Abstract

Generation of broadband electrostatic noise (BEN) in the plasma sheet boundary layer (PSBL) in terms of electron-acoustic solitons and double layers is proposed. The PSBL is treated as a multi-component magnetized plasma consisting of background electrons, counter-streaming electron beams and ions. The model is based on the multi-fluid equations and the Poisson equation, and uses the Sagdeev pseudo-potential technique. For the plasma parameters at the time of BEN in the PSBL observed by Cluster on 22 September 2004, the model predicts solitons/double layer with electric field \( \sim (0.01-30) \) mV/m. It is proposed that the model can be a good candidate for explaining the generation of BEN in the PSBL.

1 Introduction

Matsumoto et al. [1] showed for the first time, from the analysis of the high time resolution of the plasma wave data from GEOTAIL, that the BEN in the PSBL actually consists of short electrostatic solitary waves (ESWs) whose Fourier spectrum give rise to the broadband nature of the noise. The electrostatic solitary structures can have either positive or negative potentials, and their electric field amplitude can vary from a fraction of a mV/m in the PSBL to a few 100 mV/m in the dayside auroral zone. The velocities of ESWs can vary from \( \sim \) a few hundred to a few thousand km s\(^{-1}\), and their parallel scale sizes from \( \sim 100 \) m to a few 1000 m.

The most common interpretations for the ESWs observed in the PSBL are based on the nonlinear evolution of a bump-on-tail instability/electron two stream instability [2] leading to the formation of Bernstein-Greene-Kruskal (BGK) potential structures which reproduce well the observed electrostatic solitary wave forms. Other interpretations put forward for the generation of ESWs are based on electron-acoustic solitary waves [3,4,5]. Recently, the plasma measurements made on the Cluster spacecraft in the PSBL have shown that broadband (\( \sim 2-6 \) kHz) electrostatic noise is associated with cold counterstreaming electron beams flowing through the hot Maxwellian plasma [6,8]. The observed plasma parameters were (from Ref. [6]): total electron density, \( n_0=(0.27-0.32) \) cm\(^{-3}\), core electron temperature, \( T_c=600-800 \) eV, ion temperature, \( T_i=3.7-5.0 \) keV, electron beams drift speed, \( v_B=7000-12000 \) km s\(^{-1}\), electron beams temperature, \( T_B=10-100 \) eV, and magnetic field, \( B_0=8-12 \) nT. It is interesting to note these parameters are quite different than those employed in most theories for the PSBL ESWs based on electron-acoustic solitary waves [3,4,5]. Recently, the plasma measurements made on the Cluster spacecraft in the PSBL have shown that broadband (\( \sim 2-6 \) kHz) electrostatic noise is associated with cold counterstreaming electron beams flowing through the hot Maxwellian plasma [6,8]. The observed plasma parameters were (from Ref. [6]): total electron density, \( n_0=(0.27-0.32) \) cm\(^{-3}\), core electron temperature, \( T_c=600-800 \) eV, ion temperature, \( T_i=3.7-5.0 \) keV, electron beams drift speed, \( v_B=7000-12000 \) km s\(^{-1}\), electron beams temperature, \( T_B=10-100 \) eV, and magnetic field, \( B_0=8-12 \) nT. It is interesting to note these parameters are quite different than those employed in most theories for the PSBL ESWs based on electron-acoustic solitary waves [3,4,5].

2 Electron-acoustic Soliton/Double Layer Model

We model the PSBL plasma by a homogeneous, collisionless, and magnetized four component plasma consisting of hot core electrons \((N_c, T_c, v_c)\), two cold electron beams \((N_1, T_1, v_1)\) and \(N_2, T_2, v_2)\) propagating along the magnetic field, and hot ions \((N_i, T_i, v_i)\), where \(N_j, T_j, v_j\) represent the equilibrium values of the density, temperature and beam velocity (along the direction of the ambient magnetic field) of the species.
the conditions that respect to the ion thermal velocity. Then, solving for perturbed densities, putting these expressions in the Poisson equation \[5\]. We transform these to a stationary frame moving with velocity \(V\), the solitary wave velocity, i.e., \(\xi = (x - Mt)\) where \(M = V/C_t\) is the Mach number with respect to the ion thermal velocity. Then, solving for perturbed densities, putting these expressions in the Poisson equation, and assuming appropriate boundary conditions for the localized disturbances along with the conditions that \(\phi = 0\), and \(d\phi/d\xi = 0\) at \(\xi \to \pm, \infty\), we get the following energy integral \[5\],

\[
\frac{1}{2} \left( \frac{\partial \phi}{\partial \xi} \right)^2 + \psi(\phi, M) = 0
\]

where, \(\psi(\phi, M) = \mu n_i^{0} \left\{ (M-v_c)^2 - \frac{(M-v_c)}{\sqrt{2}} B_1^{1/2} \right\} + n_i^{0} T_i \left\{ 1 - 2\sqrt{2}(M-v_c)^3 B_c^{-3/2} \right\} + \mu n_i^{0} \left\{ (M-v_i)^2 - \frac{(M-v_i)}{\sqrt{2}} B_2^{1/2} \right\} + n_i^{0} \left\{ 1 - 2\sqrt{2}(M-v_i)^3 B_i^{-3/2} \right\}
\]

is the pseudopotential, also known as the Sagdeev potential. Here,

\[
B_c = A_c \pm \sqrt{A_c^2 - \frac{12T_c(M-v_c)^2}{\mu}}, \quad B_1 = A_1 \pm \sqrt{A_1^2 - \frac{12T_1(M-v_1)^2}{\mu}},
\]

\[
B_2 = A_2 \pm \sqrt{A_2^2 - \frac{12T_2(M-v_2)^2}{\mu}}, \quad B_i = A_i \pm \sqrt{A_i^2 - 12(M-v_i)^2},
\]

\[
A_c = (M-v_c)^2 + \frac{3T_c}{\mu} + \frac{2\phi}{\mu}, \quad A_1 = (M-v_1)^2 + \frac{3T_i}{\mu} + \frac{2\phi}{\mu},
\]

\[
A_2 = (M-v_2)^2 + \frac{3T_2}{\mu} + \frac{2\phi}{\mu}, \quad A_i = (M-v_i)^2 + 3 - 2\phi, \quad \mu = \frac{m_e}{m_i}.
\]

In Eq. (2), \(n_j^{0} = N_j/N_i\) such that \(n_i^{0} + n_i^{1} + n_i^{2} = n_i^{0} = 1\), and the temperatures of the species are normalized with the ion temperature, velocities with the ion thermal velocity \(C_i = (T_i/m_i)^{1/2}\), time with the inverse of ion plasma frequency, \(\omega_{pi} = (4\pi N_i e^2/m_i)^{1/2}\), the lengths with the ion Debye length, \(\lambda_D = (T_i/4\pi N_i e^2)^{1/2}\), the electrostatic potential \(\phi\) by \(T_j/e\), and the thermal pressures \(P_j\) with \(N_i T_i\). Also, we have considered the same adiabatic index, i.e., \(\gamma = 3\), for all the species in the equation of state.

### 3 Soliton and Double Layer Solutions

Equation (1) will give a soliton solution when the pseudo-particle is reflected in the pseudo-potential field and returns to its initial state (zero potential drop). Therefore, for soliton solutions, the Sagdeev potential \(\psi(\phi, M)\) must satisfy the following conditions: \(\psi(\phi, M) = 0, d\psi(\phi, M)/d\phi = 0, d^2\psi(\phi, M)/d\phi^2 < 0\) at \(\phi = 0\), \(\psi(\phi, M) = 0\) at \(\phi = \phi_0\), and \(\psi(\phi, M) < 0\) for \(0 < |\phi| < |\phi_0|\). The double layer solutions could also exist at the upper limit on the Mach number \(M = M_{DL}\) provided one more additional condition, namely, \(d\psi(\phi, M)/d\phi = 0\) at \(\phi = \phi_{DL}\) and \(M = M_{DL}\), is satisfied. In such a case, the pseudo-particle is not reflected at \(\phi = \phi_{DL}\) because of the vanishing pseudo-force and pseudo-velocities. Instead, it goes to another state producing an asymmetrical double layer (DL) with a net potential drop of \(\phi_{DL}\), where \(\phi_{DL}\) is the amplitude of the double layer. From Eq. (2), it is seen that \(\psi(\phi, M)\) and its first derivative with respect to \(\phi\) vanish at \(\phi = 0\).
4 Numerical results

We have numerically solved Eq. (2) for the Sagdeev potential, \( \psi(\phi, M) \), as a function of \( \phi \) for various values of Mach numbers for the case of counterstreaming electron beams having equal densities (i.e., \( N_1=N_2=N_B \)) and temperature (i.e., \( T_1=T_2=T_B \)), and equal and opposite streaming velocities (i.e., \( v_1=-v_2=v_B \)). Based on the observations provided in Teste and Parks [6], we consider the following normalized parameters for the numerical computations: \( v_c = v_i=0.0, \frac{T_c}{T_i}=0.12-0.22, \frac{T_B}{T_i}=0.002-0.005, \frac{v_B}{c_i}=10.0-20.0, \) and \( N_c/n_0=0.1-0.7 \) (correspondingly \( N_B/n_0=0.45-0.15 \)). Figure 1 shows the variation of Sagdeev potential \( \psi(\phi, M) \) versus the normalized electrostatic potential \( \phi \) for various values of the Mach number for the case when the core electron density is equal to the density of both electron beams, i.e., \( N_c=2N_B \). The curves for \( M=25.10 \) and 25.175 correspond to the electron-acoustic soliton solution, and the curve \( M=25.2161 \) to the double layer solution. There are no soliton or double layer solutions for the Mach numbers exceeding 25.2161. Figure 2 shows the profiles of normalized electrostatic potential, \( \phi \) (solid curves) and electric field, \( E \) for the electron-acoustic solitons/double layer for the parameters of Figure 1. The curves 1, 2 and 3 are the Mach number \( M=25.10, 25.175, \) and 25.2161.

Figure 1: Shows variations of Sagdeev potential, \( \psi(\phi, M) \), versus electrostatic potential, \( \phi \), for electron-acoustic solitons/double layer for the identical counterstreaming electron beams with plasma parameters: \( v_c = v_i=0.0, \ v_1 = -v_2 = v_B=10.0, \ T_c=0.175, \ T_B=0.005, \ N_c/n_0=0.5, \ N_B/n_0=0.25 \) and for \( M=25.10, 25.175, \) and 25.2161.

Figure 2: The profiles for electrostatic potential, \( \phi \) (solid curves) and the electric field, \( E \) for the electron-acoustic solitons/double layer for the parameters of Figure 1. The curves 1, 2 and 3 are the Mach number \( M=25.10, 25.175, \) and 25.2161.

5 Discussion

The model allows the existence of electron-acoustic solitons and double layers for the PSBL parameters reported by Teste and Parks [6]. Unfortunately, the Cluster WBD waveform data for this event discussed by Teste and Parks [6] are not available to confirm the presence of bipolar and monopolar pulses predicted by our analysis. However, we note that the pulse durations of \( \sim (0.1-4.5) \) ms, typical for the PSBL as shown...
Table 1: Variations of soliton velocity (V), electric field (E), soliton width (W) and pulse duration (τ), for various values of the core and beam electron temperatures for the equal density counterstreaming electron beams for the PSBL parameters: \( n_0 = 0.3 \text{ cm}^{-3} \), \( T_i = 4 \text{ keV} \), \( T_c/T_i = 0.175 \), \( T_B/T_i = 0.005 \), \( N_c/n_0 = 0.5 \), \( N_B/n_0 = 0.25 \), \( V_B/C_i = 10.0 \).

\[
\begin{array}{cccccc}
T_c/T_i & V (10^3 \text{ km s}^{-1}) & E (\text{mV/m}) & W (\text{km}) & \tau (\text{ms}) \\
0.125 & 13.4-13.6 & 0.01-12.6 & 38.2-12.5 & 2.9-0.9 & 13.6-13.7 & 0.004-9.8 & 45.2-11.6 & 3.3-0.8 \\
0.150 & 14.4-14.6 & 0.01-12.6 & 39.0-12.8 & 2.7-0.9 & 14.6-14.7 & 0.01-10.3 & 46.8-14.3 & 3.2-1.0 \\
0.175 & 15.3-15.5 & 0.01-12.9 & 52.2-16.0 & 3.4-1.0 & 15.5-15.6 & 0.01-10.8 & 58.3-12.9 & 3.77-0.8 \\
0.200 & 16.2-16.4 & 0.01-13.3 & 73.5-16.1 & 4.5-1.0 & 16.4-16.5 & 0.01-11.4 & 59.2-20.7 & 3.6-1.3 \\
0.225 & 17.0-17.2 & 0.01-13.8 & 77.9-18.5 & 4.58-1.0 & 17.1-17.3 & 0.01-12.1 & 97.8-19.6 & 5.7-1.1 \\
\end{array}
\]

in Figs 3 and 4 of Pickett et al. [8] from Cluster WBD measurements, could give rise to BEN ranging from \( \sim 220 \text{ Hz} \) to \( 10 \text{ kHz} \). Further, the amplitude of the intense electrostatic emissions (\( \sim 0345 - 0346:50 \text{ UT} \) on 22 September 2004, see Figure 2 of Teste and Parks, [6] is seen to be \( \delta E \sim (0.05 - 0.1) \text{ mV/m} \). It is noticed that both the observed frequency range (\( \sim 2-6 \text{ kHz} \)) and electric field amplitude are within the limit of predicted values by the electron-acoustic soliton model. In fact the predicted electric fields, corresponding to the lower range of soliton velocity ranges in Table 1, seem to be in excellent agreement with the observed \( \delta E \). To summarize, the generation of BEN observed in the PSBL by Teste and Parks [8] may be explained by the electron-acoustic soliton/double layer model discussed here.

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7 References