Electromagnetic signals propagating in the ionosphere can exhibit rapid variation in amplitude and phase, referred to as scintillation. UHF radars that operate in the polar region can experience scintillation anytime of the day or night, particularly in the winters around solar maximum. Unless accounted for, scintillation is likely to disturb many radar functions, especially those that rely on the measurement of radar cross section. This paper discusses several estimators, based on radar track data, of the $S_4$ scintillation index that characterize the severity of amplitude scintillation.

Estimates of $S_4$ are based on data (referred to as radar returns) obtained during track of targets which may have intrinsic RCS fluctuations. Key to this work is the consideration of thresholding, which is used in many radars to remove (from further processing) signals whose signal-to-noise ratio (SNR) is considered too low. For example, in the early warning radar (EWR) located at Thule, the instantaneous signal-to-noise ratio on every return is compared to a threshold before the return is applied in additional radar processing (track acquisition, track, etc.). Returns below threshold are discarded from further processing. We consider several estimators here. The direct estimator attempts to estimate the scintillation index through the direct calculation of the mean and standard deviation of the RCS from a number of radar returns. The maximum likelihood estimator uses multiple hypothesis testing to estimate the scintillation index that best fits the radar returns from some number of pulses. This estimator assumes that the statistics of the scintillation are well represented by the Nakagami-$m$ distribution and uses a count of the number of pulses transmitted and the number of returns above threshold.

Based on this work, we developed a modified maximum likelihood estimator with no pulse counter. We applied the modified estimator to several years of radar tracks in the polar region to obtain the statistics of UHF scintillation as viewed from the EWR at Thule, Greenland. One-way $S_4$ was measured from five thousand horizon-to-horizon tracks of large calibration satellites during a two-year period after solar maximum in May 2000. The data is analyzed to quantify the exceedance, or the level of scintillation experienced at various probabilities.

1 Introduction

In this work we analyze several years of radar tracks of calibration satellites to measure the $S_4$ scintillation index, which describes the variation in the received amplitude due to ionospheric propagation. In the following we describe a simulation used to study the performance of several estimators of scintillation index. We apply the simulation to compare the performance of the direct estimator (based on the usual definition of $S_4$) to that of a maximum likelihood estimator which has knowledge of the number of radar transmissions and the number of returns above and below the radar threshold. We then develop a modification of this estimator which operates without knowledge of the number of missed returns (below threshold). Since the mean SNR of the data is sufficiently high (well above the threshold), the modified estimator is found acceptable to measure the statistics of $S_4$ for several years of radar data collected near solar maximum.

2 Radar returns simulation

To study the performance of the $S_4$ estimators we developed a simulation of radar returns. The received signal is generated as the sum of two terms. The first term represents the voltage after two-way propagation through the ionosphere. This term gives a voltage with a two-way Nakagami-$m$ probability distribution function with a user
determined mean power. To this term is added in-phase and quadrature-phase white Gaussian noise. The simulation then generates independent samples of the received signal power plus noise for Monte-Carlo analysis. The user specifies the desired SNR of the radar return and the value of the one-way scintillation index. The samples are generated as two-way Nakagami-\(m\) variates and include the enhancement in average power due to two-way monostatic radar propagation [1].

### 3 Direct estimation of the scintillation index

The value of \(S_4\) is defined by the equation

\[
S_4 = \left\{ \frac{\langle P^2 \rangle - \langle P \rangle^2}{\langle P \rangle^2} \right\}^{\frac{1}{2}}
\]

(1)

where the ensemble averages (angle brackets) are replaced by the sample averages given by

\[
\langle P \rangle = \frac{1}{N} \sum P_i
\]

(2)

\[
\langle P^2 \rangle = \frac{1}{N} \sum P_i^2
\]

(3)

The quantity \(P_i\) is the radar cross section from a return. In our implementation of the direct estimator, we also corrected for the contribution to \(S_4\) from the known radar noise power. Fig. 1 shows the performance of the direct estimator as a function of the number of independent radar returns used for the estimate. In this example, the true value of \(S_4\) is unity. The figure shows the mean and standard deviation of the estimate for values of the mean SNR of 20, 30, and 40 dB, respectively. In all cases in this example, the threshold for SNR is 10 dB. Each point plotted in the figure is obtained from 100,000 measurements of \(S_4\) generated via Monte Carlo simulation. In the simulation, the two-way value of \(S_4\) is measured from \(N\) returns, corrected for the effects of the noise, and then converted to one-way \(S_4\) via the assumption of Nakagami-\(m\) statistics. In spite of the high values of mean SNR, the estimator is biased, even for as many as 500 independent samples of the return. The bias is a function of the SNR, decreasing as the value of SNR increases. The standard deviation is a very weak function of SNR for these high values of SNR.

![Figure 1: Performance of the direct estimator of \(S_4\).](image)

The direct estimator of scintillation is accurate for the case where there is no threshold and there are many returns or samples from which to estimate \(S_4\). However, the direct estimator is flawed (especially for strong scintillation) if deep fades fall below the radar threshold and are ignored.
4 Maximum likelihood estimation of the scintillation index

The channel estimation problem here is similar to that of multiple hypothesis testing as described in [2]. In the multiple hypothesis testing problem, a number of noisy measurements are available from a source. The goal of multiple hypothesis testing is to determine which of several source “signals” caused the observed measurements. Here the measurements are the observed radar returns. The measured returns include the effects of target information, the channel scintillation, the radar noise, and the radar SNR threshold. We separately estimate the mean SNR and RCS. For this work we assume the target RCS is constant (as it is for radar calibration targets). Given $N$ independent values of the radar RCS or SNR (these are proportional if the range, transmission frequency, threshold, pulse duration, etc. do not change), we compare the measured distributions to many possible (hypothesized) distributions calculated beforehand for all possible values of $S_4$. The best match gives the estimate of the value of $S_4$.

Fig. 2 shows the performance of the maximum likelihood estimator for the case of four values of mean SNR from 13-30 dB. The true value of $S_4$ is unity and the radar threshold for SNR is 10 dB. Note that good measurement accuracy is obtained with as few as 100 independent radar returns even for the case that the mean SNR is only 3 dB above the threshold. For this estimator, the radar knows the number of transmitted pulses and the number of returns above and below threshold.

5 Revised maximum likelihood estimator

During the data collection activity, there was no pulse counter to note the occurrence of a radar return below threshold for a given transmitted pulse. Therefore we developed a revised maximum likelihood estimator that ignores the number of missing returns and determined its performance as a function of mean SNR, the number of returns, and the true value of $S_4$. Generally, the accuracy was very good for true $S_4$ less than 0.8 as long as the mean SNR exceeded 25 dB. Almost all the Thule data had mean SNR in excess of 25 dB, so the revised estimator could be applied accurately to the data analysis.

6 Results

The radar data consist of the RCS, SNR, and the time of the return. The transmission frequency and SNR threshold are constant. The pulse duration changes at most one time during a satellite pass. We used a moving 30-second averaging window, moving the window by 5 seconds to obtain the next processing window. For each window, we estimate the mean SNR and RCS, and used all the returns in the window to obtain an estimate of $S_4$. In our hypothesis
testing, we used possible values of mean SNR ranging from 15 to 55 dB (in steps of 1 dB) and $S_4$ from 0 to 1.4 (steps of 0.02). The radar pulse repetition frequency varied and the number of returns in a 30-sec window ranged from approximately 50 to 200, sufficient for good estimates of $S_4$. For each month we found the cumulative distribution of $S_4$ and then found the values of $S_4$ that are likely to be exceeded with certain probabilities. The data set extended from October 2001 through April 2003.

![Figure 3: Exceedance values estimated using the modified maximum likelihood estimator.](image)

Fig. 3 shows the values of $S_4$ that are exceeded with a certain probability (denoted by the color) as a function of the month and year. No data was taken in the months where there are no colored boxes. From the figure, in January 2002, the value of $S_4$ of 0.75 is likely to be exceeded 0.05 percent of the time. The value of $S_4$ of 0.3 is likely to be exceeded 50 percent of the time in the same month.

Consideration of the tops of the red boxes (the 5 percent level) leads to an important result for polar region scintillation observed by UHF radar. For example, from Thule during the period from December 2001 through February 2002, the probability exceeds five percent that the one-way scintillation index at UHF is greater than 0.7. On a daily basis this translates into 1.2 hours a day of strong scintillation. In the next polar winter (2002) the probability exceeds five percent that the scintillation index is greater than 0.47. That is, there is a significant probability of severe scintillation for several winters following solar maximum. Unfortunately, there is insufficient data to obtain equivalent information for the winter of solar maximum (2000) when the scintillation is most severe.

7 References
