Calculation of the Double Scattering from Lossy Dielectric Cylinders

Qianyi Zhao[1] ***and Roger H. Lang***[1]

[1]Department of Electrical and Computer Engineering, The George Washington University, Washington DC 20052, USA, zhao@gwu.edu, lang@gwu.edu

Abstract

A numerical Fresnel Double Scattering (FDS) method is presented in this paper to accurately calculate the bistatic cross sections due to double scattering between two lossy dielectric cylinders in the Fresnel zone of each other. The cylinders have a comparable size to the wavelength for L band frequencies. It is demonstrated that the FDS results reduce to the far field results when the scatterers are sufficiently far apart. The FDS method can be employed to study the double scattering effects between tree branches in microwave forest scattering models.

1. Introduction

Forests provide a sink for carbon and their health should be monitored globally. Both active and passive sensors at frequencies from P band to X band can provide useful data to understand forest properties. Microwave models of forests provide an important means for interpreting experimental results. These models replace trunks, branches and leaves by finite lossy dielectric cylinders and disks. Both coherent [1, 2] and incoherent [3] single scattering techniques have been used to compute the radar backscatter from forests and to estimate their emissivity. These theories assume that the forest scatterers are independent of one another, i.e., it is assumed there is no correlation between forest elements. As the frequency increases, multiple scattering becomes more important. The point at which multiple scattering effects start to appear depends on the size of the scattering elements and their density. Radiative transport theory has been used to calculate these effects [4, 5]. Transport theory is an incoherent method and thus coherent effects (enhancement) are not included. More important, however, is that transport theory assumes scatterers to be in the far field of one another. At L band frequencies, this is certainly not the case for most large branches in an individual tree. The question is: *Are these non-far field effects important?*

To attempt to answer this question, researchers have constructed a simulated tree and have used a numerical modeling technique to compute the scatter [6]. The problem is that the number of nodes needed is so large that only a small section of the tree can be considered. The authors of the present paper have used an alternative approach [7]. In our approach only first and second order scattering is considered. This restriction is reasonable since second order scattering will be the first multiple scattering effect to appear as the frequency is increased. We consider the problem of two large branches in close proximity to each other, i.e., the branches are within a Fresnel zone of each other. The calculation is carried out at L band. The exact solution of the two scatterer problem (treated as one body) is obtained by a numerical volume integral technique. This exact result is then compared with a second order scattering approximation that consists of single and double scattering from the two cylinders. In computing the double scattering, the Fresnel Double Scattering (FDS) technique, to be described in the next section, has been employed. It has been shown [7] that the exact and the approximate techniques give almost the same result for inter-cylinder spacing of one to many Fresnel zones. Although this does not answer the question posed above, it does provide a mechanism for testing the effect of double scattering in a large tree by a numerically efficient method. In this paper it will be shown that the FDS result tends to the transport result when the scatterers are in the far field of each other.

2. Fresnel Double Scattering Method

The Fresnel double scattering (FDS) method is introduced in this section to accurately calculate the double scattering from two dielectric cylinders. It is assumed that the cylinders are located one to many Fresnel zones from each other. The procedure of implementing the FDS method is as follows: (1) Assume plane wave $\mathbf{E}_{inc}$ is incident on the first scatterer and solve for its interior field assuming that the second scatterer is not present. (2) Use the interior fields in the first scatterer, to calculate its scattered field. This scattered field is then the incident wave on the second scatterer. (3) Solve for the interior field in the second scatterer (without the first scatterer being present) and calculate

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the scattered wave $\mathbf{E}_s^{DS}$ and the scattering amplitude dyadic scattering amplitude, $\underline{\underline{f}}^{DS}$, of double scatterer as shown as shown in Figure 1. The bistatic cross section σ_{pq}^{DS} due to double scattering is

$$\sigma_{pq}^{DS} = 4\pi \left| \mathbf{p} \cdot \underline{\underline{f}}^{DS} \cdot \mathbf{q} \right|^2 = \lim_{R \to \infty} \frac{R^2 \left| \mathbf{p} \cdot \mathbf{E}_s^{DS} \right|^2}{\left| \mathbf{q} \cdot \mathbf{E}_{inc} \right|^2} \qquad p, q \in \{h, v\}$$

where p and q are the polarizations for the scattered wave and the incident wave, respectively; R is distance between the observation point and the origin. Here the origin is located in the vicinity of the scatterers. A numerical volume integral technique: the Discrete Dipole Approximation (DDA) [8] is applied to solve for the interior electric fields inside the cylinders. Without assuming the cylinders are in the far field of each other, the FDS method provides an accurate calculation of double scattering between the two cylinders. This approach is fully polarimetric and bistatic. It can be extended to build a double scatter model for a collection composed of more than two scatterers by taking two at a time.

The results obtained by the FDS method are compared with the far field approximation usually used in the transport theory. Applying the far field approximation, the bistatic cross section $\sigma_{pq}^{DS(FAR)}$ due to double scattering from the first scatterer to the second scatterer in Figure 1 is

$$\sigma_{pq}^{DS(FAR)} = 4\pi \frac{\left| \mathbf{p} \cdot \underline{\underline{f_1}} \cdot \underline{\underline{f_2}} \cdot \mathbf{q} \right|^2}{s^2} \qquad (s \gg \frac{D^2}{\lambda})$$

where $\underline{\underline{f_1}}$ and $\underline{\underline{f_2}}$ are the dyadic scattering amplitudes of the first and second scatterers respectively; D is the length of the longer cylinder; λ is the wavelength in free space. The inter-cylinder separation s is measured as the distance between the centers of these two scatterers. The dyadic scattering amplitudes of the individual cylinders are also computed by the DDA method.

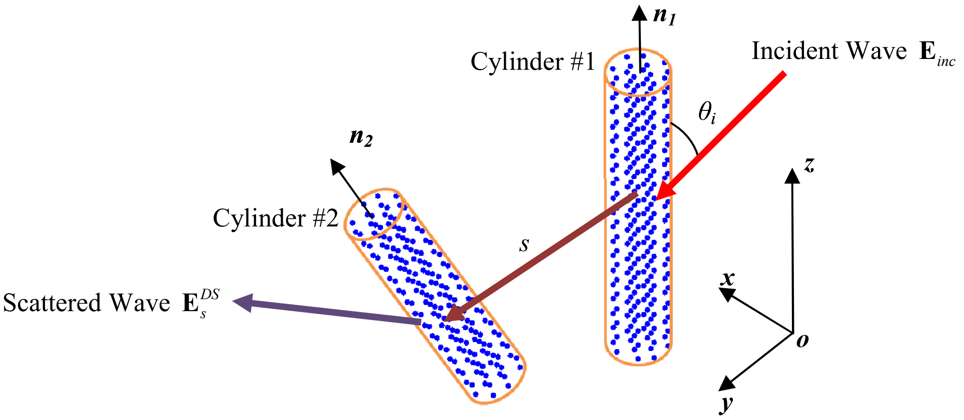


Figure 1. Geometry of double scattering from two lossy dielectric cylinders. Points represent the dipoles inside each cubic cell of the cylinders.

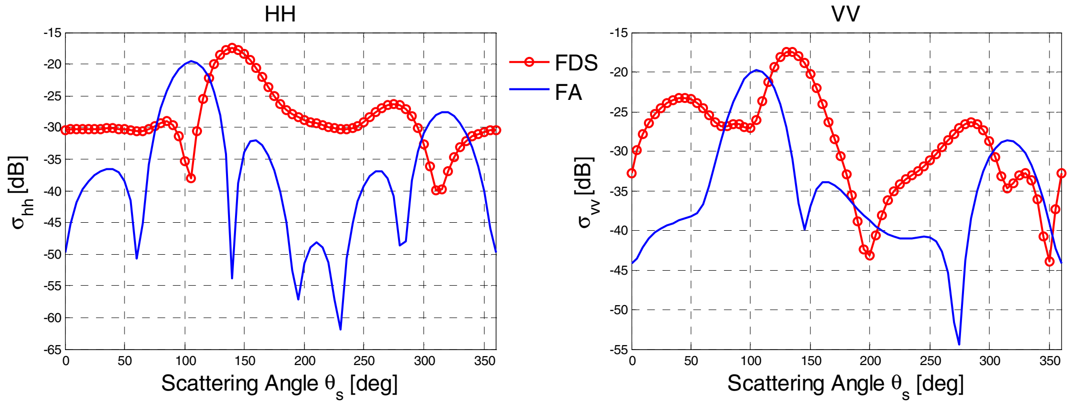
In this paper, the study focuses on the calculation of bistatic cross sections of double scattering from two dielectric cylinders. The FDS results will be compared with the far field results for various cylinder separations in the next section. They will show that if s gets larger, the outcomes of the FDS method will approach the far field results. When the scatterers are close to each other, the far field approximation is no longer valid and its values differ substantially from the output of the FDS.

3. Results

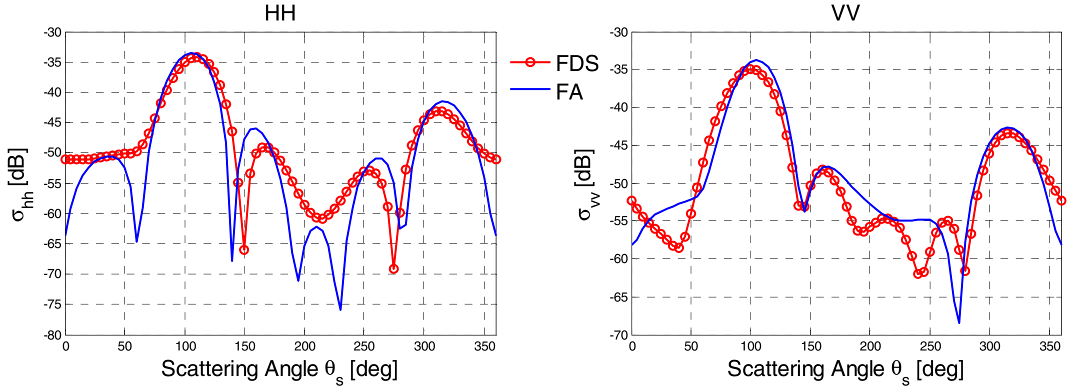
As a specific example, consider two lossy dielectric cylinders that are not necessarily in the far field of each other as shown in Figure 1. A plane wave is incident on two cylinders whose size and orientation information is given in Table 1. The incident wave has a frequency = 1.4 GHz and propagates parallel to the y0z plane with an elevation angle of 30^o ($\theta_i = 30^o$, $\varphi_i = 270^o$). Here φ_i is the azimuthal angle used in spherical coordinates. Cylinders are meshed into cubic cells with a dimension of 1cm×1cm×1cm for each cell to implement the DDA method.

	Size		Normal Vector **n**	
	Radius	Length	θ	φ
Cylinder #1	4cm	40cm	0^o	90^o
Cylinder #2	4cm	30cm	30^o	90^o

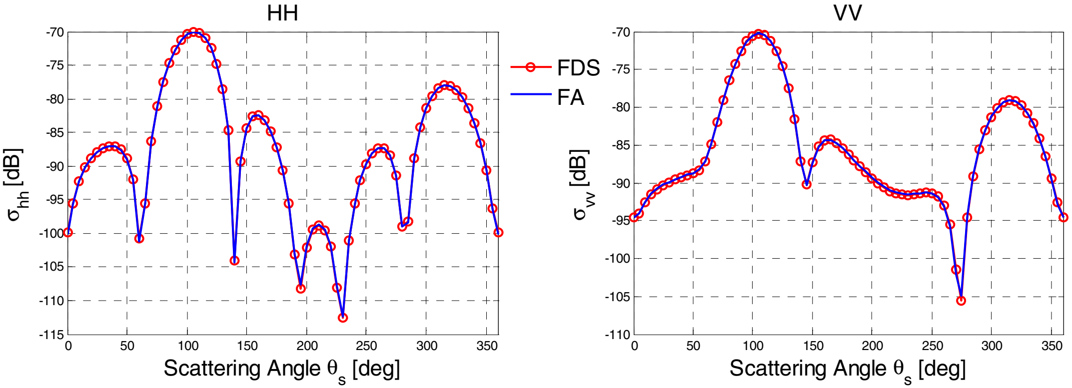
Table 1. Size and orientation of the cylinders in Figure 1. Vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ lie along the axis of each cylinder.



(a) $s = 0.3D^2/\lambda$



(b) $s = 1.5D^2/\lambda$



(c) $s >> D^2/\lambda$

Figure 2. Bistatic cross sections of double scattering from the first cylinder to the second one. FA denotes the far field approximation results and FDS denotes the Fresnel double scattering results.

The bistatic scattering cross sections due to the double scattering are observed at φ_s = 270^o and the plots are shown in Figure 2 as a function of the scattering elevation angle, θ_s, for both the HH and VV polarizations. The plots show that the FDS results (red cycles) are quite different from the far field approximate results (blue solid lines) when the branches are separated by s=0.3D^2/λ as shown in Figure 2(a). The far field method does not predict the interactions between the adjacent cylinders correctly and produces totally different patterns compared to the FDS scattering cross sections both in the HH and VV polarization. The difference between results using the two techniques is very large. They can differ by more than 30dB for certain scattering angles. As the separation between the cylinders increases to 1.5D^2/λ as shown in Figure 2(b), the FDS results are approaching but not exactly agreeing with the far field bistatic cross sections. In Figure 2(c), the distance between the centers of these two cylinders is sufficiently large, $s >> D^2/\lambda$. Therefore, the cylinders are in the far field of each other. The bistatic scattering cross sections due to double scattering obtained by both the FDS and the far field methods agree with each other. The far field approximation is valid and provides accurate results in this case.

From results presented in this section, it is clear that if the scatterers are not separated by many Fresnel zones, the far field approximation fails to give accurate double scattering calculations for both the HH and VV pol. Only in cases where the separation is sufficiently large is the far field approximation valid as expected. In general, the double scattering effects are small for two cylinders with comparable sizes to the wavelength. However, when considering a tree with thousands of branches, the double scattering effects may be quite noticeable and cannot be neglected.

4. Conclusion

The numerical Fresnel Double Scattering (FDS) method has been developed in this paper to model the double scatter from two lossy dielectric cylinders. The FDS method is based on the physical mechanism of double scattering from two scatterers. The discrete dipole approximation (DDA) is used to solve for the interior fields numerically. The FDS method does not assume that the scatterers are in the far field of one another, thus this method can be used when the scatterers are not in the far field of each other. The results have shown that as the distance between two cylinders increases, the FDS results approach the far field approximation results. The FDS method can be employed as a correction to the vegetation scattering models that only consider the single scattering. Future research will extend the FDS method to model a tree by taking the double scattering between tree components into account.

5. References

1. N.S. Chauhan and R.H. Lang, “Radar backscattering from alfalfa canopy: a clump modeling approach,” *Int. J. of Remote Sensing*, **20**, no. 11, July 1999, pp. 2203-2220.

2. R.H. Lang, C. Utku and P. de Matthaeis, “ESTAR and model brightness temperatures over forests: effects of soil moisture,” *Proc. IGARSS*, Jul. 2001 pp.1300-1302.

3. F.T. Ulaby, K. Sarabandi, K. McDonald and M. C. Dobson, “Michigan microwave canopy scattering model,” *Int. J. of Remote Sensing*, **11**, no. 7, July 1990, pp. 1223-1253.

4. H.J. Eom and A.K. Fung, “A scatter model for vegetation up to Ku-band,” *Remote Sensing of Environment*, **15**, 1984, pp. 185-200.

5. P. Ferrazzoli and L. Guerriero, “Passive microwave remote sensing of forests: a model investigation,” *IEEE Trans. Geosci. Remote Sens.*, **34**, no.1, Mar. 1996, pp.433-443.

6. T. Dufva, J. Praks, S. Järvenpää and J. Sarvas, “Scattering model for a pine tree employing VIE with a broadband MLFMA and comparison to ICA,” *IEEE Trans. Geosci. Remote Sensing*, **48**, no. 3, Mar. 2010, pp.1119-1127.

7. Q. Zhao and R.H. Lang, “Numerical analysis of scattering from a cluster of branches using the second order multiple scattering approximation,” *USNC-URSI National Radio Science Meeting*, Boulder CO, Jan. 2011.

8. B. T. Draine and P.J. Flatau, “Discrete dipole approximation for scattering calculations,” *J. Opt. Soc. Am. A* **11**, 1994, pp.1491-1499.