NUMERICAL CALCULATION OF RESONANT FREQUENCIES AND MODES QUALITY OF IONOSPHERIC-MAGNETOSPHERIC ALFVEN RESONATOR

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The model of environment close to a reality is considered. This model supposes only a numerical solution of a problem. The numerical integration of ODE for the spherical impedance is executed. Eigenfrequencies IMAR are found by Newton's method. Resonant frequencies and quality modes of ionospheric-magnetospheric Alfven resonator (IMAR) are resulted in a range [0, 1] Hz at magnetic latitude 30^0.

1. Introduction

We will continue research of ionospheric-magnetospheric Alfven resonator in this article. We considered a two-layer model of the environment earlier [1], which allows virtually qualitatively trace the development of spectrum formation IMAR, depending on different parameters [2], [3] problem. We will consider model of the environment significantly closer to reality in the present work, it will be described in detail below. The problem is solved numerically for this model. Resonant frequencies and quality modes of IMAR are found.

2. Solution of the problem

The equation for the impedance of the spherical Alfven waves recorded along the geomagnetic field line for the integration variable θ has the form:

$$\frac{dU}{d\theta} = (U^2k^2+1)a\sin\theta\frac{\sqrt{1+3\cos^2\theta}}{\sin^2\theta_a} - \frac{3U\sin(2\theta)}{2(1+3\cos^2\theta)}. \tag{1}$$

2.1. Model of environment

$k(\theta) = 2\pi f\dfrac{\sqrt{1+i\dfrac{\nu(\theta)}{2\pi f}}}{(c_a(\theta)/L)}$ - complex wave number of environment depending on θ and frequencies f.

a=6400/L – normalize of radius of the Earth.

$$c_a(\theta) = \begin{cases} c_{a1}(\theta),\ if\ \pi-\theta_1 \ge \theta \ge \theta_1, \\ c_{a2}(\theta),\ if\ \theta \le \theta_1, \\ c_{a3}(\theta),\ if\ \theta \ge \pi-\theta_1. \end{cases} \qquad \nu(\theta) = \begin{cases} \nu_1(\theta),\ if\ \pi-\theta_1 \ge \theta \ge \theta_1, \\ \nu_2(\theta),\ if\ \theta_0 \le \theta \le \theta_1, \\ \nu_3(\theta),\ if\ \pi-\theta_0 \ge \theta \ge \pi-\theta_1, \\ 0,\ if\ \theta \le \theta_0, \\ 0,\ if\ \theta \ge \pi-\theta_0. \end{cases} \tag{2}$$

$$c_{a1}(\theta) = c_a(\theta_1)\sqrt{M+1}\Bigg/\sqrt{1+M\Big/ch^2\left(\frac{a(\sin^2\theta-\sin^2\theta_1)}{L_1\sin^2\theta_1}\right)}, \tag{3}$$

$$c_{a2}(\theta) = c_a(\theta_1)\sqrt{M_1+1}\Bigg/\sqrt{1+M_1\Big/ch^2\left(\frac{a(\sin^2\theta-\sin^2\theta_1)}{L_2\sin^2\theta_1}\right)}, \tag{4}$$

$$c_{a3}(\theta) = c_a(\theta_1)\sqrt{M_1+1}\Bigg/\sqrt{1+M_1\Big/ch^2\left(\frac{a(\sin^2\theta-\sin^2(\pi-\theta_1))}{L_2\sin^2(\pi-\theta_1)}\right)}. \tag{5}$$

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$$\nu_1(\theta)=\nu(\theta_1)\exp\left(-\beta_1\frac{\sin^2(\theta)-\sin^2(\theta_1)}{\sin^2(\theta_1)}\right),\ \nu_2(\theta)=\nu(\theta_1)\exp\left(-\beta_2\frac{\sin^2(\theta)-\sin^2(\theta_1)}{\sin^2(\theta_1)}\right),$$

$$\nu_3(\theta)=\nu(\theta_1)\exp\left(-\beta_2\frac{\sin^2(\theta)-\sin^2(\pi-\theta_1)}{\sin^2(\pi-\theta_1)}\right). \tag{6}$$

The model parameters corresponding to the conditions of night and the maximum solar activity are selected as follows: β_1=21,3; β_2=427; L_1=300 km; L_2=25 km; L=300 km; M=144, M_1=4,87·10^5; $\nu(\theta_1)$=0,3 s^{-1}; $\nu(\theta_0)$= 3,15·10^{3}s^{-1}; $c_{a2}(\theta_1)$=430 km/s; a_1- corresponds to coordinate of a maximum of layer F2, θ_0 – corresponds to border an ionosphere – vacuum. Thus, in the description of the model includes 13 basic parameters. Their number can be doubled, making the northern and southern ionosphere-magnetosphere dissymmetric. The specific parameters of the layers of Epstein used in this problem are chosen, for example, using the method of least squares. As it has been made by us in work [4]. With that said, we note that the dependence of the Alfven speed is shown in Fig.1 corresponds to a similar dependence of the reduced work [5].

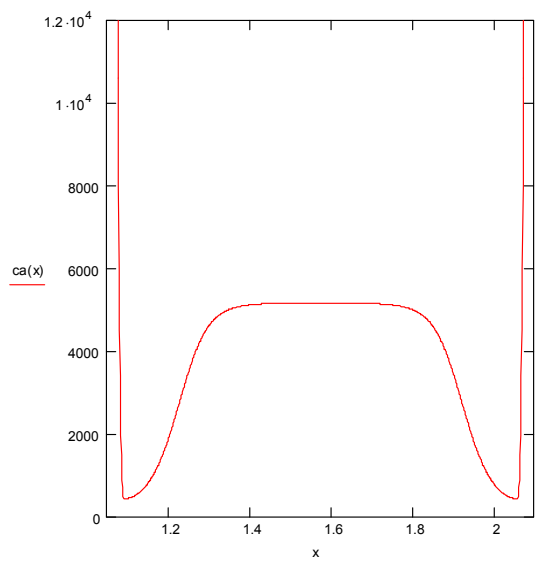

Figure1. The dependence of the Alfven speed of angle $x=\theta$.

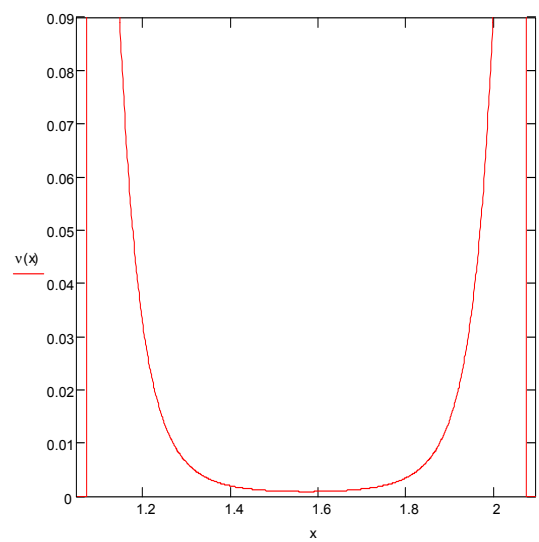

Figure 2. The dependence of the collision frequencies from $x=\theta$.

2.2. The numerical solution of the equation for the impedance

We construct a numerical solution based on the equation (1). We used the Runge-Kutta method fourth order at 7·10^4 points for integration of this equation. First boundary condition $U(\omega, a, \theta_a)$=0. Second boundary condition $U(\omega, a, \pi-\theta_a)$=0. The first boundary condition is the starting. It holds for any cyclic frequency ω when we integrate. For performance of the second boundary condition it is necessary to find a set of complex frequencies ω_1, ω_2, ω_3 … etc. at which it takes place. It has been made numerically by Newton's method. We have found 10 eigenfrequencies in the frequency range [0, 1] Hz, which are presented in the table below:

Table1. The frequency range [0, 1] Hz

f_n	1	2	3	4	5	6	7	8	9	10
Ref_n	0,126	0,142	0,312	0,384	0,457	0,601	0,676	0,789	0,902	0,994
Imf_n	0,0142	0,0163	0,0104	0,0185	0,0154	0,0161	0,0189	0,0157	0,0182	0,0182
Q_n	4,43	4,37	15,0	10,4	14,8	18,7	17,9	25,1	24,8	27,3

In the numerical solution of the problem, we relied on the results of analytical studies performed by us [3]. It has allowed us as a whole to understand and use the features of the problem being solved.

Let's result for descriptive reasons on fig.3 the graph of dependence of good quality mode from its number. For smoothing, which we have used cubic splines.

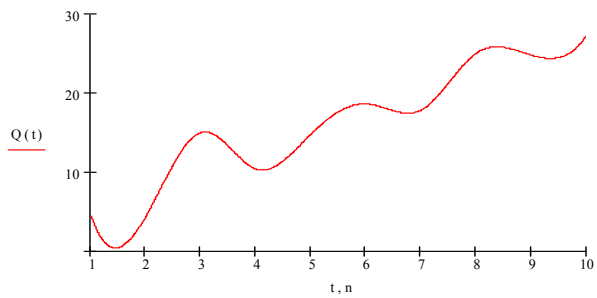

Figure3. The graph of dependence of good quality mode from its number

Local maxima in the graph correspond to the modes closer in properties to the modes of the magnetospheric Alfven resonator with typical Q values of 15 to 27. The first three minima in the table can be correlated with the modes of the ionospheric Alfven resonator (IAR). Size of good quality, which from several units to ten. Good qualities of other modes are defined as properties of an ionosphere, and properties of a magnetosphere. As a whole, analyzing the properties of the modes can say that in this model, they are formed only when the account close of ionospheric-magnetospheric interaction. Some modes of type IAR having small good qualities ~ 4, quickly fade and don't make a significant contribution to the total signal. With increasing numbers of mod and, consequently, its frequency of good quality of modes, increase attenuations to proportionally factor iv/ω because of dispersion.

The following figures show the impedances of the second, third and fourth modes along the geomagnetic field lines.

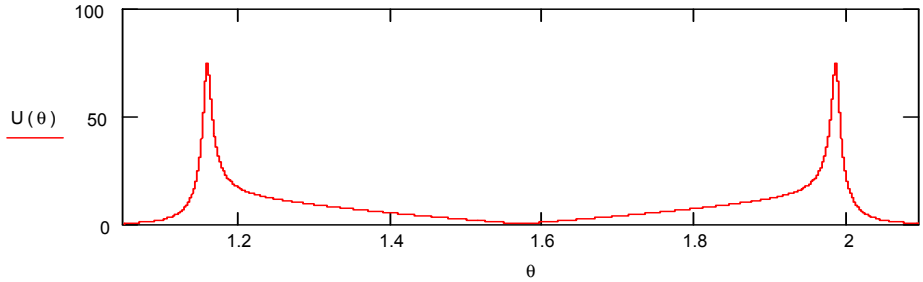

Figure 4. The impedance of the second mode

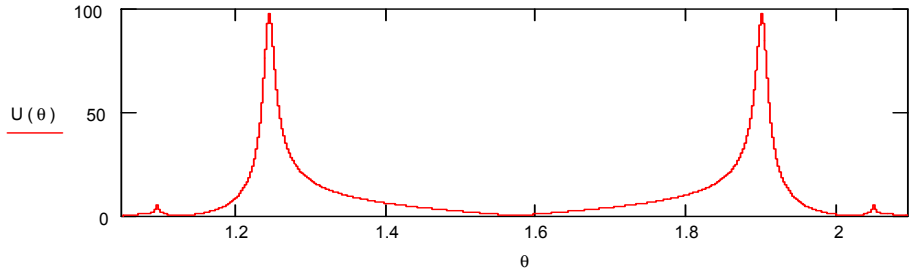

Figure 5. The impedances of the fourth mode

As can be seen from these figures that the number of local maxima is exactly equal to the mode number. This allows at the numerical calculation to correctly identify the number of resonant modes.

3. Conclusion

1. We managed to find the natural frequencies of multi-parameter problems for spherically layered model that includes two layers of Epstein with different parameters, taking advantage of the numerical investigation of the problem.
2. It is shown that good quality of modes, calculated for this model, reach a significant magnitude.
3. Considering the near-Earth space plasma for Alfven waves as a single entity IMAR, we can perform adequate calculations of its resonance frequencies.
4. The received results can be a starting point for solving the problem of finding the resonant frequencies of IMAR in the absence of symmetry of the problem from the angle φ – the spherical coordinate system.

4. References

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