A method of calculation of electron density profiles from \( h'(f) \) traces of vertical sounding

**O. A. Laryunin, V. I. Kurkin**

ISTP SB RAS (Institute of Solar-Terrestrial Physics of Siberian Branch of Russian Academy of Sciences),
Irkutsk, 126a, Lermontov Street, 664033, Russian Federation
mail to: laroleg@inbox.ru

**Abstract**

A necessity to develop a new method to calculate the electron density profiles has been vital due to inaccuracy when calculating the profiles through the widespread Huang-Reinisch method (including the one used at ISTP SB RAS) [1]. The algorithm we offer calculates electron density distribution in the middle ionosphere containing maximum in the E-layer (or without it) by height-frequency \( h'(f) \) traces of vertical sounding.

**1. Introduction**

In this paper we describe a method developed at ISTP SB RAS to calculate the electron density profile in the anisotropic ionosphere from vertical sounding. The input data in the problem provided are magnetic inclination and \( h'(f) \) traces for ordinary and extraordinary wave. The output data are the electron density profile (or the plasma frequency profile). The program implementing this algorithm has been tested for a great number of model profiles, and the profiles calculated provide a coincidence accurate enough with the original ones except a valley region discrepancy inevitable for any approximating method.

**2. Determining the heights of the E and F layer peak**

At the first stage, the program calculates the \( E \) and \( F \) layer peak heights. For this purpose, one can use the known empirical formula of the view:

\[
h_{0E}F2 = \frac{1490}{M(3000)F2 + \Delta M} - 176
\]

which is known, at least, in its three versions:
- the Shimazaki formula [2], for which \( \Delta M = 0 \);
- the Bradley-Dudeney formula [3], where \( \Delta M = \frac{0.18}{X_E - 1.4} \);
- the Dudeney formula [4]: \( \Delta M = \frac{0.253}{X_E - 1.215} \).

Here \( X_E = f_0F2 / f_0E \).

The parameter \( M(3000)F2 = \frac{MUF(3000)}{f_0F2} \) is determined from the \( h'(f) \) trace by the Smith method [5] in the following sequence: from dependence \( h'(f) \) (for this instance, the \( h'(f) \) trace in Figure 1b was simulated from the profile in Figure 1a with the \( F \) layer critical frequency \( f_0F2 = 4.58MHz \) and peak height \( h_{0F}F2 = 206km \)) we obtain the dependence of the incidence angle on the frequency at oblique sounding \( \varphi(f)[6] \):
\[
\varphi(f) = \arctg \left[ \frac{\sin \left( \frac{D}{2R} \right)}{1 + \frac{h'(f)}{R} - \cos \left( \frac{D}{2R} \right)} \right]
\]

(2)

where \( D = 3000 \text{km} \) is the path distance, \( R \) is the Earth radius. Further, we find the frequency at oblique sounding (Figure 1c):

\[
f_{\text{cos}}(f) = \frac{k \cdot f}{\cos \varphi(f)}
\]

(3)

where \( k = 1.116 \) is the factor of the Earth sphericity for \( D = 3000 \text{km} \).

Finally, \( MUF(3000) \)-factor is equal to the maximum of function \( f_{\text{cos}}(f) \) (Figure 1c).

![Figure 1](image1.png)

Figure 1. Determining the \( MUF(3000) \)-factor by the Smith method.

The listed ways to determine the layer peak height by using the \( h'(f) \) function have been tested for a great number of the profiles taken from DPS-4, obtained by the IRI-2000 model or made by analytic functions. Analyzing the errors of the three formulas, the authors decided to use the Shimazaki formula to calculate the \( E \) and \( F \) layer peak heights.

3. Calculation of the electron density profile

The gyrofrequency is determined from the relation

\[
f_{\text{gyro}} = f_{\text{co}} - f_{\text{co}} \approx \frac{f_{\text{HI}}}{2} \quad [7].
\]

To approximate the electron density profile below the \( E \) layer peak we use the function of the following view
\[
f_p(z) = f_E \exp \left( - \left( \frac{h_m E - z}{l_E} \right)^{k_E} \right), \quad z \leq h_m E
\]  

(4)

where \( f_E \) - is the E layer critical frequency determined by the O \( h'(f) \) function, and the E layer peak height \( h_m E \) is calculated by the method described above. \( l_E \) and \( k_E \) are found as follows: for each fixed degree \( k_E \) we carry out enumeration of \( l_E \) scale values within the 10 to 25 km range in 1 km increments whereas in the outer cycle \( k_E \) accepts values between 1,5 and 2,5 in 0,1 increments. We consider the ranges indicated cover the values \( l_E \) and \( k_E \), necessary to approximate the overwhelming majority of the profiles below the E layer peak by a function of the view (4). Further, for each pair of values \( (l_E, k_E) \) the program simulates the O \( h'(f) \) trace in 0,1 MHz increments over the range between the minimum frequency \( f_{\text{min}} \) recorded in an ionogram and the E layer critical frequency \( f_E \). Afterwards, using the least square method, we search for the closest coincidence of the \( h'(f) \) trace simulated with the original one, i.e. the minimum of the values is searched for

\[
S(i, j) = \sum_j \left( h'_\exp(f) - h'^{\text{sim}}(f) \right)^2
\]  

(5)

where \( h'_\exp(f) \) - is the virtual height for the experimental \( h'(f) \) trace at frequency \( f \), \( h'^{\text{sim}}(f) \) - is virtual height for the \( h'(f) \) trace, simulated from the profile with parameters \( (l_E, k_E) \). The pair of \( (l_E, k_E) \), corresponding to the minimum \( S(i, j) \), is the required one for substitution into Expression (4).

Further, to calculate the profile at heights \( z \geq h_m E \) the approximating function of the following view is used:

\[
f_p(z) = f_{c0} \exp \left( - \left( \frac{h_m F2 - z}{l_0} \right)^{k_0} \right) + (f_E - \Delta f) \exp \left( - \left( \frac{z - h_m E - \Delta z}{l_E} \right)^{k_E} \right) + f_E \exp \left( - \left( \frac{z - h_m E}{l_E} \right)^{k_E} \right)
\]  

(6)

In the first, basic summand (6) \( l_0 \) and \( k_0 \) are determined by the least square method as described above, with the only difference that the range of their possible values is extended to \( 45km \leq l_0 \leq 130km \) and \( 1,5 \leq k_0 \leq 3,5 \), and comparison of group delays occurs then in the frequency interval \( f_E < f < f_{c0} \). The third summand (6) optimizes the approximation in the valley region: height \( h_z \) is the local minimum point of function

\[
f_{c0} \exp \left( - \left( \frac{h_m F2 - z}{l_0} \right)^{k_0} \right) + f_E \exp \left( - \left( \frac{z - h_m E}{l_E} \right)^{k_E} \right)
\]  

in the interval \( h_m E < z < h_m F2 \). Small corrections \( \Delta f \) and \( \Delta z \) in the addend emerge because appearance of the first and third summand in Expression (6) results in violation of the indispensable conditions \( f_p(h_m E) = f_E \) and \( \frac{df}{dz} (h_m E) = 0 \), typical of Relation (4).

In Figure 2, we present an example of the algorithm operation where the original profile (dashed line) was taken from an IRI-2000 model. Inevitable discrepancy in the valley region takes place as well as it does at small heights.
Figure 2. Example of the profile calculation: dotted line shows the original profile, solid line represents the calculated one.

4. References


