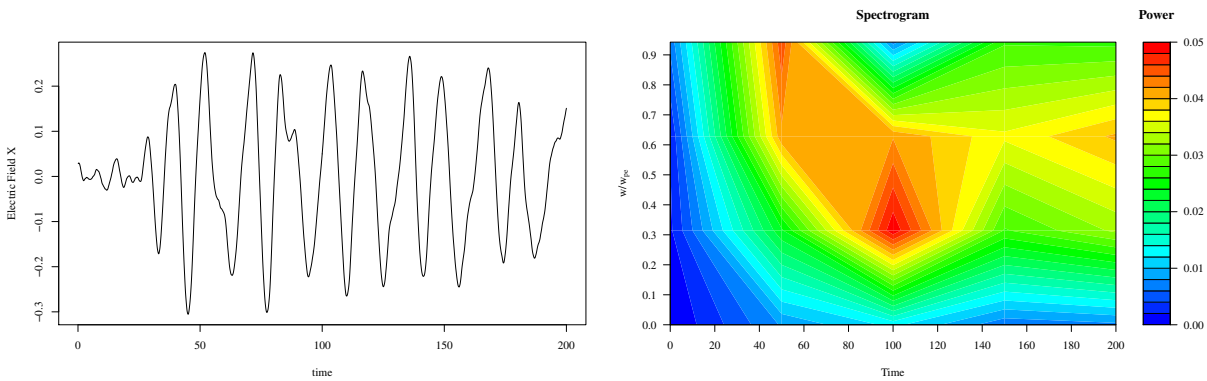

Figure 5: (a) Left: A portion of the electric field versus time. (b) Right: power spectrogram of frequency versus time.

5 Conclusion

PIC simulations prove to be successful for simulating the one dimensional case of the electron beam instability where both the background and beam have the same density. Potential holes are formed in phase space and the electron beam gets distorted. Waves are generated later as the electron beam moves through the cold plasma. This simple scenario has been used to validate the initial version of a code which is being developed to model VLF chorus generation.

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