Underdense, Overdense, and Bragg Scattering in Radar Meteors

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Abstract

Many models have been proposed to interpret the radar scattering represented in the radar meteor head- and trail-echoes. These models often invoke the concepts of "overdense" versus "underdense" (relative to plasma frequency) scattering and of Bragg scattering or filtering in describing details of the scattering structure. We argue that, while appropriate to volume filled incoherent scattering, Bragg scattering is of limited use in interpretation of radar meteors. We also present a stricter definition of underdense versus overdense scattering in terms of the net scattered E-field in the plasma relative to the incident E-field. We give modeling examples supporting this approach.

1. Introduction

From the earliest days of radar meteor observations it was assumed that both underdense and overdense scattering occurred in both trail- and head-echoes [1-3]. That overdense echoes were the explanation for complex trail-echoes was disputed by *Elford and Campbell* [4] who concluded that the observed effects were due to meteoroid fragmentation. Recently several authors [5-8] have observed and modeled the effects of fragmentation and flare-trails on head-echoes. In this they have concluded that the meteor head-echo results from underdense scattering that effectively originates from a point target located at the phase center of the scattering region. Here we utilize numerical models to explore scattering from meter-scale distributions of plasmas that are smooth and small relative to the scales necessary for Bragg or incoherent scattering in the meteor zone [9].

2. Bragg Scattering

The issue of requiring Bragg scale—$\lambda_0/2$ for backscattering where λ_0 is the radar wavelength—irregularities to obtain radar backscattering meteor head- or trail-plasmas stems from misunderstanding how "deep" the medium must be before Bragg filtering is essentially perfect. We briefly explore this question via 1-D backscattering of plane waves from first a uniform medium and then from a medium with concentration variations at the Bragg scale of $\lambda_0/2$ as illustrated in Figure 1. In particular, as long as the medium is perfectly uniform and uniformly illuminated, the net

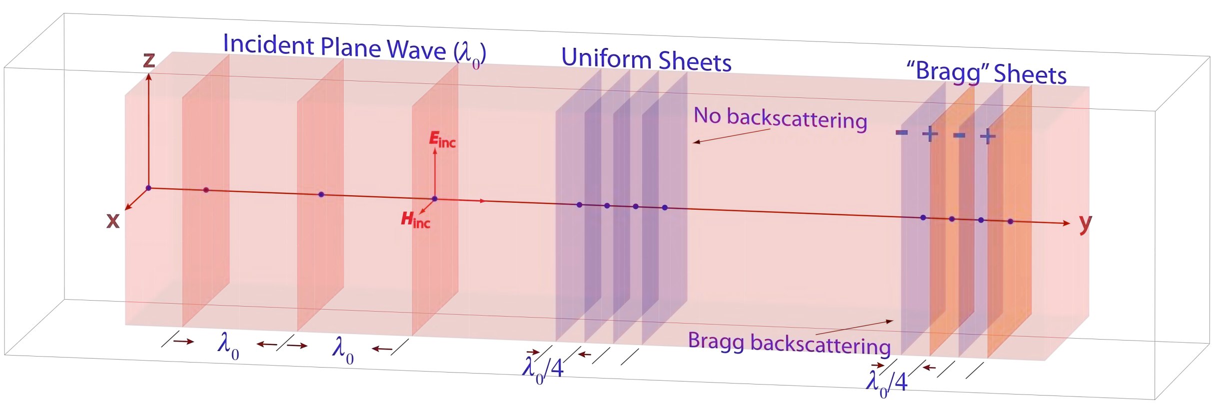

Figure 1. Examples of Bragg scattering or filtering effects. For a plane wave incident on an optically thin "plasma", there is no backscattering from a uniform plasma as scattering from sheets separated by $\lambda_0/2$ cancel. There is backscattering from a medium that has inhomogeneities on the Bragg scale. In particular, as illustrated above, a longitudinal wave of wavelength $\lambda_0/2$ gives rise to net backscattering. This is the usual incoherent scatter case.

body scattering is always zero. This is because the medium can always be divided into uniform sheets separated by $\lambda_0/4$ such that the scattering from all such pairs of sheets always cancel as they are 180° out of phase—the roundtrip phase path from one sheet to the other is 180°. It then follows that a longitudinal wave or other disturbance that

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produces perturbations on this scale—the Bragg scale—will result in net scattering as also illustrated in Figure 1. It seems intuitively clear from Figure 1 that scattering density variations on other scales will also produce net scattering unless the depth of medium over which scattering occurs is sufficient for contributions from these other scales to cancel. For example, a slight linear increase in scattering density will always produce net scattering for any radar for which the medium is optically thin. Additionally, any "small" and "smooth" optically thin scatterer will also produce scattering over a large range of frequencies as we illustrate in Figure 2.

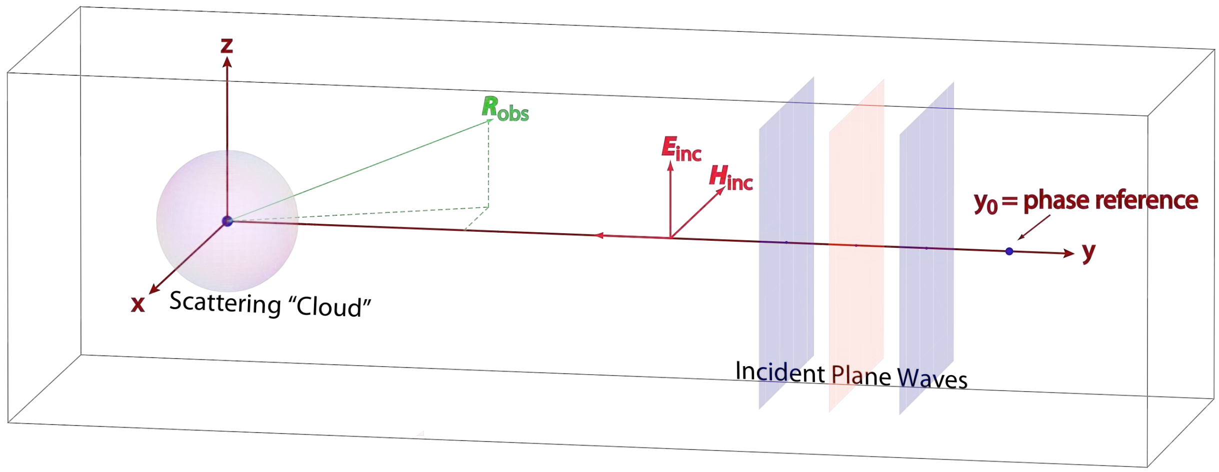

Figure 2. The geometry for which the net E-field scattered from the plasma "cloud" is found using eqns. (1)-(4).

3. Underdense Scattering

Figure 2 shows the geometry of a numerical experiment for determining if scattering from a plasma sphere is underdense or overdense. Here the scattering "cloud" is a uniform plasma at a specified plasma frequency. The sphere is illuminated by a uniform plane wave and, assuming all electrons are uniformly illuminated, the net scattered electric field is calculated anywhere in the sphere, on the surface of sphere, or at any angle/distance from the sphere. We argue that if the ratio of the net scattered E-field to the incident E-field is small in the sphere or at the surface of the sphere, then the scatterer is in fact underdense—multiple scattering does not occur—even if the plasma frequency is above the incident wave frequency.

Our model is based on solution of the following equations for the scattered E-field where (1) describes the plane wave incident on the spherical plasma, (2) is the non-relativistic acceleration of the electron by the incident E-field, and (3) gives the scattered E-field at the observer location where (4) is the unit vector in the direction of the observer. These equations are for single electron scattering where ω is angular frequency, $k=2\pi/\lambda$ is the wavenumber, and v, e, m_e are the electron speed, charge, and mass, respectively. Additionally, E_0 is the incident E-field magnitude, ε_0 is the dielectric constant of free space, and c is the speed of light. The net E-field scattered by the plasma, $\underline{E}_{Scattered}$, is found by dividing the spherical volume into differential elements that are very small compared with the wavelength. All the electrons in each differential element are then considered to scatter in phase—thus the net scattered E-field is just equation (3) times the number of electrons in that differential element and net E-field is found by summing over all elements with the net phase delay for each path—this is equivalent to the correct retarded time.

$$\underline{E}_{inc} = E_0 \hat{z} e^{j(\omega t + ky)} \tag{1}$$

$$\underline{\dot{v}} = \frac{e}{m_e} \underline{E}_{inc} \tag{2}$$

$$\underline{E}_{Scattered} = \frac{e}{4\pi\varepsilon_0 c^2 R_{obs}} \left[\hat{q} \times \hat{q} \times \underline{\dot{v}} \right]_{\text{retarded time}} \tag{3}$$

$$\hat{q} = \underline{R}_{obs} / \left|\underline{R}_{obs}\right| \tag{4}$$

The result of applying the above described numerical solution to the Figure 2 geometry is given in Figure 3. Here the ratio of scattered-to-incident E-field is found at the center and the surface of a 0.5 meter radius sphere with an electron concentration corresponding to a 50 MHz plasma frequency. As can seen, the maximum scattering ratio of slightly more than 0.09 occurs at 10 MHz (30 m wavelength) with the corresponding backscatter ratio at the surface of 0.073. These results then confirm the original conjecture that all electrons are nearly equally illuminated and that multiple scattering effects are of little or no importance. These results further confirm that the often assumed reflection of the radar signal from a region of plasma with plasma frequency greater than radar frequency depends on the size of the plasma in wavelengths. Additionally note that in the Figure 2 geometry, the Figure 3 results occur without need to invoke any Bragg criterion considerations. We will explore the required size/depth of the plasma giving rise to both Bragg and plasma frequency based reflections in a future paper.

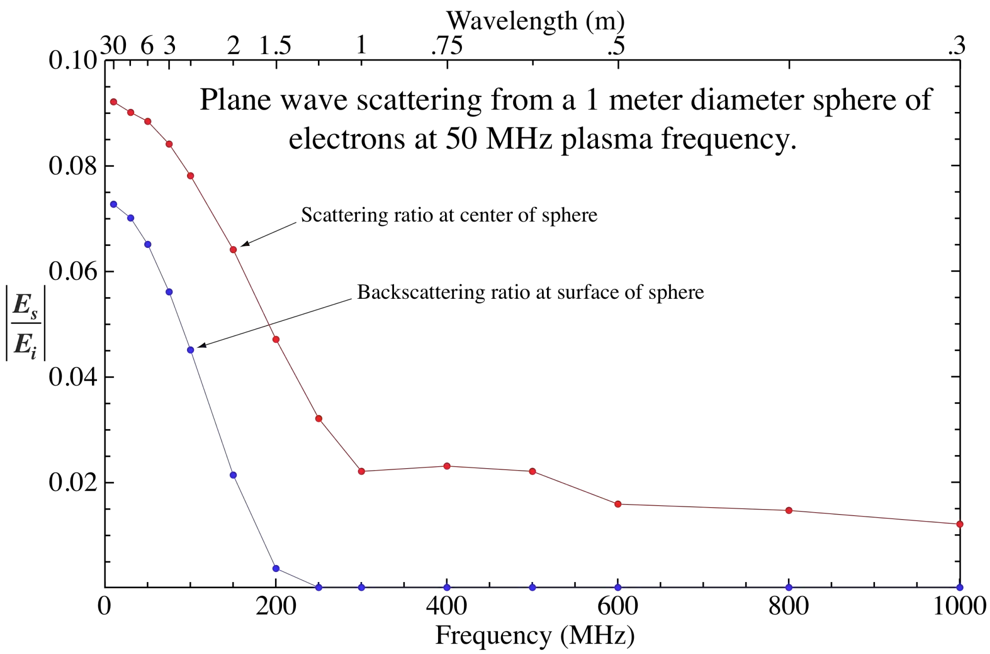

Figure 3. Ratio of net scattered-to-incident E-field at the center and surface of a spherical plasma.

4. Summary & Conclusions

Many models have been proposed to interpret the radar scattering represented in the radar meteor head-echo and in the many non-classical trail-echoes such as Range-Spread Trail Echoes (RSTE) [10, 11]. *Mathews et al.* [6] to some extent bridge the head/trail-echo gap by reporting flare-trails that are seen in common volume using the Arecibo VHF radar but are not seen at UHF thus providing additional clues to the radio science of the scattering. Their result is at least partially addressed by Figure 3 where the backscatter E-field ratio at the surface of the sphere is above 0.06 at 50 MHz but too small to be resolved at 430 MHz simply because the plasma is optically thin and front-to-back interference of the scattered wave occurs as the sphere dimension is greater than the wavelength at 430 MHz. Additionally, scattering models often invoke the concept of Bragg scattering or Bragg filtering that, while appropriate to volume filled incoherent scattering, is we argue, of limited use for interpretation of head-echoes and, per above, trail-echoes. We presented in Figure 1 two simple examples of Bragg scattering or filtering on the way to demonstrating a scattering model comprised of a simple spherical plasma distributions for which we determine the importance of "underdense" scattering in a situation that includes what might be expected to be "overdense" cases. We find that, even though regions of the plasma surrounding the meteoroid may exhibit plasma frequencies above the radar frequency, the plasma remains underdense. That is, all electrons are equally illuminated and the scattered E-field remains small compared with the incident E-field everywhere in the scattering plasma. Additionally, we find that, as a result, the net radar scattering cross-section depends in detail on the actual distribution of the plasma surrounding the meteoroid. Further we note that scattering from these underdense plasmas exhibit a well defined "phase center" or net origin of the scattering. This implies that scattering from these plasmas can be treated as point scatterers and that the net scattering cross-section will be the result of coherent scattering in the sense that all the electrons near the phase center scatter in-phase. This result is important to ready interpretation of these returns in terms of the Doppler frequency offset [12] as the spectral return is quite narrow thus further distinguishing these returns from background ionospheric scattering such as Bragg scattering from volume-distributed instability structures and from incoherent scattering.

Acknowledgement

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