Solitary Potential Structures in Presence of Non-thermal Electrons

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Abstract

Arbitrary amplitude electron-acoustic solitary waves are studied in an unmagnetized plasma consisting of non-thermally distributed electrons, fluid cold electrons, electron beam and ions using the Sagdeev psuedo-potential method. Positive potential solitary structures are obtained for the typical auroral region parameters. The electric field amplitude amplitude of these potential structures ranges from few mV/m to 200 mV/m.

1 Introduction

Broadband electrostatic noise (BEN) emissions have been observed everywhere in the Earth's magnetosphere, e.g., magnetopause, auroral field lines, polar cusp, magnetosheath and plasma sheet boundary layer (PSBL) etc. Frequencies of these emissions range from ion plasma and lower hybrid frequencies to electron plasma frequency and sometimes upto the cyclotron frequencies. Analysis of the high time resolution plasma wave data from GEOTAIL have shown for the first time, that BEN in the plasma sheet boundary layer actually consists of electrostatic solitary waves (ESWs) and its Fourier spectrum gives rise to the broadband nature of the noise [1]. The electrostatic solitary waves can have either positive or negative potentials, and their electric field amplitude can vary from a few mV/m in the PSBL to a few 100 mV/m in the dayside auroral zone. The velocities of ESWs can vary from $\sim$ a few hundred to a few thousand km s^{-1}. Electrostatic solitary structures have been observed in the almost all regions of the Earth's magnetosphere [1-5].

Electrostatic solitary waves are observed in conjuction with the electron beams. Therefore, in this paper, we extend the work of Singh et al.[6] by including an electron beam and study the properties of the electron-acoustic solitons in such plasmas. In the next section, formulation of the model is presented, numerical results are presented in section 3 and results are concluded in the last section.

2 Formulation

We consider a homogeneous, unmagnetized four component plasma consisting of non-thermal hot electrons, fluid cold electrons, electron beam and ions. The non-thermal distribution for the electrons is given by [7]

$$f_{0\,h}(v) = \frac{n_{oh}}{\sqrt{2\pi v_{th}^2}} \frac{(1+\frac{\alpha v^4}{v_{th}^4})}{(1+3\alpha)} exp\left(-\frac{v^2}{2v_{th}^2}\right) \tag{1}$$

where n_{oh} is the equilibrium hot electron density, v_{th} is the thermal speed of the hot electrons and α is a parameter which determines the population of energetic non-thermal electrons. The distribution of electrons in the presence of non-zero potential can be found by replacing $\frac{v^2}{v_{th}^2}$ by $\frac{v^2}{v_{th}^2} - \frac{2e\phi}{T_h}$. Thus, integration over the resulting distribution function gives the following expression for the electron density (Cairns et al., 1995)

$$n_h = n_{oh}(1 - \beta\phi + \beta\phi^2)exp(\phi), \tag{2}$$

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and the other governing equations of the model are given by

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j v_j) = 0, \tag{3}$$

$$\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial x} + \frac{1}{\mu_j n_j}\frac{\partial P_j}{\partial x} - \frac{Z_j}{\mu_j}\frac{\partial \phi}{\partial x} = 0, \tag{4}$$

$$\frac{\partial P_j}{\partial t} + v_j \frac{\partial P_j}{\partial x} + 3P_j \frac{\partial v_j}{\partial x} = 0 \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_h + n_b + n_c - n_i \tag{6}$$

where $j = c, b, i$ represents cold electrons, beam electrons and ions, respectively, $Z_j = \pm 1$ for electrons and ions, respectively, and $\mu_j = m_j/m_e$, $\beta = \dfrac{4\alpha}{(1+3\alpha)}$, where α determines the population of non-thermal electrons. It must be pointed out that equations (2)-(6) are normalized equations. We have normalized the densities by $n_o = n_{oc} + n_{oh} + n_{ob} = n_{oi}$, velocities by thermal velocity of hot electrons, $v_{th} = \sqrt{T_h/m_e}$, lengths by effective hot electron Debye length defined as $\lambda_{dh} = \sqrt{T_h/4\pi n_o e^2}$, temperature by hot electron temperature T_h, time by inverse of electron plasma frequency $\omega_{pe}^{-1} = \sqrt{m_e/4\pi n_o e^2}$, the potential by T_h/e, and, the thermal pressure by $n_o T_h$

In order to study the properties of arbitrary amplitude electrostatic solitary waves, we transform the above set of equations (2)-(6) to a stationary frame moving with velocity V, the phase velocity of the wave, i.e., ξ = (x- M t), where M=V/v_{th} is the Mach number with respect to the hot electron thermal velocity. Then, we solve for perturbed densities using equations (2)-(5) and substitute these expressions in the Poisson equation (6). Assuming appropriate boundary conditions for the localized disturbances along with the conditions that ϕ =0, and dϕ /dξ =0 at $\xi \to \pm\infty$, we obtain the following energy integral,

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0, \tag{7}$$

where $V(\phi, M)$ is the Sagdeev potential given by

$$\begin{aligned}
V(\phi) &= n_{oh}\left\{1 + 3\beta - (1 + 3\beta - 3\beta\phi + \beta\phi^2)e^{\phi}\right\} \\
&+ n_{oc}\left[M^2 - \frac{M}{\sqrt{2}}\left\{M^2 + 3T_c + 2\phi \pm \sqrt{(M^2 + 3T_c + 2\phi)^2 - 12T_c M^2}\right\}^{\frac{1}{2}}\right] \\
&+ n_{oc}T_c\left[1 - 2\sqrt{2}M^3\left\{M^2 + 3T_c + 2\phi \pm \sqrt{(M^2 + 3T_c + 2\phi)^2 - 12T_c M^2}\right\}^{-\frac{3}{2}}\right] \\
&+ n_{ob}\left[(M - v_o)^2 - \frac{(M - v_o)}{\sqrt{2}}\left\{(M - v_o)^2 + 3T_b + 2\phi \pm \sqrt{\left\{(M - v_o)^2 + 3T_b + 2\phi\right\}^2 - 12T_b(M - v_o)^2}\right\}^{\frac{1}{2}}\right] \\
&+ n_{ob}T_b\left[1 - 2\sqrt{2}(M - v_o)^3\left\{(M - v_o)^2 + 3T_b + 2\phi \pm \sqrt{\left\{(M - v_o)^2 + 3T_b + 2\phi\right\}^2 - 12T_b(M - v_o)^2}\right\}^{-\frac{3}{2}}\right] \\
&+ \mu_{ie}\left[M^2 - \frac{M}{\sqrt{2}}\left\{M^2 + \frac{3T_i}{\mu_{ie}} - \frac{2\phi}{\mu_{ie}} \pm \sqrt{\left(M^2 + \frac{3T_i}{\mu_{ie}} - \frac{2\phi}{\mu_{ie}}\right)^2 - \frac{12T_i M^2}{\mu_{ie}}}\right\}^{\frac{1}{2}}\right] \\
&+ T_i\left[1 - 2\sqrt{2}M^3\left\{M^2 + \frac{3T_i}{\mu_{ie}} - \frac{2\phi}{\mu_{ie}} \pm \sqrt{\left(M^2 + \frac{3T_i}{\mu_{ie}} - \frac{2\phi}{\mu_{ie}}\right)^2 - \frac{12T_i M^2}{\mu_{ie}}}\right\}^{-\frac{3}{2}}\right],
\end{aligned} \tag{8}$$

where $\mu_{ie} = m_i/m_e$. The first term on the right hand side (r.h.s.) of (8) represents the hot electron contribution to the Sagdeev potential. In the absence of non-thermal electrons, i.e., for $\alpha = \beta = 0$, the term reduces to usual Boltzmann distributed hot electron contribution [8-9]. The second and third terms on the r.h.s. of (8) represent the cold electron contribution, and the next two terms are due to beam electrons. The last two terms on the r.h.s. of (8) give the contribution of ions to the Sagdeev potential. To solve equation (8), one has to choose the plus (+) or minus (-) appearing in various terms on the r.h.s. of this equation very carefully as these expressions are related to the densities of the jth species. In order that the associated densities attain their undisturbed values in the limit of $\phi \to 0$ at $\xi \to \pm\infty$, we must use the + (plus) sign when the condition $(M - v_j)^2 + 2\frac{\phi}{\mu_j} > 3\frac{T_j}{\mu_j}$ is satisfied, and the -(minus) sign when $(M - v_j)^2 + 2\frac{\phi}{\mu_j} < 3\frac{T_j}{\mu_j}$ is satisfied [10].

3 Numerical results

We have numerically solved Eq. (8) for the Sagdeev potential, $V(\phi)$, as a function of real potential ϕ for various values of Mach numbers and for some typical normalized plasma parameters, namely, cold electron density, n_{oc}=0.7, hot electron density n_{oh}=0.05, beam electron density n_{0b}=0.25, electron beam speed, v_0=0.1, ratio of beam to hot electron temperatures, T_b/T_h=0.4, ratio of cold to hot electron temperatures, T_c/T_h=0.01=T_i/T_h (ratio of ion to hot electron temperatures) and non-thermal parameter $\alpha = 0.2$ The results are shown in Figure 1 with corresponding Mach numbers labelled on the curves . The solitary wave solutions of electron-acoustic waves are found when the Mach numbers exceed the critical values ($M > M_0$). It may be pointed out here that the soliton solutions exist for a narrow Mach number range of $1.05 < M < 1.18$ and for positive potential structures for the parameters considered above. The maximum electrostatic potential ϕ_0 increases with the increase of the Mach number, M, as can be seen from the curves. There is an upper limit on value of M, say M_{max}, above which soliton solutions do not exist. It may be recalled that only negative potential structures could be found in three-component plasma model of Singh and Lakhina [6] consisting of cold and non-thermal electrons and ions.

In Figure 2, variation of Sagdeev potential $V(\phi)$ with normalized potential ϕ is shown for different values of the electron beam speed v_0 as indicated on the curves. The chosen parameters are same as of Figure 1 and Mach number M=1.15. From the figure it is evident that maximum amplitude of the positive potential structures reduces as electron beam speed increases. These results are consistent with Singh et al. [9] results for the negative potential structures in a four-component plasma model consisting of cold, beam and hot electrons (Maxwelian) and ions.

4 Discussion

Properties of the electron-acoustic solitary waves in non-thermal, unmagnetized four-component plasma have been examined. Present theoretical model is the extension of the model used by Singh and Lakhina [6] by including the beam electrons. Inclusion of electron beam in the model does reduce the maximum amplitude of the electron-acoustic solitary waves. For the parameters representative of auroral zone, both positive and negative potential electron-acoustic solitons are obtained.

Numerical integration of equation (7) directly yields the normalized electric field amplitude, $d\phi/d\xi$ of the electron-acoustic solitary waves. The calculated electric field amplitude of these structures comes out to be in the range of (80-200) mV/m for the representative parameters, namely, n_{oc}=0.7, n_{oh}=0.05, n_{0b}=0.25, v_0=0.1, T_b/T_h=0.4, T_c/T_h=0.001=T_i/T_h and non-thermal parameter $\alpha = 0.2$ for the mach number $1.1 < M < 1.15$ for hot electron temperature $T_h = 250eV$ and total electron density, n_0=3.5 cm^{-3}. The widths of the solitary structures for the above mentioned parameters ranges fall in the range of (225-300)m and the soliton velocity are $\sim$ (7000-7700)km s^{-1}.

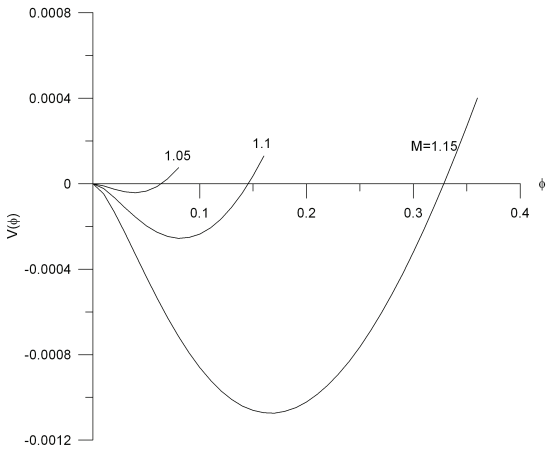

Figure 1: Sagdeev potential, $V(\phi)$ versus potential ϕ for various values of Mach numbers as indicated on the curves. Other normalized plasma parameters are n_{oc}=0.7, n_{oh}=0.05, n_{0b}=0.25, v_0=0.1, T_b/T_h=0.4, T_c/T_h=0.001,$T_i/T_h = 0.01$ and $\alpha = 0.2$

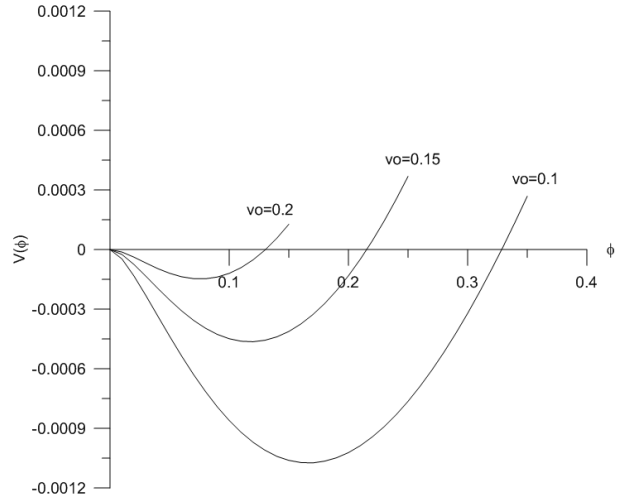

Figure 2: Sagdeev potential, $V(\phi)$ versus potential ϕ for various values of beam speed as indicated on the curves and for M=1.15. Other normalized plasma parameters are same as in figure 1

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6 References

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