Particle-Wave-Particle Interactions Involving Whistler-Mode Waves in the Magnetosphere

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Abstract

Search for and understanding of mechanisms for particle energization is a key problem in physics of the Earth's radiation belts. A good deal of suggested mechanisms is related to resonant interactions between waves and energetic particle. In the plasmaspheric region of the magnetosphere, the energy density variation of resonant particles is often much larger than the wave energy density which contradicts energy conservation under the prevalent assumption that the wave serves as the energy source or sink. By examples of whistler-mode wave-electron interactions we emphasize that, in many cases, the energy increase (or decrease) of resonant particles is related to energy transfer from (to) other group of resonant particles, while the wave basically mediates the energization process. The importance of energy transfer between different electron populations for energization processes in the radiation belts has been pointed out in [1, 2].

1 Introduction

A systematic investigation of resonant wave-particle interaction in the magnetosphere has been undertaken in 70th, and the idea of electron acceleration caused by interaction with quasimonochromatic whistler-mode waves can be traced back to the corresponding works from this period. We will refer to a review paper [3] where the references to the most important studies on this subject fulfilled by that time can be found. Recently, the interest to this issue has been recommenced in connection with the problem of spacecraft safe functioning in the Earth's radiation belts. The idea of electron acceleration by whistler-mode waves has been developed and enriched by including relativistic effects into consideration [4, 5], and by considering electron acceleration by whistler-mode waves of varying frequency [4, 6].

In many cases, the interacting system may be viewed as consisting of the wave with energy density U that includes oscillation energy of "cold" (non-resonant) particles, and two groups of resonant particles with kinetic energy densities W_1 and W_2 varying oppositely. As is well known, such groups of particles may be introduced in the case of Landau damping of Langmuir wave. In the case of parallel propagating whistler-mode wave, when only one Doppler-shifted cyclotron resonance is in effect, those may be particles with close parallel velocities, but essentially different pitch angles [1]; in the case of oblique propagation those may be particles interacting with the wave at different resonances, say, at the first cyclotron resonance and the Cerenkov resonance. Another example is provided by a whistler-mode wave propagating along a non-uniform magnetic field, when two groups are formed by phase-trapped and phase-untrapped particles.

The energy conservation requires that the variations of the quantities defined above satisfy the equation:

$$\Delta U + \Delta W_1 + \Delta W_2 = 0; \tag{1}$$

There are only two possibilities to satisfy this equation: either all three quantities are of the same order, or one is much smaller than other two, which are close in magnitude, but have different signs. The question is, which of these two possibilities is in point of fact in each particular case. This is not an idle question, and the answer to it is not obvious. We will now show that in two cases of interest for magnetospheric applications, namely, for parallel and obliquely propagating whistler-mode waves, the following relations hold:

$$\Delta U \ll (\Delta W_1\ ,\ W_2)\ ;\quad \Delta W_1 \simeq -\Delta W_2\ .$$

Thus, in both cases, the energy exchange is basically between two groups of particles, while the wave mainly mediates the process of energy transfer.

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2 P-W-P interaction in the case of parallel propagating whistler-mode wave

The variation of electron kinetic energy w caused by the interaction with the wave propagating along the ambient magnetic field $\mathbf{B}_0 \parallel \mathbf{z}$ is described by the equation:

$$\frac{dw}{dt} = \frac{e\omega B}{kc} v_\perp \cos\zeta \ ; \quad \zeta = \int^z k(z')dz' - \omega t + \varphi \ , \tag{2}$$

where B is the amplitude of the wave magnetic field, φ is particle gyrophase, and the rest of notation here and hereinafter is standard. For resonant particles of interest whose parallel velocity $v_\parallel$ is close to the resonance value $v_{\rm R} = (\omega - \omega_c)/k$, the variation of transversal adiabatic invariant $\mu = mv_\perp^2/2\omega_c$ is connected with the variation of kinetic energy w by the following integral of motion: $w - \omega\mu \equiv C^2 = \text{const.}$, while the phase ζ obeys the equation (e.g. [7, 8]):

$$\frac{d^2\zeta}{dt^2} = \frac{1}{\tau^2}\cos\zeta - \alpha \ , \tag{3}$$

where nonlinear time τ and inhomogeneity parameter α are determined by the expressions:

$$\frac{1}{\tau^2} = hkv_\perp\omega_c \ ; \quad h \equiv \frac{B}{B_0} \ ; \quad \alpha = \frac{k}{2}\left(\frac{dv_{\rm R}^2}{dz} + \frac{v_\perp^2}{\omega_c}\frac{d\omega_c}{dz}\right) \ . \tag{4}$$

Equation (3) describes particle motion in effective potential $P = \alpha\zeta - \sin\zeta/\tau^2$. For $\alpha\tau^2 < 1$, P has potential wells and, hence, there are phase-trapped particles (hereinafter *trapped particles* for short). Since for such particles the coordinate ζ varies in a limited interval, the quantity $d\zeta/dt$ is zero on average; thus, particle parallel velocity oscillates around resonance value $v_{\rm R}$ (see (2)). As in an inhomogeneous plasma the quantity $v_{\rm R}$ varies in space monotonously, the same is true for the average value of trapped particle parallel velocity while it moves along the geomagnetic field line in the wave packet: $\overline{v}_\parallel = v_{\rm R}(z)$. This relation together with above mentioned integral of motion C^2 permit to determine kinetic energy of a trapped particle as a function of coordinate, namely:

$$w = C^2 + \frac{\omega}{\omega_c(z) - \omega}\left[C^2 - \frac{mv_{\rm R}^2(z)}{2}\right] \ . \tag{5}$$

As has been mentioned above, phase trapping is possible only for $\alpha\tau^2 < 1$. However, for $\alpha\tau^2 = 1$, the phase volume of trapped particles is equal to zero; it gradually increases with decreasing of $\alpha\tau^2$. We will define trapping region by the inequality: $\alpha\tau^2 < 1/3$. Relation (5) permits to estimate the variation of trapped particle energy density, $\Delta W_{\rm tr}$, while they move inside the trapping region from a pole side toward the equator. Assuming the relation between electron plasma and cyclotron frequencies in the form $\omega_p \propto \omega_c^\eta$, $(0 < \eta \lesssim 1/2)$, we will find after straightforward calculations:

$$\Delta W_{\rm tr} = n_{\rm tr}\Delta w_{\rm tr} \ ; \quad n_{\rm tr} \simeq \frac{4n_{\rm h}}{\pi k\tau v_{\rm Th}} \ , \quad \Delta w_{\rm tr} \simeq \frac{m}{2}\frac{\omega}{\omega_{ceq} - \omega}\frac{h^2 v_{\perp eq}^2 \omega_{ceq}^2 (LR)^2}{(v_{\perp eq}^2 + bv_{{\rm R}eq}^2)} \ , \quad b \equiv \frac{3\omega_{ceq}}{\omega_{ceq} - \omega} - \eta \ , \tag{6}$$

where $n_{\rm h}$ is the density of energetic electrons, $v_{\rm Th}$ is their thermal velocity, L is McIlwain's parameter, R is the Earth's radius, and subscript "eq" denotes the equatorial value. Let us compare $\Delta W_{\rm tr}$ with the wave energy density $U = (B^2/8\pi)[\omega_c/(\omega_c - \omega)]$. Using the values of cold plasma and energetic particle parameters typical of $L = 4$, namely,

$$\omega_c = 8.5 \cdot 10^4 \ \text{rad/s} \ ; \quad \omega_p = 7.9 \cdot 10^5 \ \text{rad/s} \ ; \quad n_{\rm h} = 0.2 \ \text{cm}^{-3} \ ; \quad v_{\rm Th} = 2.7 \cdot 10^9 \ \text{cm/s} \ ,$$

wave frequency $\omega = 3.14 \cdot 10^4$ rad/s ($f = 5$kHz) , and wave amplitude $B = 3 \cdot 10^{-7}$gauss (30 pT), we find:

$$U = 5.7 \cdot 10^{-15} \ \text{erg/cm}^3 \ ; \quad \Delta W_{\rm tr} = 2.3 \cdot 10^{-13} \ \text{erg/cm}^3 \ ,$$

while the total energy density of trapped particles $W_{\rm tr} \sim 3 \cdot 10^{-12}$ erg/cm^3. The question arises as to where the energy increase of trapped particles comes from. The answer to this question, which follows from detailed analysis of wave-particle interaction in the case under discussion, consists in the following. Along with the trapped particles, there are untrapped resonant particles whose contribution to wave-particle interaction is equally important as that of trapped ones. While trapped particles remain in resonance with the wave for a long time and undergo significant energy variation, phase volume of untrapped particles is continuously renewing. The rate of this renewing is proportional to the magnitude of the inhomogeneity parameter α. Energy variation of an untrapped particle during the time of resonant interaction with the wave is much smaller than for trapped particles. In return, the total number of untrapped resonant particles is much larger, while the rates of energy variation and the phase volumes of trapped and untrapped particles interacting with the wave at a given instant of time are of the same order. A peculiarity of resonance interaction in an inhomogeneous plasma is that, on the average, the energy variation has different sign for trapped and untrapped particles. The key to understanding this feature consists in that the phase volumes of trapped and untrapped particles are not symmetrical with respect to the phase ζ, while the energy variation is proportional to $\cos\zeta/\alpha\tau^2$ (see (2)). A strict prove of the features mentioned above can be found, e.g. in [8]. The fact that the wave energy variation, determined by growth (or damping) rate γ, is much smaller than the rate of energy variation of trapped particles, which is necessary for the inequality $U \ll \Delta W_{\rm tr}$ to be fulfilled, implies that energy exchange between trapped and untrapped particles is much more significant than energy exchange between wave and particles. Thus, if the energy of trapped particles increases, the source of energy is not the wave, as was habitually assumed, but the untrapped particles, while the wave only mediates the energy transfer.

3 Oblique whistler-mode wave in marginally unstable plasma

In this Section, we will assume the unperturbed distribution to be unstable against excitation of whistler-mode waves. In general, wave-particle interactions are described by nonlinear set of equations that consists of Maxwell's equations and Boltzmann-Vlasov equation with a self-consistent electromagnetic field. This nonlinear set of equations has an exact integral, which, in the case of a single wave, may be interpreted as energy conservation in the interacting system consisting of a wave and resonant particles. (As is well known, in the case of oblique propagation, all cyclotron resonances come into play). Using this integral, we will now show that, in the course of development of plasma instability, a significant energy transfer may take place between resonant particles interacting with the wave at different cyclotron resonances and, thus, belonging to different populations of energetic particles. As was argued by many authors, the first cyclotron ($n = 1$) and the Cerenkov ($n = 0$) resonances are most essential for wave-particle interactions, since they correspond to lower values of particle parallel velocities, and since the number of particles usually decreases fast with increasing particle energy. Another feature of whistler-mode kinetic instabilities typical of a wide class of distributions with a loss-cone or/and temperature anisotropy is that the first cyclotron resonance gives rise to wave growth, while the Cerenkov resonance leads to wave damping. Introducing the wave growth rate γ in a usual way, and dividing it into contributions from two resonances, we can express energy conservation in the form of differential equation system:

$$\frac{1}{U}\frac{dU}{dt} \equiv 2\gamma = 2(\gamma_1 + \gamma_0) \; ; \quad \frac{dW_1}{dt} = -2\gamma_1 U \; ; \quad \frac{dW_0}{dt} = -2\gamma_0 U \; , \tag{7}$$

with $\gamma_1 > 0$, $\gamma_0 < 0$, and $\gamma \equiv \gamma_1 + \gamma_0 > 0$, the last inequality ensuring an overall wave growth. A formal solution to the set of equations (7) may be represented in the form:

$$\begin{aligned} U &= U_i e^{2\Psi(t)} \; ; \quad \Psi(t) \equiv \int_0^t [\gamma_1(\tau) + \gamma_0(\tau)] d\tau \; ; \\ \Delta W_1 &= -2\int_0^t \gamma_1(\tau) U_i e^{2\Psi(\tau)} d\tau \simeq -\left\langle \frac{\gamma_1}{\gamma_1 + \gamma_0} \right\rangle_t (U - U_i) \; ; \\ \Delta W_0 &= -2\int_0^t \gamma_0(\tau) U_i e^{2\Psi(\tau)} d\tau \simeq -\left\langle \frac{\gamma_0}{\gamma_1 + \gamma_0} \right\rangle_t (U - U_i) \; , \end{aligned} \tag{8}$$

where U_i is the initial wave energy density, ΔW_1, ΔW_0 are, respectively, the energy density variations of particles that interact with the wave at the first cyclotron resonance (hereinafter *first-cyclotron-resonance particles* for short), and of particles that interact with the wave at the Cerenkov resonance (*Cerenkov-resonance particles*), and $< ... >_t$ denotes a time average value of the corresponding quantity calculated with the weighting function dU/dt which is assumed to be positively defined. The formal solution given above satisfies the energy conservation $U - U_i = -\Delta W_1 - \Delta W_0$, which is required by the initial set of equations (7). Assuming $U_i \ll U$ and taking into account the signs of γ_1 and γ_0, we may rewrite (8) in the form showing the sought-for result most obviously:

$$\Delta W_1 \simeq -\left\langle \frac{\gamma_1}{\gamma_1 - |\gamma_0|} \right\rangle_t U \ ; \quad \Delta W_0 \simeq \left\langle \frac{|\gamma_0|}{\gamma_1 - |\gamma_0|} \right\rangle_t U \ ; \quad U = |\Delta W_1| - \Delta W_0 \ . \tag{9}$$

We see that for marginal instability conditions, i.e.

$$\gamma_1 + \gamma_0 \equiv \gamma_1 - |\gamma_0| \ll \gamma_1, \ |\gamma_0|$$

the energy variation of resonant particles greatly exceeds the wave energy, i.e. the wave mainly mediates the energy transfer from the first-cyclotron-resonance particles to the Cerenkov-resonance particles. We should mention that the state of plasma marginal instability, particularly in the magnetosphere, is not an exceptional, but a natural one, as has been pointed out already in the classical works [9, 10]. Thus, wave-particle interactions under conditions of marginal instability provide a regular mechanism of energy transfer between energetic particle populations, the process being mediated by slowly growing unstable waves.

4 References

1. R. B. Horne, R. M. Thorne, S. A. Glauert, J. M. Albert, N. P. Meredith, and R. R. Anderson, "Timescale for radiation belt electron acceleration by whistler mode chorus waves," *J. Geophys. Res.*, **110**, 2005, A03225, doi:10.1029/2004JA010811.

2. Santolík, O., et al., "Wave-particle interactions in the equatorial source region of whistler-mode emissions," *J. Geophys. Res.*, **115**, 2010, A00F16, doi:10.1029/2009JA015218.

3. H. Matsumoto, "Nonlinear whistler-mode interaction and triggered emissions in the magnetosphere: a review," in P. J. Palmadesso and K. Papadopoulos (eds.), *Wave instabilities in space plasmas,* D. Reidel Pub., 1979, pp. 163–190.

4. A. G. Demekhov, V. Y. Trakhtengerts, M. J. Rycroft, and D. Nunn, D. "Electron acceleration in the magnetosphere by whistler-mode waves of varying frequency," *Geomagnetism and Aeronomy*, **46**, 2006, pp. 711–716, doi:10.1134/S0016793206060053.

5. Y. Omura, N. Furuya, and D. Summers, "Relativistic turning acceleration of resonant electrons by coherent whistler mode waves in a dipole magnetic field," *J. Geophys. Res.*, **112**, 2007, A06236, doi:10.1029/2006JA012243.

6. V. Y. Trakhtengerts and M. J. Rycroft, *Whistler and Alfvén mode cyclotron masers in space*, Cambridge University Press, 2008.

7. D. Nunn, "A self-consistent theory of triggered VLF emissions," *Planet. Space Sci.*, **22**, 1974, pp. 349–78.

8. D. R. Shklyar H. and Matsumoto, "Oblique whistler-mode waves in the inhomogeneous magnetospheric plasma: resonant interactions with energetic charged particles," *Surv. Geophys.*, **30**, 2009, pp. 55–104, doi:10.1007/s10712-009-9061-7.

9. A. A. Andronov and V. Y. Trakhtengerts, "Kinetic instability of the earths outer radiation belt," English Transl., *Geomagnetism and Aeronomy*, **4**, 1964, pp. 181–188.

10. C. F. Kennel and H. E. Petschek, "Limit on stably trapped particle fluxes", *J. Geophys. Res.*, **71**, 1966, pp. 1-28.