

# Revisiting Ion Acoustic Solitary Waves in magnetized and Un-magnetized Plasmas

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## Abstract

Ion acoustic solitary waves have been studied in detail, using Sagdeev pseudopotential technique. It has revealed interesting features, like coexistence of positive and negative amplitude solutions for ion acoustic solitary waves in a magnetized plasma and anomalous width variations for large amplitude solutions.

## 1 Introduction

Ion acoustic solitary waves have been studied extensively by many authors and found to be relevant for large amplitude, dynamic structures observed at the auroral region. In recent days, there are renewed interest to study the inherent properties of solitary waves for various plasma models [1]. In the present work, ion acoustic solitary waves have been studied using Sagdeev pseudopotential technique. It is observed that both positive and negative amplitude solitary waves may coexist in a magnetized plasma. It has also been shown that the analytical estimation of width-amplitude variation agreed qualitatively with satellite observations.

## 2 Derivation of the Sagdeev Pseudopotential

In the present analysis, fully nonlinear solutions for rarefactive ion acoustic solitary waves have been obtained for a two electron temperature magnetized plasma where the magnetic field is parallel to  $z$  direction and the wave is propagating obliquely in the  $y$ - $z$  plane with an angle  $\theta$  with the ambient magnetic field. The parameter  $\alpha$  denotes the ratio of the ion cyclotron and ion plasma frequencies ( $\alpha = \omega_{ci}/\omega_{pi}$ ). A parallel beam of  $O^+$  ions has been considered moving with a beam velocity  $u$ . The bulk of the plasma is consisted of  $H^+$  ions which are magnetized. Since the  $O^+$  ions are much heavier, they are assumed to be unmagnetized. A Sagdeev pseudopotential technique has been adopted to get the required solutions [2]. The set of the normalized fluid equations are —

$$\frac{\partial N_b}{\partial t} + \nabla \cdot (N_b \mathbf{v}_b) = 0, \quad (1)$$

$$\frac{\partial \mathbf{v}_b}{\partial t} + (\mathbf{v}_b \cdot \nabla) \mathbf{v}_b = -Q [\nabla \phi + 3\sigma N_b \nabla N_b], \quad (2)$$

$$\frac{\partial N_i}{\partial t} + \nabla \cdot (N_i \mathbf{v}_i) = 0, \quad (3)$$

$$\frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla) \mathbf{v}_i = [\nabla \phi + 3\sigma N_i \nabla N_i] + \alpha (\mathbf{v}_i \times \mathbf{b}); \quad (4)$$

with an electron density of

$$n_e = n_{ec} + n_{ew} = \mu e^{\frac{\phi}{\mu + \nu \beta}} + \nu e^{\frac{\beta \phi}{\mu + \nu \beta}}, \quad (5)$$

where  $N_{i,b} = n_{i,b}/\rho_{i,b}$  respectively,  $\rho_{i,b}$  being the ambient densities of  $H^+$  ions and  $O^+$  beam ions respectively. The subscripts  $i$  and  $b$  denote  $H^+$  ions and  $O^+$  beam ions respectively and the usual normalization process has been adopted [4]. Since the bulk is assumed to be consisted of  $H^+$  ions, the parameters are normalized by  $H^+$  ion mass ( $Q = m_b/m_i$ ). To evaluate (1–4), we assume quasineutrality condition. All other parameters are described in [2]

Assuming the equation of state  $[p_{i,b} \propto n_{i,b}^3]$ , and the charge neutrality condition  $[n_i = n_e - n_b]$ , the stationary state solution is obtained for the transformation  $\eta = k_y y + k_z z - Mt$  where  $M$  is the Mach no. and  $k_{y,z}$  are the corresponding direction cosines for the oblique propagation.

$$\frac{d}{d\eta} \left[ \frac{1}{2} \frac{1}{N_i} \frac{d^2}{d\eta^2} \left\{ \left( \frac{M}{N_i} \right)^2 + 3\sigma N_i^2 + 2\phi \right\} + \alpha^2 \left\{ \frac{1}{N_i} + \frac{k_z^2}{M^2} (\sigma N_i^3 + N_i') \right\} \right] = 0, \quad (6)$$

where

$$N_i' = \int N_i d\phi.$$

Solving the coupled equations (??) and (6) we get —

$$\frac{1}{2} \left( \frac{d\phi}{d\eta} \right)^2 + \psi_{ion}(\phi) = 0, \quad (7)$$

where  $\psi_{ion}(\phi)$  is the corresponding Sagdeev pseudopotential for an ion acoustic wave.

$$\psi_{ion}(\phi) = V(\phi) \frac{L(\phi)}{H(\phi)}, \quad (8)$$

$V(\phi)$ ,  $L(\phi)$ ,  $H(\phi)$  being functions of  $\phi$ ,  $M$  and other parameters. Solitary wave solutions are obtained for usual boundary conditions

$$\psi_{ion}(0) = \frac{\partial \psi_{ion}(0)}{\partial \phi} < 0 \quad ; \quad \frac{\partial^2 \psi_{ion}(0)}{\partial \phi^2} = 0; \psi_{ion}(\phi_0) = 0; \psi_{ion}(\phi) < 0 \text{ for } 0 < |\phi| < |\phi_0|, \quad (9)$$

$|\phi_0|$  being the amplitude of the corresponding solitary wave solution.

### 3 Result and Discussion

Figure 1 shows the Sagdeev pseudopotential curves for an ion acoustic wave. It reveals that both the positive and negative polarity waves coexist for the same parameter space. Fig. 2 shows corresponding rarefactive ion acoustic solitary wave solutions obtained numerically by integrating (7) with (8). The potential profiles presented in Fig. 2 readily shows an increase in the width with increasing amplitude. The result was further corroborated in Fig. 3 presenting the analytical estimation of width–amplitude variations (solid curves) with recent POLAR observations by Dombeck et al. (presented by the points) [3]. The qualitative agreement between the two gives further credential to the anomalous width variation (increase of the width with increasing amplitude for large amplitude solutions) emphasizing characteristic differences between fully nonlinear and weakly nonlinear solutions.

## 4 Conclusion

Ion acoustic solitary waves have been studied in a magnetized, warm ion plasma. It has been shown that both positive and negative polarity waves may coexist for a specific parameter space. It has also been shown that the estimated width-amplitude variation for large amplitude solutions agree qualitatively with satellite observations. Further studies are required for understanding the characteristics of ion acoustic solitary waves in mmore detail.

## 5 Acknowledgments

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## 6 References

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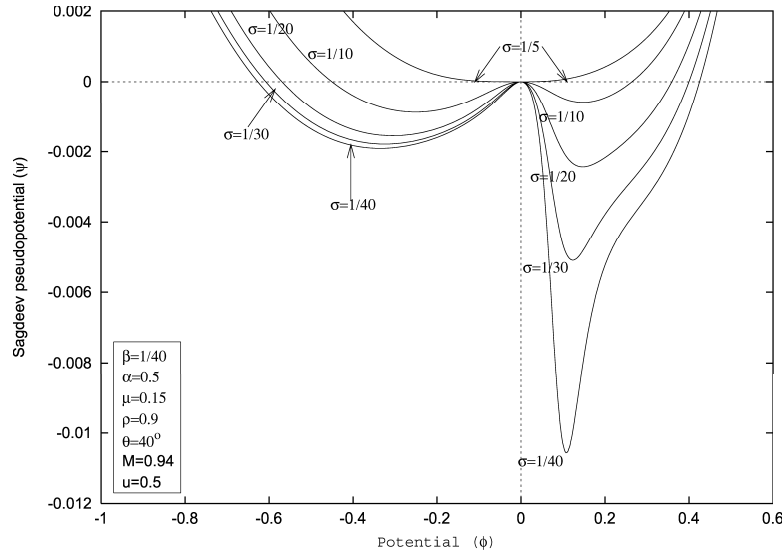


Figure 1: Sagdeev pseudopotentials for ion acoustic solitary waves in a magnetized plasma .

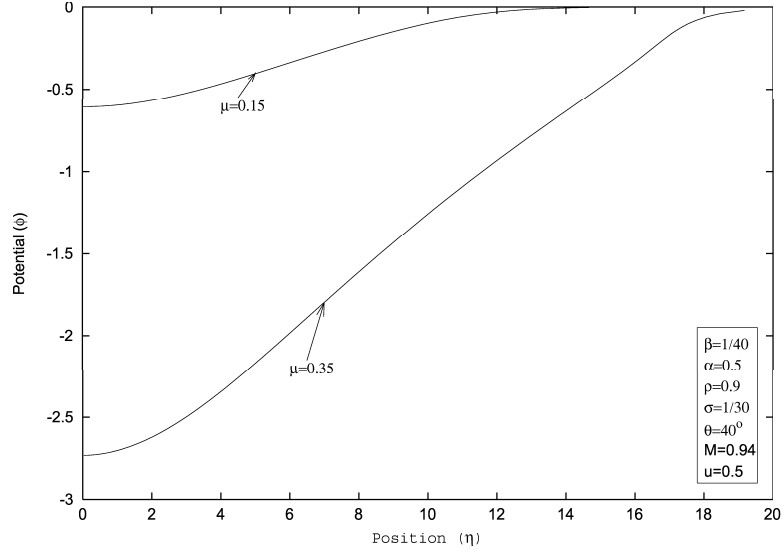


Figure 2: Potential profiles for rarefactive ion acoustic solitary waves in a magnetized plasma.

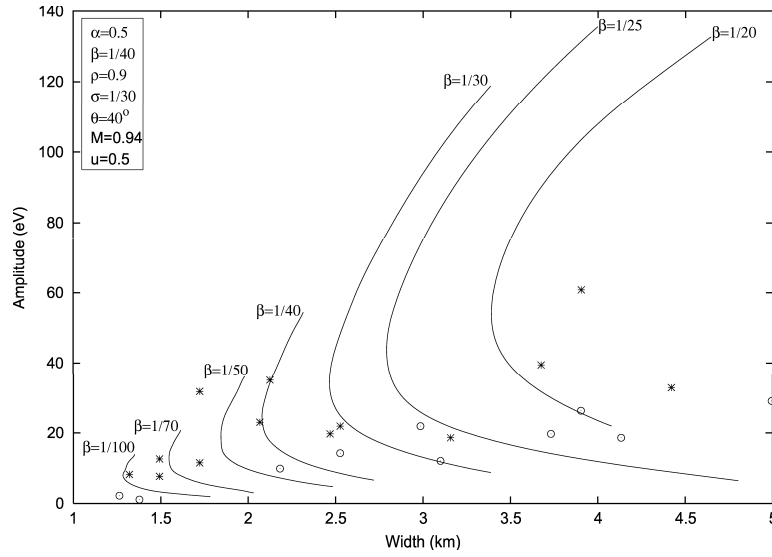


Figure 3: Width–amplitude variation profiles for rarefactive ion acoustic solitary waves in a magnetized plasma. The points represent the observational values for two different bursts as presented by Dombeck et al. [8]