Excitation of Whistler Waves in a Magnetoplasma Containing a Nonuniform Density Depletion Duct

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Abstract

Whistler wave excitation in a laboratory magnetoplasma containing a cylindrically symmetric density depletion duct is studied. Using a rigorous solution for the total source-excited field, the radiation resistance of a circular loop antenna located in such a duct is determined. Conditions are found under which the radiation resistance of the loop antenna in a nonuniform density depletion duct can be notably greater than that in a homogeneous magnetoplasma whose parameters coincide with those near the duct axis or outside the duct.

1. Introduction

Excitation of whistler waves by loop antennas in a magnetoplasma containing density ducts was studied in many works (see, e.g., [1, 2] and references therein). Most papers on the subject discuss ducts with enhanced plasma density, and it has been shown that the radiation resistance of a loop antenna located in such a duct turns out to be greater than that in the surrounding plasma and increases with increasing plasma density in the duct. It is the purpose of the present work to demonstrate that under certain conditions, the radiation resistance of a loop antenna in the whistler range can also be increased if the antenna is placed in a duct with depleted density. To analyze the features of excitation of whistler waves in this case, we use a full-wave approach and focus on cylindrical ducts which can be formed near electromagnetic sources in the laboratory magnetoplasma due to various nonlinear effects and the radii of which are usually comparable to or less than typical wavelengths of the guided modes.

2. Formulation of the Problem and Basic Equations

We consider a cold unbounded collisionless magnetoplasma containing a cylindrical duct in which the plasma density is a function of radial distance from the duct axis taken as the z axis of a cylindrical coordinate system (ρ, ϕ, z). Parallel to this axis is an external static magnetic field $\boldsymbol{B}_0 = B_0\hat{\boldsymbol{z}}_0$. The plasma is assumed to be described by the cold-plasma dielectric tensor $\boldsymbol{\varepsilon}$ whose nonzero elements are written as follows: $\varepsilon_{\rho\rho} = \varepsilon_{\phi\phi} = \epsilon_0\varepsilon$, $\varepsilon_{\rho\phi} = -\varepsilon_{\phi\rho} = -i\epsilon_0 g$, and $\varepsilon_{zz} = \epsilon_0\eta$, where ϵ_0 is the permittivity of free space. Expressions for the quantities ε, g, and η are given elsewhere [1]. It is assumed that the frequency ω belongs to the whistler range

$$\omega_{\mathrm{LH}} \ll \omega \ll \omega_{\mathrm{H}} \ll \omega_{\mathrm{p}}, \tag{1}$$

where ω_{LH} is the lower hybrid frequency, ω_{H} is the electron gyrofrequency, and ω_{p} is the electron plasma frequency.

We take the following model of a density duct in a magnetoplasma. It is assumed that the plasma density N is a constant, $\tilde{N}$, in an inner core $\rho < a_0$, a constant, N_a, in an outer region $\rho > a_1$, and varies smoothly from $\tilde{N}$ to N_a within the duct wall $a_0 < \rho < a_1$ by the law

$$N(\rho) = \{\tilde{N} + N_a + (\tilde{N} - N_a)\sin[\pi(a - \rho)/(a_1 - a_0)]\}/2, \tag{2}$$

978-1-4244-6051-9/11/$26.00

where $a = (a_0 + a_1)/2$ and $\tilde{N} < N_a$. The field is excited by a circular loop antenna the electric current in which (with the $\exp(i\omega t)$ time dependence dropped) is given by

$$\boldsymbol{J}(\rho, z) = \hat{\boldsymbol{\phi}}_0 I_0 \delta(\rho - b)\delta(z), \tag{3}$$

where δ is a Dirac function, b is the loop radius ($b < a$), and I_0 is the total antenna current.

The solution of the Maxwell equations outside the source possessing current (3), with allowance for the azimuthal symmetry of the problem, can be sought in terms of the axisymmetric modal fields

$$\begin{bmatrix} \boldsymbol{E}_{s,\alpha}(\boldsymbol{r}, q) \\ \boldsymbol{B}_{s,\alpha}(\boldsymbol{r}, q) \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_{s,\alpha}(\rho, q) \\ \boldsymbol{B}_{s,\alpha}(\rho, q) \end{bmatrix} \exp[-ik_0 p_{s,\alpha}(q) z], \tag{4}$$

where q is the transverse wave number in the ambient magnetoplasma ($\rho > a_1$), normalized to the free-space wave number $k_0 = \omega/c$; the function $p_{s,\alpha}(q)$ describes the dependence of p, the axial wave number normalized to k_0, on the transverse wave number q for the "ordinary" ($\alpha = \mathrm{o}$) and "extraordinary" ($\alpha = \mathrm{e}$) characteristic waves of the ambient uniform plasma; the subscript s denotes the wave propagation direction ($s = -$ and $s = +$ designate waves propagating in the negative and positive directions of the z axis, respectively); and $\boldsymbol{E}_{s,\alpha}(\rho, q)$ and $\boldsymbol{B}_{s,\alpha}(\rho, q)$ are the vector wave functions describing the radial distribution of the field of a mode corresponding to the transverse wave number q and the indices s and α. The functions $p_{s,\alpha}(q)$ obey the relation $p_{+,\alpha}(q) \equiv p_\alpha(q) = -p_{-,\alpha}(q)$, where

$$p_\alpha(q) = \left[\varepsilon_a - \frac{1}{2}\Big(1 + \frac{\varepsilon_a}{\eta_a}\Big) q^2 + \chi_\alpha R_p(q)\right]^{1/2}, \quad R_p(q) = \left[\frac{1}{4}\Big(1 - \frac{\varepsilon_a}{\eta_a}\Big)^2 q^4 - \frac{g_a^2}{\eta_a} q^2 + g_a^2\right]^{1/2}. \tag{5}$$

Here, ε_a, g_a, and η_a denote the quantities ε, g, and η, respectively, in the outer region, and $\chi_{\mathrm{o}} = -\chi_{\mathrm{e}} = -1$. It is assumed that $\operatorname{Re} R_p(q) > 0$ and $\operatorname{Im} p_\alpha(q) < 0$. Note that the functions $\boldsymbol{E}_{s,\alpha}(\rho, q)$ and $\boldsymbol{B}_{s,\alpha}(\rho, q)$ can be expressed in terms of two scalar functions $E_{\phi;\, s,\alpha}(\rho, q)$ and $B_{\phi;\, s,\alpha}(\rho, q)$.

It can be shown that in the considered case, a complete set of modes over which the total field can be expanded comprises the discrete spectrum of transversely localized eigenmodes, which are guided by the density duct, and the continuous spectrum of unguided modes that correspond to positive real values of q. In the uniform outer region ($\rho > a_1$), the fields of the continuous-spectrum modes are written as follows:

$$\begin{aligned} E_{\phi;\, s,\alpha}(\rho, q) &= i\left[\sum_{k=1}^{2} C_{s,\alpha}^{(k)}(q) H_1^{(k)}(k_0 q \rho) + D_{s,\alpha}(q) H_1^{(2)}(k_0 q_\alpha \rho)\right], \\ B_{\phi;\, s,\alpha}(\rho, q) &= -c^{-1}\left[\sum_{k=1}^{2} C_{s,\alpha}^{(k)}(q) n_{s,\alpha}^{(1)} H_1^{(k)}(k_0 q \rho) + D_{s,\alpha}(q) n_{s,\alpha}^{(2)} H_1^{(2)}(k_0 q_\alpha \rho)\right]. \end{aligned} \tag{6}$$

Here, $H_1^{(1,2)}$ are Hankel functions of the first and second kinds, $C_{s,\alpha}^{(1,2)}$ and $D_{s,\alpha}$ are coefficients to be determined, and

$$n_{s,\alpha}^{(1,2)}(q) = -\frac{\varepsilon_a}{p_{s,\alpha}(q) g_a}\Big[(q_\alpha^{(1,2)})^2 + p_\alpha^2(q) + \frac{g_a^2}{\varepsilon_a} - \varepsilon_a\Big],\ q_\alpha(q) = \left[\varepsilon_a - p_\alpha^2(q) - \frac{g_a}{\varepsilon_a}\left(g_a - \frac{\eta_a p_{s,\alpha}(q)}{n_{s,\alpha}^{(1)}(q)}\right)\right]^{1/2}, \tag{7}$$

where $q_\alpha^{(1)} = q$, $q_\alpha^{(2)} = q_\alpha(q)$, and $\operatorname{Im} q_\alpha(q) < 0$.

In the nonuniform part of the duct, the field equations can be solved only numerically. There are four independent solutions for the field in the region $\rho < a_1$, of which two solutions, hereafter denoted as $\tilde{E}_{\phi;\, s,\alpha}^{(1)}(\rho, q)$, $\tilde{B}_{\phi;\, s,\alpha}^{(1)}(\rho, q)$ and $\tilde{E}_{\phi;\, s,\alpha}^{(2)}(\rho, q)$, $\tilde{B}_{\phi;\, s,\alpha}^{(2)}(\rho, q)$, are regular at $\rho = 0$. Then the solution for the field in the region $\rho < a_1$ is written as

$$E_{\phi;\, s,\alpha}(\rho, q) = i\sum_{k=1}^{2} A_{s,\alpha}^{(k)}(q) \tilde{E}_{\phi;\, s,\alpha}^{(k)}(\rho, q), \quad B_{\phi;\, s,\alpha}(\rho, q) = -c^{-1}\sum_{k=1}^{2} A_{s,\alpha}^{(k)}(q) \tilde{B}_{\phi;\, s,\alpha}^{(k)}(\rho, q), \tag{8}$$

where $A_{s,\alpha}^{(1,2)}$ are coefficients to be determined. In the uniform inner core ($\rho < a_0$), the particular solutions $\tilde{E}_{\phi;\, s,\alpha}^{(k)}(\rho, q)$ and $\tilde{B}_{\phi;\, s,\alpha}^{(k)}(\rho, q)$ can be represented explicitly in terms of cylindrical functions [1]. Knowing the values of these functions and their derivatives at $\rho = a_0$, one can easily find the functions $\tilde{E}^{(k)}_{\phi;\,s,\alpha}(\rho, q)$ and $\tilde{B}^{(k)}_{\phi;\,s,\alpha}(\rho, q)$ in the nonuniform region $a_0 < \rho < a_1$ by numerically solving the wave equations for each q. Such a procedure automatically ensures the continuity of the tangential field components at $\rho = a_0$. Next, satisfying the continuity conditions for the tangential field components at $\rho = a_1$, we arrive at the system of linear equations for unknown coefficients $A^{(1,2)}_{s,\alpha}$, $C^{(1,2)}_{s,\alpha}$, and $D_{s,\alpha}$. This system can be represented in matrix form as $\mathbf{S}\cdot\boldsymbol{G} = C^{(1)}_{s,\alpha}\,\boldsymbol{F}$, where the elements of the column vector $\boldsymbol{G}$ are given by the expressions $G_{1,2} = A^{(1,2)}_{s,\alpha}$, $G_3 = C^{(2)}_{s,\alpha}$, and $G_4 = D_{s,\alpha}$. The elements of the matrix $\mathbf{S}$ and the components of the column vector $\boldsymbol{F}$, which are not written here for brevity, are expressed in an obvious manner via particular field solutions entering the representations of the tangential fields on both sides of the interface $\rho = a_1$. The above-mentioned matrix equation gives four linear relationships for five coefficients $A^{(1,2)}_{s,\alpha}$, $C^{(1,2)}_{s,\alpha}$, and $D_{s,\alpha}$, so that one of these coefficients can be taken arbitrary [1]. For numerical calculations, it is most convenient to put $C^{(1)}_{s,\alpha} = \det||\boldsymbol{S}||$ and then determine the remaining coefficients.

The transversely localized eigenmodes (also called the discrete-spectrum modes) correspond to discrete complex quantities q_n ($n = 0, 1, 2, \ldots$). These quantities are zeros of the coefficient $C^{(1)}_{s,\alpha}$ and have the negative imaginary part (i.e., $\operatorname{Im} q_n < 0$). The substitution of q_n into $p_{s,\alpha}(q)$ yields the axial wave numbers $p_{s,n}$ of the eigenmodes. It is adopted that $p_{\pm,n} = \pm p_n$. For shortening the writing, we represent the fields of the discrete-spectrum modes as $\boldsymbol{E}_{s,n}(\boldsymbol{r}) = \boldsymbol{E}_{s,\alpha}(\rho, q_n)\exp(-ik_0 p_{s,n} z)$ and $\boldsymbol{B}_{s,n}(\boldsymbol{r}) = \boldsymbol{B}_{s,\alpha}(\rho, q_n)\exp(-ik_0 p_{s,n} z)$.

With allowance for the performed analysis, the field excited by an axisymmetric source in the presence of a density duct is given by the following expansion:

$$\begin{bmatrix}\boldsymbol{E}(\boldsymbol{r})\\ \boldsymbol{B}(\boldsymbol{r})\end{bmatrix} = \sum_n a_{s,n}\begin{bmatrix}\boldsymbol{E}_{s,n}(\boldsymbol{r})\\ \boldsymbol{B}_{s,n}(\boldsymbol{r})\end{bmatrix} + \sum_\alpha \int_0^\infty a_{s,\alpha}(q)\begin{bmatrix}\boldsymbol{E}_{s,\alpha}(\boldsymbol{r},q)\\ \boldsymbol{B}_{s,\alpha}(\boldsymbol{r},q)\end{bmatrix} dq, \tag{9}$$

where $a_{s,n}$ and $a_{s,\alpha}$ are the excitation coefficients of the respective modes. In expansion (9), one should put $s = +$ for $z > 0$ and $s = -$ for $z < 0$. Next, following the well-known technique for finding the excitation coefficients of modes of open waveguides [1], we can obtain for source (3) that $a_{\pm,n} = 2\pi b I_0 N_n^{-1} E^{(\mathrm{T})}_{\phi;\,\mp,n}(b)$ and $a_{\pm,\alpha}(q) = 2\pi b I_0 N_\alpha^{-1}(q) E^{(\mathrm{T})}_{\phi;\,\mp,\alpha}(b, q)$, where the superscript (T) denotes fields taken in a medium described by the transposed dielectric tensor $\boldsymbol{\varepsilon}^{\mathrm{T}}$, and the normalization quantities N_n and $N_\alpha(q)$ for modes are given by

$$\begin{aligned} N_n &= \frac{2\pi}{\mu_0}\int_0^\infty \left[\boldsymbol{E}_{+,n}(\rho)\times\boldsymbol{B}^{(\mathrm{T})}_{-,n}(\rho) - \boldsymbol{E}^{(\mathrm{T})}_{-,n}(\rho)\times\boldsymbol{B}_{+,n}(\rho)\right]\cdot\hat{\boldsymbol{z}}_0\,\rho\,d\rho, \\ N_\alpha(q) &= -\frac{16\pi}{Z_0 k_0^2}\left(\frac{dp_\alpha(q)}{dq}\right)^{-1}\left[1 + \eta_a^{-1}\left(n^{(1)}_{+,\alpha}\right)^2\right] C^{(1)}_{+,\alpha}(q) C^{(2)}_{+,\alpha}(q), \end{aligned} \tag{10}$$

where μ_0 and Z_0 are the permeability and wave impedance of free space, respectively.

The total radiation resistance of the antenna with current (3) is written as $R_\Sigma = 2P_\Sigma/|I_0|^2 = \sum_n R_n + R_{\mathrm{cs}}$, where P_Σ is the total radiated power and

$$R_n = -2\pi b\,\mathrm{Re}\,[I_0^{-1} a_{s,n} E_{\phi;\,s,n}(b)], \quad R_{\mathrm{cs}} = -2\pi b\,\mathrm{Re}\int_0^\infty I_0^{-1} a_{s,\mathrm{e}}(q) E_{\phi;\,s,\mathrm{e}}(b,q)dq. \tag{11}$$

The quantities R_n and R_{cs} are the partial radiation resistances corresponding to the discrete- and continuous-spectrum modes, respectively. Note that the "ordinary" wave is evanescent in range (1) and does not contribute to the radiation.

3. Numerical Results

An examination shows that in range (1), a finite number of volume eigenmodes with the indices $n = 1, 2, \ldots$ can exist in density depletion ducts. The normalized axial wave numbers of such modes lie in the range $2\tilde{\varepsilon}^{1/2} < p < 2\varepsilon_a^{1/2}$. Hereafter, the tilde quantities refer to the inner region of the density duct. In addition, such structures can guide no more than one axisymmetric eigenmode of the surface type with the index $n = 0$ and the axial wave number $p < 2\tilde{\varepsilon}^{1/2}$. However, modes of the surface type are of little interest since they are excited inefficiently by source (3).

Figure 1 shows the results of numerical computations of the total radiation resistance R_Σ of the loop antenna with current (3) as a function of the antenna radius b in the presence of a density depletion duct and the analogous dependences of the partial radiation resistances R_n for the volume eigenmodes. Note that the values of dimensionless parameters chosen for numerical calculations are appropriate to conditions typical of the corresponding modeling laboratory experiments (see [1] and references therein). For comparison, Fig. 1(a) also shows the total radiation resistance of the same antenna immersed in a homogeneous magnetoplasma with the electron plasma frequency $\tilde{\omega}_{\mathrm{p}}$ (the curve of $R_\Sigma(b)$ for a homogeneous magnetoplasma with the electron plasma frequency ω_{p} corresponding to density N_a is very close to curve *2*, and is not shown in Fig. 1(a)). We can infer from Fig. 1 that for the chosen parameters, the radiation resistance of the loop antenna located in a weakly nonuniform density depletion duct can be several times greater than the radiation resistance of the same source immersed in a homogeneous plasma with inner or background density. Note that in the considered case, the surface eigenmode and the continuous-spectrum modes give the very small contribution to R_Σ. It turns out that the contribution of the continuous-spectrum modes to the total radiation resistance is negligible if $\tilde{\omega}_{\mathrm{p}} b/c > 1$. Thus, the maxima of R_Σ in our case are determined by the maxima of R_n for the dominant volume modes. With increasing duct radius a, the number of guided modes accordingly increases, the maxima of R_Σ become smoothed, and the gain in the total radiation resistance no longer takes place.

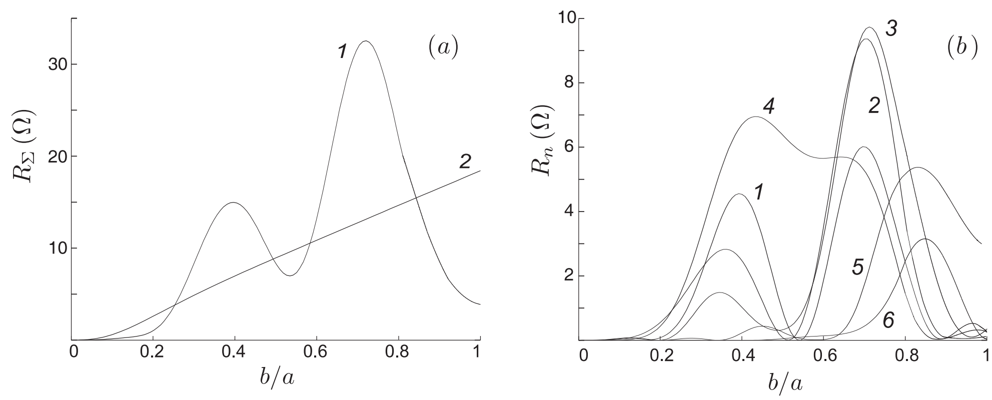

Figure 1. (a) Total radiation resistance of the loop antenna as a function of its radius b in the cases where the antenna is located in a density depletion duct with the above-described density profile (curve *1*) and in a homogeneous magnetoplasma whose density coincides with that in the inner region of the duct (curve *2*). (b) Partial radiation resistances for the eigenmodes as functions of the loop radius b (the curves are labeled in order of increasing axial wave numbers of the eigenmodes). $\omega/\omega_{\mathrm{H}} = 0.3$, $\tilde{\omega}_{\mathrm{p}}/\omega_{\mathrm{p}} = 0.82$, $\omega_{\mathrm{p}}/\omega_{\mathrm{H}} = 29.3$, $\omega_{\mathrm{H}} a/c = 0.42$, $a_0/a = 0.8$, and $a_1/a = 1.2$.

4. Conclusion

We have studied the features of whistler wave excitation in a magnetoplasma containing a weakly nonuniform duct with depleted density. It has been established that under conditions of the laboratory plasma, the radiation resistance of the loop antenna located in such a guiding structure can be notably greater than that in a homogeneous magnetoplasma whose parameters coincide with those near the duct axis or outside the duct.

5. Acknowledgments

This work was supported by the Russian Foundation for Basic Research (project No. 09–02–00164-a) and the Russian Federal Program “Scientific and Education Personnel of the Innovative Russia” (contract No. P313).

6. References

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