Ionospheric Irregularities and Potentialities of Multifrequency Correction in Global Navigation Satellite Systems

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Abstract

Measurements of propagation time or phase of a signal at two frequencies enable us to eliminate the first ionospheric correction and improve accuracy of the Global Navigation Satellite System (GNSS) from tens of meters to centimetres. This paper explores possibilities of eliminating higher-order ionospheric errors in going to measurements at more frequencies. It is shown that the second-order correction, associated with the geomagnetic field effect on the refractive index, may be taken into account already in dual-frequency measurements. Further improvement of accuracy in the triple-frequency GNSS in view of phase scintillations is complicated by diffraction effects.

1. Introduction

As is known, the main contribution to the error of the Global Navigation Satellite Systems (GNSS) is made by the inhomogeneous ionosphere containing most of the GNSS signal propagation path. Anisotropic ionospheric plasma effects in phase or signal delay at high frequencies may be represented as a rapidly decreasing series in inverse powers of frequency. This dispersive property of ionospheric plasma helps to eliminate most of its effect in dual-frequency measurements. However, this accuracy is not sufficient for high-precision measurements. In this report, we study the possibility of improving the GNSS measurement accuracy as the number of frequencies employed increases.

2. The Geometrical Optics Approximation for the GNSS Signal Phase

We will employ a coordinate system $\{x, y, z\} = \{\boldsymbol{\rho}, z\}$ with the axis z passing through the radiation $\mathbf{r}_0 = \{0, z_0\}$ and reception $\mathbf{r}_t = \{0, z_t\}$ points. At a frequency f, the GNSS signal phase path $\varphi_g(f)$, obtained when solving the eikonal equation in the second-order approximation of the perturbation theory, has the form [1]

$$\varphi_g(f) = D^{(1)} = D - \Delta D_{1g}^{(1)}(f) - \Delta D_{2g}^{(1)}(f) - \Delta D_{3g}^{(1)}(f) = D - 40.3 I_{1g} f^{-2} - 40.3 I_{2g} f^{-3} - (40.3)^2 I_{3g} f^{-4} . \quad (1)$$

Here D includes the true distance $|z_t - z_0|$ as well as all frequency-independent errors in phase measurements; $\Delta D_{1g}^{(1)}(f)$, $\Delta D_{2g}^{(1)}(f)$, $\Delta D_{3g}^{(1)}(f)$ are single-frequency corrections of the first, second and third orders, respectively; $\Delta D_{1g}^{(1)}(f) = 40.3 f^{-2} I_{1g} = 40.3 f^{-2} \int_{z_0}^{z_t} N(z') dz'$, $N(\mathbf{r})$ is the electron density of ionospheric plasma;

$$\Delta D_{2g}^{(1)}(f) = 40.3 f^{-3} I_{2g} = 40.3 f^{-3} \int_{z_0}^{z_t} N(z') f_H(z') \cos\theta(z') dz' , \quad (2)$$

$f_H(z')$ is the electron gyrofrequency, $\theta(z')$ is the angle between the directions of wave propagation and geomagnetic field line; $\Delta D_{3g}^{(1)}(f) = (40.3)^2 f^{-4} I_{3g} = f^{-4} \int_{z_0}^{z_t} p_N^2(z) dz / 2$, $\mathbf{p}_N(z) = -40.3 \int_{z_0}^{z_t} \partial N(\boldsymbol{\rho}, z') / \partial \boldsymbol{\rho}\big|_{\boldsymbol{\rho}=0} dz'$.

Leaving only the largest first-order correction in (1), in dual-frequency measurements we have

$$\varphi_1 = \varphi_g(f_1) = D - 40.3 I_{1g} / f_1^2, \quad \varphi_2 = \varphi_g(f_2) = D - 40.3 I_{1g} / f_2^2 \quad (3)$$

From System (3) it is easy to find the distance D and the total electron content I_{1g}. For the distance, in particular, we have the ionosphere-free formula:

$$D \approx D^{(2)}(f_1, f_2) = \left(\varphi_1 f_1^2 - \varphi_2 f_2^2\right) / \left(f_1^2 - f_2^2\right) \quad . \quad (4)$$

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A residual error of the dual-frequency measurements can be found by substituting (1) into (4):

$$D^{(2)}(f_1, f_2) \approx D - \Delta D_{2g}^{(2)}(f_1, f_2) - \Delta D_{3g}^{(2)}(f_1, f_2)\,, \tag{5}$$

where

$$\Delta D_{2g}^{(2)}\left(f_1, f_2\right) = -40.3 I_{2g} / \left[\left(f_1 + f_2\right) f_1 f_2\right],\ \ \Delta D_{3g}^{(2)}\left(f_1, f_2\right) = -\left(40.3\right)^2 I_{3g}\left(f_1 f_2\right)^{-2}. \tag{6}$$

Should the first three terms be held in the right side of (1), we derive

$$\varphi_g\left(f\right) \approx D - 40.3 I_{1g} f^{-2} - 40.3 I_{2g} f^{-3}. \tag{7}$$

The geomagnetic field in the ionosphere changes slowly. Expression (2) may therefore be written as [2]:

$$\Delta D_{2g}^{(1)}(f) = 40.3 f^{-3} I_{2g} \ \simeq \Delta D_{2gap}^{(1)}(f)\,, \tag{8}$$

where

$$\Delta D_{2gap}^{(1)}(f) = 40.3 f^{-3} f_H(z_m) \cos\theta(z_m) I_{1g}\,, \tag{9}$$

z_m - is the height of the principal ionospheric maximum.

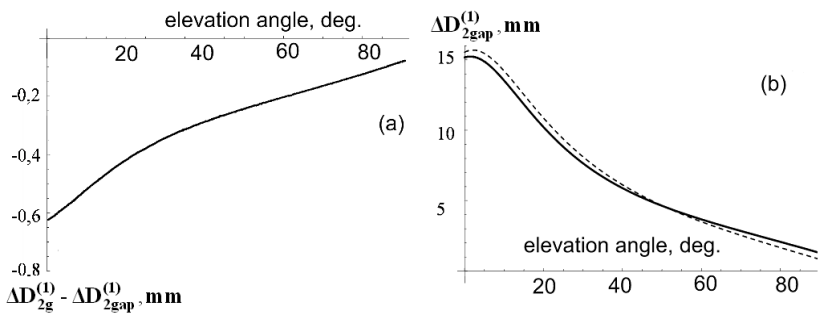

Fig. 1. The error in calculating the second-order correction from approximate formula (8) (a) and the comparison between calculations of the second-order correction (b) for the IGRF (solid curve) and the magnetic dipole (dashed curve) models as a function of the angle of elevation.

Our simulation has revealed that (see also [2-3]) approximation (8) is sufficiently accurate to account for the second-order correction in phase measurements, to within the experimental error, 1 mm. As an example, Fig. 1a shows the angle-of-elevation dependence of the difference between the second-order error (2) and its approximation (8) with the geomagnetic field considered to be the magnetic dipole field situated at the Earth's centre. The error was calculated for the azimuth of 180°, when the receiver was at the latitude of 60° and the longitude of 0°.

Taking into consideration geomagnetic field effects from formulas (7)–(9) requires information not only on satellite angular coordinates, but on the magnetic field as well. Fig. 1b illustrates the role of this geomagnetic field model, using results of the second-order error calculations made for the same conditions as in Fig. 1a by means of two models – the IGRF model (solid curve) and the magnetic dipole model (dashed curve) – applied as the geomagnetic field model. It is evident that because of the small second-order correction, the refinement of the magnetic field model has little effect on the calculations. Approximation (9) enables us to simplify expression (7) given that the ionospheric gyrofrequency is much less than the GNSS signal frequency $f_H << f$:

$$\varphi_g\left(f_e\right) \approx D - 40.3 I_{1g} f_e^{-2}. \tag{10}$$

where $f_e = f - f_H(z_m)\cos\theta(z_m)/2$.

Formula (10) allows us to account for the second-order effects in dual-frequency measurements:

$$D \approx D^{(2)}(f_1, f_2) = \left(\varphi_1 f_{e1}^2 - \varphi_2 f_{e2}^2\right) / \left(f_{e1}^2 - f_{e2}^2\right). \tag{11}$$

Taking the second-order effects into consideration, as in Formula (11), (1) can be rewritten as

$$\varphi_g\left(f\right) \approx D - 40.3 I_{1g} f_e^{-2} - \left(40.3\right)^2 I_{3g} f_e^{-4}. \tag{12}$$

Given phase measurements at three frequencies, we can write a system of equations:

$$\begin{cases} \varphi_1 = \varphi_g(f_1) = D - 40.3 I_{1g} f_{e1}^{-2} - (40.3)^2 I_{3g} f_{e1}^{-4}, \\ \varphi_2 = \varphi_g(f_2) = D - 40.3 I_{1g} f_{e2}^{-2} - (40.3)^2 I_{3g} f_{e2}^{-4}, \\ \varphi_3 = \varphi_g(f_3) = D - 40.3 I_{1g} f_{e3}^{-2} - (40.3)^2 I_{3g} f_{e3}^{-4}. \end{cases} \quad (13)$$

By solving system (13), we derive a triple-frequency distance formula

$$D \approx D^{(3)}(f_1, f_2, f_3) = a_1 f_{e1}^4 \varphi_1 + a_2 f_{e2}^4 \varphi_2 + a_3 f_{e3}^4 \varphi_3 , \quad (14)$$

where $a_1 = \left[\left(f_{e1}^2 - f_{e2}^2\right)\left(f_{e1}^2 - f_{e3}^2\right)\right]^{-1}$, $a_2 = \left[\left(f_{e2}^2 - f_{e1}^2\right)\left(f_{e2}^2 - f_{e3}^2\right)\right]^{-1}$, $a_3 = \left[\left(f_{e3}^2 - f_{e1}^2\right)\left(f_{e3}^2 - f_{e2}^2\right)\right]^{-1}$.

Thus, the triple-frequency measurements enable us to account for the first-, second- and third-order corrections in the GO approximation, using formula (14).

3. Diffraction Effects in GNSS Measurements

Diffraction effects are associated only with random irregularities. We ignored, therefore, these effects when calculating the second-order correction, bearing in mind the small amplitude of these irregularities. The diffraction variant of formulas for third-order errors has been derived in a second-order Rytov approximation [1]:

$$\varphi_d(f) = D^{(1)} = D - \Delta D_{1d}^{(1)}(f) - \Delta D_{2g}^{(1)}(f) - \Delta D_{3d}^{(1)}(f) = D - \frac{40.3}{f^2} I_{1d}(f) - \frac{40.3}{f^3} I_{2g} - \left(\frac{40.3}{f^2}\right)^2 I_{3d}(f), \quad (15)$$

where

$$I_{1d} = \int_{z_0}^{z} dz' \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\kappa N_\kappa(\boldsymbol{\kappa}, z') \cos\left\{\kappa^2 (z - z')/(2k)\right\} , \quad (16)$$

$$I_{3d} = -\frac{1}{2}\int_{z_0}^{z} dz' \int_{z_0}^{z'}\int_{z_0}^{z'} dz_1 dz_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\kappa_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\kappa_2 \boldsymbol{\kappa}_1\boldsymbol{\kappa}_2 N_\kappa(\boldsymbol{\kappa}_1, z_1) N_\kappa(\boldsymbol{\kappa}_2, z_2) \cos\left\{S(\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2, z', z_1, z_2)\right\} \quad (17)$$

$$S(\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2, z', z_1, z_2, k) = \left[\kappa_1^2 (z - z_1) + \kappa_2^2 (z - z_2) + 2\boldsymbol{\kappa}_1\boldsymbol{\kappa}_2 (z - z')\right]/(2k), \quad \tilde{N}_\kappa(\boldsymbol{\kappa}, z) = (2\pi)^{-2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\rho\, \tilde{N}(\boldsymbol{\rho}, z) \exp\{-i\kappa\boldsymbol{\rho}\}.$$

is the 2D spectrum of the electron density $\tilde{N}(\boldsymbol{\rho}, z)$ of ionospheric plasma irregularities. In (16)-(17), $N(\boldsymbol{\rho}, z) = \bar{N}(\boldsymbol{\rho}, z) + \tilde{N}(\boldsymbol{\rho}, z)$, $\left\langle \tilde{N}(\boldsymbol{\rho}, z) \right\rangle = 0$. Given the minimum size of irregularities larger than the Fresnel scale, diffraction formulas (15)-(17) transform to corresponding GO expressions (1). By substituting (15) into (4) and taking the second-order correction into consideration, as in Section 2, we can obtain the first- and third-order corrections of the dual-frequency reception with regard to diffraction effects [1]. And the substitution of (15) into (14) may give corrections for the triple-frequency reception [1]. In the GO approximation, these corrections up to the third order are eliminated by linear combination (14). However, taking account of diffraction effects yielded that expression (15) as opposed to (1) was not of the form of a series in inverse power of f. Thus, the linear combinations of dual-frequency (4) and triple-frequency (14) phase measurements retain not only the third-order corrections, but also a part of the first-order error.

We can get statistical characteristics for these corrections. The results of the numerical simulation of the first- and third-order corrections of dual-frequency (thin lines) and triple-frequency (thick lines) phase measurements in the turbulent ionosphere with the 5 % standard deviation of electron density fluctuations and the external scale of $L_0 = 20km$ are presented in Figs. 2. GPS frequencies $f_1 = 1572.42 \times 10^6 Hz$, $f_2 = 1227.6 \times 10^6 Hz$, $f_3 = 1176.45 \times 10^6 Hz$ were taken as emission frequencies; the Chapman layer with a critical frequency $f_c = 15 \times 10^6 Hz$, the characteristic scale of 70 km and a height of maximum of 320 km, as the ionospheric layer. Fig. 2a shows the average third-order corrections when the inner scale l_m is 1 km that corresponds to the case where

the inner scale exceeds the Fresnel scale r_f. Fig. 2b presents the results of calculating standard deviations of the first-order (dashed line) and third-order (solid line) corrections for the same parameters as in Fig. 2a. It is obvious that the third-order corrections make the largest contribution to these parameters. It is evident from Fig. 2a and Fig. 2b that in this case, when going to triple-frequency measurements (thick lines), the mean of errors (bias) of the first- and third-order and their variances decrease.

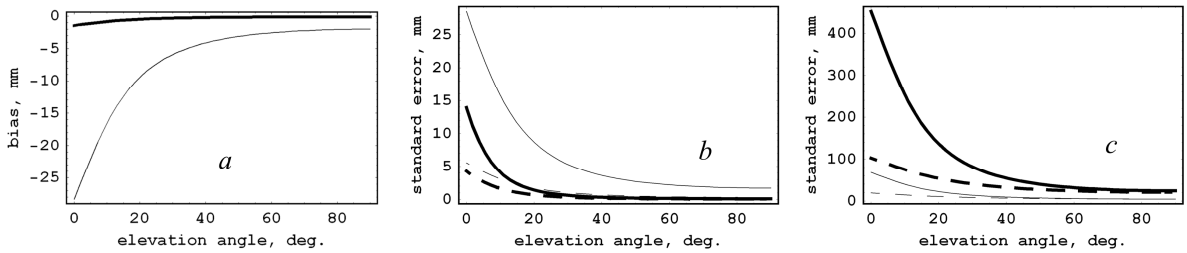

Fig. 2. The angle-of-elevation dependencies of the average third-order correction (a) and of the standard deviations of the first-order (dashed line) and third-order (solid line) corrections at $l_m = 1km$ (b) and $l_m = 0.07km$ (c). Thin and thick lines correspond to the dual-frequency and three-frequency GNSS, respectively.

Now let us examine a situation when the inner scale l_m is less than the Fresnel scale r_f. In this case, the results of calculating the mean error differ slightly from those presented in Fig. 2a; we do not show them here. Thus, the bias decreases during the transition from dual-frequency to three-frequency measurements, independently of the relation between the Fresnel scale and the inner scale. However, the behaviour of the error variance is different. Fig. 2c shows the results of calculating the standard deviations of the first- and third-order corrections in conditions similar to those in Fig. 2b, but at $l_m = 0.07km << r_f$. It is evident that the decrease in the inner scale l_m led to the increase in the correction variance during the transition to the triple-frequency measurements.

4. Conclusion

The numerical simulation was used to demonstrate the possibility of decreasing the systematic error in triple-frequency GNSS. The standard error associated with phase fluctuations (phase scintillations) decreases during the transition to triple-frequency measurements, if the inner scale of the ionospheric irregularity spectrum exceeds the Fresnel scale. Otherwise, the residual, diffraction phase scintillations in triple-frequency measurements may increase.

5. Acknowledgments

This study was supported by the Ministry of Education and Science of the Russian Federation (project No. 14.740.11.0078 of the Federal Special-Purpose Program «Human Resources for Science and Education in Innovative Russia» for 2009-2013).

6. References

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