**Theory and Simulations of Discrete VLF Emissions in the Magnetosphere**

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## Abstract

We consider the results of theoretical and numerical studies of the of generation discrete VLF emissions in the Earth's magnetosphere. The physics of cyclotron wave-particle interactions will be reviewed and current ideas on the mechanism forming the discrete spectrum of whistler-mode waves will be discussed. Two most important classes of the discrete emissions, i.e., triggered and chorus signals will be considered, and their common and specific features will be analyzed. We discuss the relationship between the model results and observations, and also possible application of similar mechanisms to the phenomena in the ion-cyclotron frequency range.

## 1 Introduction

The specific features of VLF chorus emissions in the Earth's magnetosphere [1–4] are their discrete (narrow-band at each time moment) spectrum in the form of repetitive short (with the duration $\tau \sim 0.1$–$0.2$ s) elements with fast frequency drift (the frequency drift rate $|\mathrm{d}f/\mathrm{d}t| \sim 10^4$ Hz/s) and, as a rule, with the frequency increasing in time. The wave growth rate of a chorus element is very large ($\gamma \gtrsim 10^2$ s$^{-1}$), and unrealistically high energetic electron fluxes are required to explaine such high values observed at $L \simeq 4$–$5$. It has been established to date [4–6] that the chorus is generated near the top of a geomagnetic field line in the region with the length along the magnetic field $l \sim 2000$–$5000$ km, which is much less than the characteristic field-line length.

To explain the parameters of chorus emissions in the Earth's magnetosphere, a model was proposed in [7], which uses the transition from convective to absolute instability in the presence of a step-like deformation in the distribution function of energetic electrons in the velocity component parallel to the geomagnetic field. Such a feature arises in a natural way in the course of cyclotron generation of noise-like emissions [8–11]. In this case, the absolute instability has the similar nature as in the backward wave oscillator (BWO), i.e., it arises due to the opposite direction of the wave group velocity with respect to the velocity of resonant particles. The BWO model explains the observed large growth rates

Nonlinear stage of chorus generation has similar underlying physics to another type of discrete emisions, i.e., triggered emissions. In this paper, we discuss the formation of chorus frequency spectrum on the basis of analytical considerations and numerical simulations.

## 2 Analytical estimates of parameters of chorus emissions

It was shown in [7, 12] that, the characteristic length $l_{\text{eff}}$ of the magnetospheric cyclotron maser in the BWO regime equals to the distance at which the increment of the wave phase with respect to the resonant particles reaches $\pi$. For the parabolic dependence of the magnetic field on the logitudinal coordinate $z$ this length is

$$l_{\text{BWO}} \simeq 3{,}5\, l_* = 3{,}5\, a \left(\frac{V_*}{a\omega_{HL}}\right)^{1/3}, \qquad (1)$$

where $a \simeq \sqrt{2}R_0L/3$ for the dipole magnetic field, and $R_0$ is the planet radius. For whistler-mode waves in the Earth's magnetosphere, $ka \sim 10^3 \div 10^5$, i.e., $l_{\text{BWO}}/a \sim 10^{-2} \div 10^{-1} \ll 1$.

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The growth rate in the BWO regime can be approximately calculated according to the formula [12, 13]

$$\gamma_{\rm BWO} \approx \frac{\pi}{T_{\rm BWO}} (q^{1/2} - q^{-1/3}), \tag{2}$$

where

$$T_{\rm BWO} = l_{\rm BWO}(1/V_* + 1/v_g) \tag{3}$$

is the feedback delay time in the system taking into account the particle motion and wave propagation, $q = S/S_{\rm thr}$ is the dimensionless parameter determining the excess of the energetic-electron flux $S$ over the threshold value for the BWO regime $S_{\rm thr}$. Note that Eq. (2) is valid in a wide range of the system parameters including large values of $q$, i.e., $q \gg 1$. As we see below, chorus emissions are formed only if $q \gg 1$, and the parameters of the nonlinear stage are determined by the linear growth rate. Therefore, the above result is very important for the analysis of the nonlinear regime of wave generation in the magnetospheric BWO.

A strict analytical theory of the nonlinear stage of the instability in the BWO regime has not yet been developed, and there exist only phenomenological approaches to this problem and some results of numerical modeling. Quantitative consideration of this problem [14, 15] is based on the assumption on the frequency drift as a consequence of sideband instability in this system. Subsequent generation of sidebands each of which is shifted by the nonlinear trapping frequency $\Omega_{\rm tr}$ with respect to the previous one and is delayed by the time of about $2\pi/\Omega_{\rm tr}$ during which the distribution function in the trapping region becomes plateau-like, results in the frequency drift with the rate

$$d\omega/dt \sim \Omega_{\rm tr}^2/(2\pi). \tag{4}$$

Note that similar estimate has been obtained in [16] from calculations of the maximum nonlinear growth rate of a wave for a specific distribution function with a plateau-like valley in the trapping region. Such a distribution function was chosen on the basis of the results of numerical simulations. The agreement between the estimates obtained by different methods is most probably not just a coincidence, since the sideband instability is related exactly to the formation of a plateau on the distribution function in the trapping region.

The theory of saturation of the instability of a quasimonochromatic wave upon the cyclotron interaction in a homogeneous plasma [17] yields a relation

$$\Omega_{\rm tr} \approx \cdot 32\gamma_{\rm BWO}/(3\pi)\,. \tag{5}$$

As a result we have for the frequecy drft rate:

$$\frac{\Delta f}{\Delta t} \approx 0.3\gamma_{\rm BWO}^2. \tag{6}$$

Of course the estimates given by (4) and (6) are valid only with an accuracy of about an order of magnitude. However, as it was shown in [15], they are in good agreement with observations of chorus emissions.

Note that Eq. (5) also yields an estimate of the characteristic wave amplitude, since

$$\Omega_{\rm tr}^2 = (hkv_\perp\omega_B)\,, \tag{7}$$

where $k$ is the wave number, $\omega_B$ is the electron gyrofrequency, $h = b/B$, $b$ is the wave magnetic-field amplitude, and $B$ is the geomagnetic field. As a result, we have for the wave ampitude:

$$b \approx \frac{32mc}{9\pi^2 ekV_{\perp 0}}\gamma_{\rm BWO}^2 \tag{8}$$

where $V_{\perp 0}$ is the characteristic transverse velocity of electrons.

In [18], it was shown that the conditions suitable for the BWO regime of the cyclotron instability can also be satisfied for ion-cyclotron frequency range if the flux of energetic protons exceeds a threshold value of about $S_{\rm thr} \simeq 3\cdot 10^7$ cm$^{-2}$ s$^{-1}$ for protons with energies $W \sim 10$ keV. Rather intense chorus-like emissions were indeed observed recently in the Pc1 range [19].

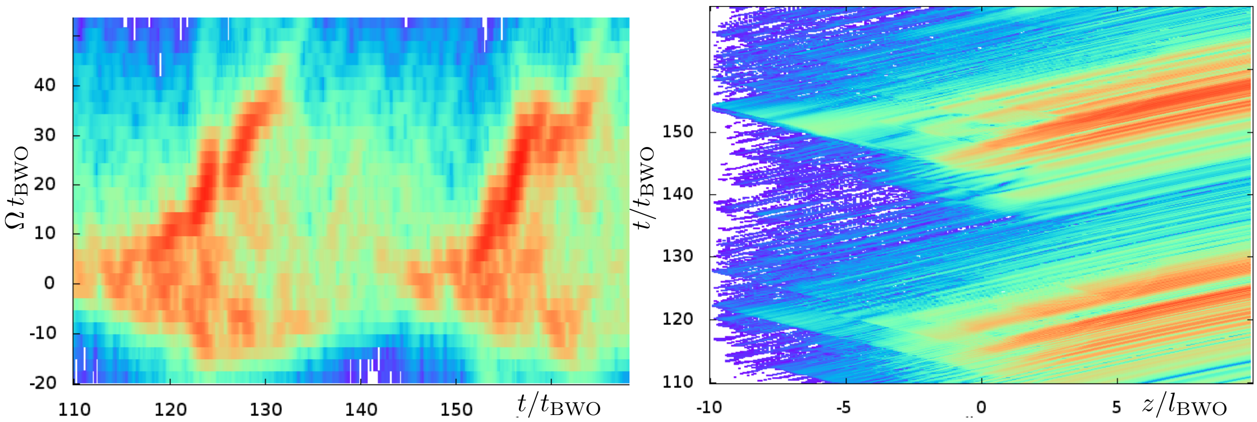


Figure 1: Results of numerical simulations of the BWO model [21] for $q \approx 12$. Left: dynamic spectrum of VLF waves at the output of the system ($z/l_{\rm BWO} = 10$); right: spatio-temporal evolution of the electric field ampltude. Hereafer, $t_{\rm BWO} = l_{\rm BWO}/V_*$, and $\Omega$ is the frequency shift with respect to the frequency of the linear instability. Under typical conditions for Cluster observations at $L = 4.4$, i.e., $N_c \approx 10$ cm $^{-3}$, we have $t_{\rm BWO} \approx 0.01$ s.

## 3 Some results of numerical simulations

A self-consistent numerical model describing the BWO dynamics in a uniform and non-uniform medium was developed in [20, 21]. It was demonstrated in [21] that chorus elements are formed only if the parameter $q$ described in the previous section exceeds some large enough value, i.e., $q \gtrsim 7$–10. It is necessary for the nonlinear effects to have time to develop, i.e., the wave amplitude has to be high enough to ensure at least more than one nonlinear oscillation of electrons during their motion through the generation region.

In this section, we describe some results of numerical simulations by using this model.

**Spatio-temporal picture of formation of discrete elements.** Figure 1 shows two chorus elements resulted from a simulation in the frequency-time and space-time representations. Hereafter, we consider waves propagating to the right and, hence, the resonant electrons move to the left. One can see in Fig. 1 that higher-frequency parts of the elements, which are observed later than the lower-frequency parts, have their origin at larger negative $z$. It corresponds well to the concept of the sideband instability, since the electrons move to the left as their distribution function gets distorted before the next sideband is generated. One can also see that a companion falling element is clearly discernible on the dynamic spectrum, although it is much weaker and with smaller frequency drift. It would be interesting to look for such companion elements in the actual high-resolution spectrograms of chorus emissions.

**Formation of falling tones.** We consider two mechanisms of formation of falling tones in chorus emissions. Both of them are based on the fact that shifting the generation region upstream with respect to the particle motion can ensure the generation by the particles moving in the decreasing magnetic field, which favors the second-order resonance conditions for falling-tone emissions. One mechanism is realized when the electron flux is increased to rather high values, so the generation starts at some distance from the equator. This mechanism was considered in [22] for triggered emissions. The second mechanism is related to the persistence of electron phase bunching during their bounce oscillations. In the first case, the falling-frequency elements have rather high frequency drift rate and short inter-element interval, while in the second case, the fallers have the absolute value of the frequency drift rate and inter-element interval close to the case of risers. More details about simulations of both these mechanisms for VLF chorus emissions can be found in [23].

## 4 Conclusions

The agreement between analytical estimates and simulation results, on the one hand, and between theoretical and experimental results, on the other hand indicate a significant progress in understanding the nature of discrete emissions in the inner magnetosphere. However, many related questions remain to be answered. In particular, the detection of a step-like feature on the distribution function remains a challenge for observations, while its existence seems necessary for explanation of chorus generation at least at $L = 4$–5. The characteristic time scales of ion-cyclotron BWO need a separate study, since the trapping frequency can become comparable to the wave frequency, and the usual small-amplitude approximations do not work for these waves. The origin of falling tones in chorus emissions is still not understood, and an experimental survey of the condition at which they are formed seems necessary and timely.

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