Trade-Off between Vertical Resolution and Accuracy in Water Vapor Retrievals from Ground-based Microwave Brightness Temperature Measurements

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1. Introduction

Thermodynamic properties of the troposphere, particularly water vapor content and temperature, change in response to physical mechanisms, including frictional drag, evaporation, transpiration, heat transfer, pollutant emission and flow modification due to terrain. The planetary boundary layer (PBL) is characterized by a greater rate of change in its thermodynamic state than higher tropospheric altitudes. Such changes in the PBL typically occur on time scales of less than one hour; whereas the upper troposphere exhibits much longer time constants. Large horizontal gradients in vertical wind speed and steep vertical gradients in water vapor and temperature in the PBL result in high-impact weather, including severe thunderstorms. Observation of these gradients in the PBL with improved vertical resolution is important for improvement of weather prediction. Additionally high vertical resolution and accuracy of measured thermodynamic profiles, especially water vapor and temperature, are important for initialization of numerical weather prediction models. Satellite remote sensing in the visible, infrared and microwave bands provides qualitative and quantitative measurements of many atmospheric properties, including cloud cover, precipitation, liquid water content and precipitable water vapor in the atmosphere above the PBL. However, its ability to characterize thermodynamic properties of the PBL is limited by the confounding factors of ground emission in microwave channels and of cloud cover in visible and IR channels, as well as limitations in the vertical resolution of the remote sensing instruments onboard the satellite. Ground-based microwave radiometers are routinely used to estimate thermodynamic profiles, but the accuracy and resolution of vertical profiles may be improperly estimated. Here a new technique has been used to improve the vertical resolution of retrieved water vapor density profiles, based on the design of the Compact Microwave Radiometer for Humidity Profiling (CMR-H) [1]. The CMR-H operates at four frequencies near the weak water vapor absorption line, namely 22.12, 22.67, 23.25, and 24.5 GHz.

2. Optimization of Vertical Resolution using the Backus-Gilbert Technique

In atmospheric remote sensing, a water vapor profile is retrieved by the inversion of radiometer brightness temperature measurements near a water vapor absorption line. The set of brightness temperature measurements T_{Bi} is related to the actual water vapor profile by the forward model, given by

$$T_{Bi} = \int K_i(z')x(z')dz' + \varepsilon \tag{1}$$

where i is the index of brightness temperatures, $K_i(z')$ [2] is the kernel function, $x(z')$ is the actual water vapor profile and ε is the measurement error. A kernel function for an atmospheric parameter (such as water vapor or temperature) is the change in the measured brightness temperature due to a unit change in that parameter, e.g. water vapor density, at a particular altitude in a 1-km thick layer. Therefore, a kernel function is the sensitivity of measured T_B to changes in that parameter, e.g. water vapor, at a particular altitude. The kernel function [3] is

$$K_{\rho v}(f,\theta,z) = \frac{\partial \kappa_a(z)}{\partial \rho_v(z)}\left[T(z) - T_B(f,\theta,z)\right]e^{-\tau(0,z;\theta;f)\sec(\theta)}\sec(\theta) \tag{2}$$

where $\kappa_a(z)$ is the absorption coefficient of the atmosphere at a height z, $\rho_v(z)$ is the water vapor density at height z, $T(z)$ is the temperature at altitude z, T_B is the brightness temperature at frequency f and elevation angle θ, and τ is the opacity. Figure 1 shows the weighting functions for the frequencies of operation of the CMR-H. In general,

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weighting functions for different frequencies have different sensitivities to changes in water vapor densities at particular altitudes than other weighting functions do. Even though the weighting functions at the four frequencies of operation are somewhat similarly sensitive to changes in water vapor at lower altitudes, the weighting functions have different sensitivities to changes in water vapor at higher altitudes. The weighting functions at 22.12 and 22.67 GHz are more sensitive to changes in water vapor densities at higher altitudes than the weighting functions at 23.25 and 24.5 GHz are [1]. The weighting functions for 23.25 and 24.5 GHz have higher sensitivity at lower altitudes than those for 22.12 and 22.67 GHz do.

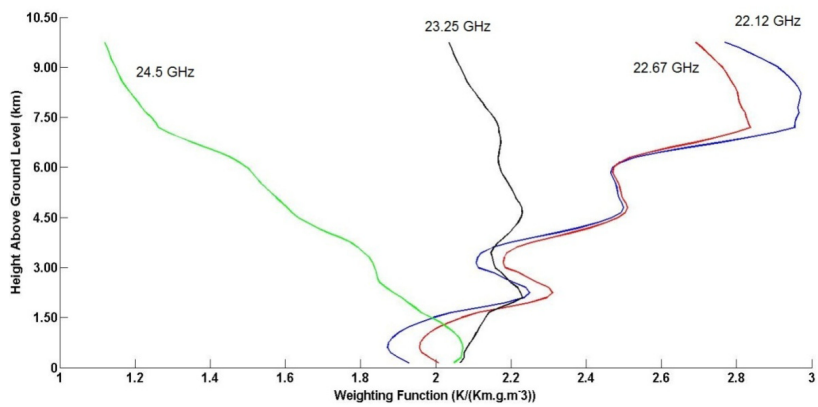

Figure 1. Weighting function at the four CMR-H frequencies

The estimated profile $\hat{x}(z)$ is related to the measured brightness temperatures by the coefficient G_i. By substituting the forward model for the brightness temperature measurements into the equation for the estimated profile, the estimated profile can be related to the actual profile by the averaging kernel. A linear combination of weighting functions also known as averaging kernels is computed at each height of retrieval. An averaging kernel [2] is given by

$$A(z,z') = \sum_{i=1}^{m} G_i(z) K_i(z') \tag{3}$$

The vertical resolution of a particular linear estimate at a given altitude is determined by the spread of the $A(z,z')$ Therefore, to have the finest vertical resolution possible, the spread of averaging kernel should be minimized. It should be noted that a Dirac delta function [2] would be an ideal averaging kernel since it has minimum spread. If the averaging kernel were a delta function, maximum information would be extracted in the form of an estimated profile. However, the shape of an averaging kernel is dependent on the finite number of weighting functions available. It would be difficult to get a delta function with a limited number of weighting functions. The spread of an averaging kernel is given by Backus and Gilbert [4] as

$$s(z) = 12\int_0^{10\,\mathrm{km}} A^2(z,z')(z-z')^2\,dz' \tag{4}$$

The maximum vertical resolution possible from a set of weighting functions can be determined from the coefficients of the averaging kernel which minimize $s(z)$ of averaging kernels subject to the condition that $A(z,z')$ is unimodular.

3. Scanning Strategy

The vertical resolution depends on the spread of the averaging kernel, which is in turn determined by the number of linearly independent weighting functions used. If the number of weighting functions is increased, the averaging kernel more closely approximates a delta function. This can be accomplished in two ways. One approach is to increase the number of measurement frequencies with high information content, and the second approach is to increase the number of elevation angles of measurements, or a combination of both. Since this study has been conducted based on the CMR-H, the number of measurement frequencies cannot be increased, but the number of elevation angles of measurement can. This can be done by observing the atmosphere at additional elevation angles. Therefore, a scanning strategy has been developed in order to maximize the number of weighting functions and thereby enhance the vertical resolution. In this approach, the atmosphere is scanned at elevation angles from 30^0 to 85^0, in increments of 5^0, with the addition of 15^0. There is no restriction on the upper bound of the elevation angles,

but the maximum elevation angle is chosen to be 90^0 because the atmosphere is assumed to be vertically stratified. Therefore, measurement at elevation angles greater than 90^0 is considered to be redundant. The lower limit of the elevation angle is set by the antenna beamwidth i.e., 3-4 degrees and the sidelobe contributions so that there is no contamination of the measured brightness temperature at lower elevation angles due to ground emission. An illustration of scanning of the atmosphere is shown in Figure 2.

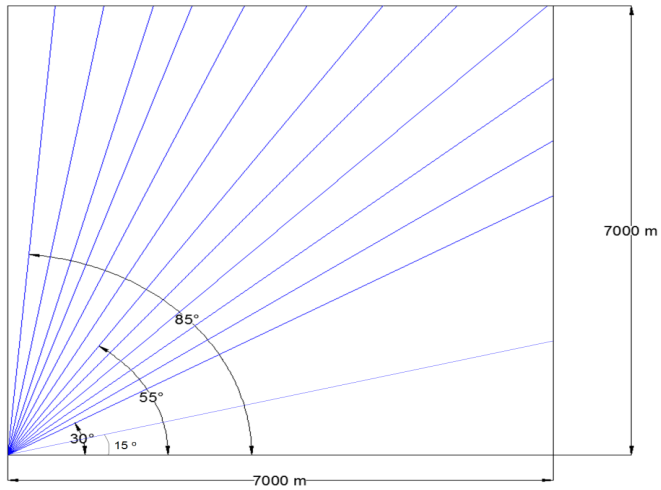

Figure 2. Scanning strategy in the vertical plane

4. Information Content Analysis

Information content analysis of measurements performed at various frequencies is important to provide insight into the profile information in a given measurement. The information content can be analyzed using weighting functions. Weighting functions form the basis for a number of methods to retrieve various meteorological parameter profiles from radiometric measurements. This analysis helps to identify frequency channels and elevation angles at which no new information is extracted, since the corresponding weighting functions are merely linear combinations of those at other frequencies, and the weighting functions can be predicted within the limits of experimental noise levels for all height levels z. Therefore, the information provided by the corresponding measurement is below the noise floor and not useful. The set of frequencies f near the weak water vapor absorption line and the elevation angle θ for scanning selected from these studies provide linearly independent observations with maximum information on the vertical profile of the water vapor density. Therefore, linearly independent weighting functions and measurements are needed to improve the vertical resolution. As a result of these studies, 12 linearly independent weighting functions corresponding to three elevation angles and four measurement frequencies are chosen. Figure 4 shows the spread values using the linearly independent weighting functions in red. Therefore, the vertical resolution that can be achieved by scanning the atmosphere is better than 500 m for the lowest 1 km and better than 750 m for the lowest 2 km of the troposphere. The spread values achieved from the technique are lower than the spread values of a currently-available inversion technique using a vertically-pointing radiometer, which are shown in blue.

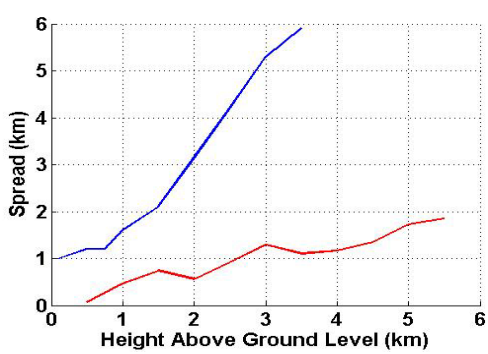

Figure 4. Spread values with respect to height above ground level for state-of-art techniques (blue curve) and a proposed new technique (red curve).

5. Trade-Off between Resolution and Accuracy

Some of the important problems in the study of atmospheric profile retrieval techniques concern the vertical resolution and accuracy of the estimated profiles. It has already been realized that improving vertical resolution from a given dataset will increase sensitivity of the retrieved profiles to random noise [5]. Analysis of a particular data set results in a trade-off between vertical resolution of the estimated profile and random noise. This can be seen in Figure 5, in which the spread versus error trade-off is evaluated for various altitudes. These trade-off curves have been evaluated using the scanning strategy mentioned in Section 4. The figure shows similar trade-off behavior at all altitudes in the lower troposphere. At each altitude, the error decreases rapidly for lower values of spread, followed by a gentler decrease for higher spread values. It can be seen that the error is maximum at lower spread values for all the altitudes. The maximum error ranges from 1 to 2 g/m^3 from 0.5 to 1.5 km altitude, and the maximum error is approximately 2.3 g/m^3 for 2 km altitude. Therefore, lower spread or improved resolution is associated with higher error due to random noise than higher spread is.

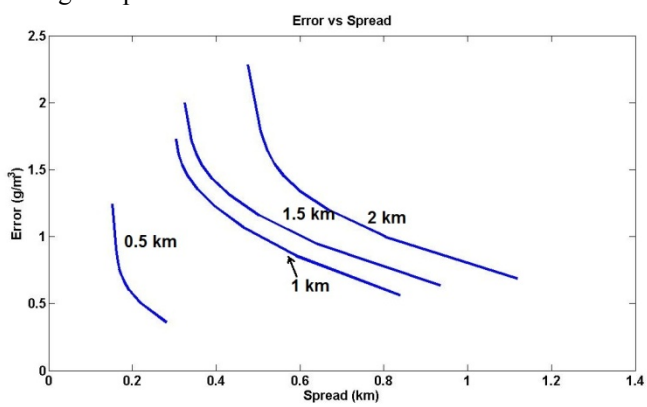

Figure 5. Trade-off of spread versus error of retrieval at various altitudes above the ground level

6. Conclusion

The results presented in this paper show that the Backus-Gilbert technique can be used to improve the vertical resolution of retrieved water vapor profiles from microwave radiometer measurements with certain limitations. The vertical resolution of retrieved water vapor profile is evaluated for a four-channel vertically scanning radiometer using the Backus-Gilbert technique. The trade-off curves obtained can be used to determine the limitations of the measurements for a given set of frequencies. Therefore, the water vapor profile can be estimated in the PBL at finer resolution by relaxing allowed errors in water vapor retrieval. Observation of these gradients in the PBL with improved vertical resolution is likely to enhance the accuracy of weather prediction.

7. References

1. F. Iturbide-Sanchez, S. C. Reising and S. Padmanabhan, "A Miniaturized Spectrometer Radiometer Based on MMIC Technology for Tropospheric Water Vapor Profiling," *IEEE Trans. Geosci. Remote Sensing*, **44**(7), July 2007, pp. 2181-2193.

2. C. D. Rodgers, *Inverse Methods for Atmospheric Sounding: Theory and Practice*, Singapore: World Scientific, 2000.

3. J. A. Schroeder and E. R. Westwater, "Guide to Microwave Weighting Function Calculations," *U.S. Dept. of Commerce, National Oceanic and Atmospheric Administration, Wave Propagation Laboratory*, July 1992.

4. G. E. Backus and F. E. Gilbert, "Uniqueness in the Inversion of Inaccurate Gross Earth Data," *Phil. Trans. Roy. Soc. London*, **A266**, 1970, pp. 123-192.

5. B. J. Conrath, ''Vertical Resolution of Temperature Profiles Obtained from Remote Radiation Measurements,'' *J. Atmos. Sci.*, **29**, 1972, pp. 1262–1271.