Stochastic Fermi acceleration in the Earth’s magnetotail current sheet: numerical studies

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Abstract

We show results from 2D and 3D test particle simulations investigating the acceleration of protons interacting with stationary and time-dependent electromagnetic fields. In the 2D simulations we mimic a Fermi-like interaction between particles and randomly positioned oscillating clouds. A constant dawn-dusk electric field and a constant out of plane magnetic field are also present. The 3D model studies that process in a more realistic configuration: a modified Harris profile has also been added. Parametric studies have been performed and the results discussed also in comparison with spacecraft observations in both the distant and the near-Earth magnetotail.

1 Introduction

In the Earth’s magnetospheric environment, a variety of accelerated particle populations is observed [1]. These particles are detected as beamlets at the lobeward edge of the plasma sheet boundary layer (PSBL) [2]. The stationary dawn-dusk electric field present in the magnetotail can be a source of acceleration for ions. However, realistic values of this large scale electric field ($E_0y \approx 0.1-0.3$ mV/m) lead to maximum potential drops of the order of 30 keV, while particles exceeding 100 keV are also observed [3]. [2] have shown that, additional acceleration mechanisms are required in order to explain such observations. In this work we propose a Fermi-like acceleration, due to the presence of time dependent electromagnetic fluctuations in the magnetotail, as an additional energization mechanism. We adopt both 2D and 3D models in order to compare the contributions to the acceleration process from the stochastic mechanism and from large scale fields. We show that the combined effect of $E_0y$ and of the moving clouds can explain a range of energetic ion observations.

2 Numerical models

A test particle numerical simulation has been performed in order to reproduce the interaction between protons and electromagnetic fields (EFs) generated by $N$ random positioned clouds in the $(x, y)$ current sheet (CS) plane [4,5]. The electric and magnetic fields are given by $B = \nabla \times A$ and $E = -\nabla \phi - \partial A / \partial t$, in the gauge where $-\nabla \phi = E_0y \varepsilon_y$, with $E_0y = \text{const}$. The vector potential has components within $(x, y)$, that is $A_x(r, t) = (-B_n/2)y + a_x(r, t)$ and $A_y(r, t) = (B_n/2)x + a_y(r, t)$ [6]. These expressions depend both on particle position $r$ in the $(x, y)$ plane and on time $t$, while $B_n$ is a constant magnetic field directed along $z$. The fluctuating terms, $a_x, a_y$, are defined as

$$a_x = a_y = a_0 \sum_i \exp \left[-|r - r_i(t)|/l_{cl}\right],$$

where $|r - r_i(t)| = \sqrt{(x - x_i(t))^2 + (y - y_i(t))^2}$ indicates the difference between the particles’ positions and the ones of the clouds of size $l_{cl}$; $a_0$ is the amplitude of the vector potential. The sum in Equation 1 is extended over all the $N$ clouds. From Equation 1, magnetic and electric fields are easily obtained (see [6]). The EFs depend on the motion of clouds having random initial positions $(x_{n0}, y_{n0})$, and performing an oscillating motion along the $x$ and $y$ directions, i.e., $x_n(t) = x_{n0} + a_{cl} \cos(\omega t + \phi_{xn})$ and $y_n(t) = y_{n0} + a_{cl} \sin(\omega t + \phi_{yn})$. In summary, those oscillating clouds mimic the time dependent electromagnetic fluctuations present in the CS of the terrestrial magnetosphere.
In the 3D model we have also included a large scale magnetic field given by a modified Harris profile [7] along $x$ (the Earth-Sun direction),

$$B_x(z) = B_l \frac{\tanh(z/\lambda) - (z/\lambda)\cosh^{-2}(L_z/2\lambda)}{\tanh(L_z/2\lambda) - (L_z/2\lambda)\cosh^{-2}(L_z/2\lambda)}. \quad (2)$$

$B_l$ represents the intensity of the large scale magnetic field in the Earth’s lobes; $B_x(z)$ is a reversal field changing polarity from the northern lobe to the southern one. The size of the $B_x(z)$ configuration along the $z$ direction (normal to the CS) is $L_z$ and the half thickness $\lambda$ of the CS is given by $\lambda = L_z/4$. The time dependent EFs have been obtained by setting in Equation 1 $|\mathbf{r} - \mathbf{r}_i(t)| = \sqrt{(x - x_i(t))^2 + (y - y_i(t))^2 + k^{-2}(z - z_i(t))^2}$; the parameter $k$ determines the thickness of the clouds along $z$. In this case, for simplicity, we set $z_i(t) = 0$, that is there is no clouds’ motion along the $z$ direction and the clouds are centered at $z = 0$ (i.e., in the CS).

Both in the 2D and in the 3D models, the equations have been normalized to characteristic values taken from observations in the magnetotail. The magnetic field is normalized to $B_0 = 2$ nT [8], so that the Larmor frequency is $\omega_0 = qB_0/m \approx 0.2$ rad/s, and $t_0 = 1/\omega_0 = 5$ sec. All the lengths are normalized to the size of the simulation box $L = 10^5$km (a value comparable with the width of the magnetotail [9]). Velocities are thus normalized to $V_0 = \omega_0 L = 2 \times 10^4$ km/s and electric fields to $B_0 \omega_0 L = 40$ mV/m. We put the oscillation frequency of the clouds as $\omega_{cl} = V_A/l_{cl}$, where $V_A \sim 400$km/s represents a typical Alfvén speed in the magnetotail [8]. In the case of the 3D model we assume for simplicity that the clouds are axisymmetric, that is, the sizes of the clouds in the $(x, y)$ plane are equal. In the EFs described above, $\sim 5 \times 10^3$ particles have been injected simultaneously in the simulation box at $t = 0$ randomly in the $(x, y)$ plane. Their initial velocities come from a Gaussian distribution with $v_{th} = 120$ km/s. The total number of clouds is $N = 100$.

### 3 Results and Discussions

Via a parametric study, we have varied the dimensionless size of the clouds $R = l_{cl}/L = 0.016, 0.032, 0.08$ as well as the oscillating frequency of the clouds $\omega = V_A/l_{cl} = 0.25, 0.125, 0.05$ s$^{-1}$ [6]. These frequencies are in the range of those observed [8].

In the case of the 2D simulations we have kept constant both $E_{0y} = 0.2$ mV/m and the magnetic field $B_n = 1$ nT. Figure 1 shows a sample trajectory both in the real (top panels) and in the velocities (middle panels) spaces along with the electric and magnetic fields seen by the particle and its normalized kinetic energy $E_k$ (bottom panels), for $R = 0.016$ (left columns) and $R = 0.08$ (right columns). Time is measured in units of $t_0$. The particle trajectory for $R = 0.016$ is strongly affected by the $\mathbf{E} \times \mathbf{B}$ drift: the particle tends to move along the positive $x$ direction. The magnetic field seen by the particle oscillates around the mean value $B_n = 1$ nT, and the kinetic energy gain is moderate. For $R = 0.08$ the influence of $B_n$ on the particle trajectory is less evident than in the other case and the particle trajectory is more complex. The plot of the normalized $E_k$ presents several increases and decreases typical of a stochastic Fermi-like process. We point out that the particle acceleration is obtained with realistic values of the fluctuating fields [8]. Figure 2 shows the comparison among the probability density functions (PDFs) of particles’ energies for three values of $R$, along with the initial Gaussian distribution (dotted line). These distributions are obtained injecting $5 \times 10^3$ particles detected at roughly $500t_0$. The energy gain increases with the size of the clouds and for $R \geq 0.032$ the energy acquired by particles is larger than the potential drop of 20 keV due to $E_{0y}$ through the simulation box (vertical dashed line). For $R = 0.08$, particles can reach energies $\geq 100$ keV. Such energies compare well with those observed in the PSBL [2].

Regarding the 3D model the magnetic clouds have also a size along the $z$ direction, which has been set to $R_z = l_{clz}/L \equiv kR$. The parameter $k$ describes the shape of the ellipsoids and it has been fixed to $k = 0.5$. We have fixed the size of the large scale magnetic field configuration along the $z$ direction to $L_z = 0.2L = 2 \times 10^4$km, the half thickness of the CS to $\lambda = L_z/4 = 5 \times 10^3$km, the asymptotic value in the lobes to $B_l = 10$ nT (see Equation 2). The analysis of particle trajectories in the 3D model has revealed that particles experience frequent interactions with the time dependent fields in the CS via the Fermi-like
Figure 1: From top to bottom: particle position as a function of time (vertical axis), particle velocities as a function of time, and time behaviors of $B_z$, of $E_x$ (solid line), $E_y$ (dashed line), and of the normalized particle kinetic energy $E_k$ for $B_n = 1$ nT, $R = 0.016$ (left column), and for $R = 0.08$ (right column) in the 2D model.

The energy gains are rather quick, occurring on times of about 10–20 $t_0 = 50–100$ seconds. These acceleration times are much shorter than the duration of disturbed periods in the magnetotail. The effect of the $E \times B$ drift in the ($x, y$) plane is rather strong. The PDFs of particles’ energies are plotted in Figure 3 for the case $R = 0.08$: they have been computed within the CS at two different times. It can be seen that the proton energy gain is less in the 3D than in the 2D case, as expected. Although in the 3D model particles having a finite initial $v_z$ have less chances to interact with the magnetic clouds at $z \sim 0$ and gain less energy, it is still possible to reach energy values beyond the 20 keV. Indeed, ions can be energized up to 40-50 keV, while in 2D the corresponding maximum energy is 80-90 keV (see Figure 2).

Summarizing, the joint effect of a Fermi-like process and the typical constant electric and magnetic fields present in the Earth’s magnetotail can explain a range of energy values reached by proton populations in the PSBL [2]. This stochastic mechanism has been found to be very efficient also when a 3D configuration of the magnetotail is considered. It is worth mentioning that in this case the gross of the acceleration process happens within the CS but the effect of the large scale fields included leads to the formation of particle beam-like structures in the direction parallel to the mean magnetic field (see [10]) which compare well with Type II beams observed in the magnetotail [2].

4 References

Figure 2: PDFs of protons at $t = 0$ (dotted line), and for $R = 0.016$ (dot-dashed line), $R = 0.032$ (dot-dot-dashed line), and $R = 0.08$ (solid line) in the 2D model. The vertical dashed line at 20 keV corresponds to the potential drop across the simulation box due to $E_{0y}$.

Figure 3: PDFs of protons at injection (dotted line), and for two times within the 3D current sheet. The format is the same of Figure 2.


