# Computation of Rain Attenuation in Tropical Region with Multiple Scattering and Multiple Absorption Effects Using Exponential Drop Size Distribution

***Fikih F. Amrullah***[1], ***Eko Setijadi***[2], ***Gamantyo Hendrantoro***[3]

[1]Department of Electrical Engineering, Institut Teknologi Sepuluh Nopember
Kampus ITS Sukolilo, Surabaya 60111, Indonesia, Email: fiam@elect-eng.its.ac.id
[2]Same as above, Email: ekoset@ee.its.ac.id
[3]Same as above, Email: gamantyo@ee.its.ac.id

## Abstract

Rain attenuation causes scattering and absorption of electromagnetic waves and could be a significant problem in radio propagation, especially in tropical region which has high rainfall rate. In this paper, raindrop was modeled using exponential raindrop size distribution and computed with multiple scattering and multiple absorption effect previously derived. It was assumed that raindrop shape is spherical and has dielectric constant following the Double Debye Model. Based on the analysis, rain attenuation effects become significant for frequencies above 10 GHz and reach the peak at about 125 GHz. Other important results are also reported.

## 1. Introduction

Rain attenuation causes scattering and absorption of electromagnetic waves. The impact is more pronounced in tropical regions due to high rainfall rate. The attenuation will be significant when considering electromagnetic wave propagation in tropical areas, particularly for frequencies above 10 GHz [1]. DSD (raindrop size distribution) model plays an important role in the estimation of rain attenuation based on scattering and absorption by raindrops. Single and multiple scattering mechanisms have been previously considered and assumed for such purposes. A previous research has formulated rain attenuation estimation method by considering the effects of multiple scattering [2]. This paper reports the computation of rain attenuation incorporating multiple scattering and multiple absorption effects to DSD models commonly proposed for tropical region. Comparisons with attenuation measured on a 28 GHz link in Surabaya, Indonesia, and with the ITU-R prediction are reported and discussed.

## 2. Raindrop Size Distribution Models

The form of DSD generally depends on rainfall intensity $R$ (mm/h). The exponential distribution model of raindrop size is obtained using the following equation [3]:

$$N(a) = N_0 \, e^{-\Lambda a} \tag{1}$$

where $N(a)$ in mm$^{-1}$m$^{-3}$ is the number of raindrops of radius $a$ per unit volume. In the Marshall-Palmer exponential distribution the value of $N_0$ is 16 000, $\Lambda = 8.2R^{-0.21}$ [3], whereas in Le-Wei Li, $N_0$ = 6359.391214 mm$^{-1}$m$^{-3}$, $\Lambda = 5.631800038\mathrm{R}^{-0.1998560971}$ [4]. The Wei Li model was derived from measurements made in Singapore and is thus representative of the tropical DSD.

To generate a raindrop of radius $a_q$ according to the exponential distribution we apply the following [2]:

$$a_q = -\frac{1}{\Lambda}\ln\left(1 - \frac{q - \frac{1}{2}}{Q}\right) \quad (q = 1,2,\ldots,Q) \tag{2}$$

Raindrops are generated randomly in the free space based on spherical coordinates having radius $a_f$ (mm). The total volume of rain, comprising a set of $Q$ drops, can be determined by an equation as follows [2]:

$$V = \frac{Q}{N(\infty)} = \frac{4}{3}\pi a_f{}^3 \tag{3}$$

978-1-4244-6051-9/11/$26.00 ©2011 IEEE

## 3. Rain Attenuation Moddelling

The numerical analysis method to compute rain attenuation uses the equation below [2]:

$$\bar{\gamma} = \frac{4343\sigma^e}{V} \text{ [dB/km]} \tag{4}$$

The method involves the absorption and scattering cross section to calculate the reduction of radio waves energy. Absorption cross section $\sigma^a$, is the ratio between the power absorbed inside the sphere with power density of incident field. Scattering cross section $\sigma^s$ is the ratio between the total scattered powers by the raindrop with power density of incident field. The sum of both is the total cross section or equal to the total power loss of radio waves due to raindrop, i.e., the extinction cross section $\sigma^e$ [5]. The value of $\sigma^e$ can be found by using the approach in the equation below [2]:

$$\sigma^e \approx -\frac{2\pi}{k_0^2}\operatorname{Re}\sum_{q=1}^{Q}\sum_{n=1}^{N_q}(2n+1)(\bar{A}_{qn}+\bar{B}_{qn}) \tag{5}$$

where $N_q$ is the truncation number. The value of $\bar{A}_{qn}$ and $\bar{B}_{qn}$ are the Mie's coefficients computed as follows [2]:

$$\bar{A}_{qn} = -\frac{J_{qn}\check{J}'_{qn} - \sqrt{\varepsilon_r}J'_{qn}\check{J}_{qn}}{\Delta_{qn}^{(1)}}, \qquad \bar{B}_{qn} = -\frac{J'_{qn}\check{J}_{qn} - \sqrt{\varepsilon_r}J_{qn}\check{J}'_{qn}}{\Delta_{qn}^{(2)}} \tag{6}$$

with $J_{qn} = k_0 a_q j_n(k_0 a_q)$, $\check{J}_{qn} = k a_q j_n(k a_q)$, $H_{qn} = k_0 a_q h_n^{(2)}(k_0 a_q)$, $\Delta_{qn}^{(1)} = H_{qn}\check{J}'_{qn} - \sqrt{\varepsilon_r}H'_{qn}\check{J}_{qn}$ and $\Delta_{qn}^{(2)} = H'_{pv}\check{J}_{qn} - \sqrt{\varepsilon_r}H_{qn}\check{J}'_{qn}$. The prime in $J'_{qn}$, $\check{J}'_{qn}$, $H'_{qn}$ denoted by replacing the variable with $k_0 r$ and $kr$ [2]. The value of $j_n$ is the spherical Bessel function of the first kind and the value of $h_n^{(2)}$ is the spherical Hankel function of the second kind. The value $k_0$ and $k$ can be replaced by:

$$k_0 = \sqrt{\varepsilon_0\mu_0}\,, \qquad k = k_0\sqrt{\varepsilon_r} \tag{7}$$

where: $\varepsilon_0 = 8.854187817 \text{ x } 10^{-12}$, $\mu_0 = 1.256637061 \text{ x } 10^{-6}$, and $\varepsilon_r$ denotes the complex permittivity of water according to the Double Debye model [6]

## 4. Numerical Result

Comparison of average radius of raindrop for the two DSD models is presented in Figure 1. The Marshall-Palmer DSD shows smaller average radius than Le-Wei Li DSD. It means that the average radius of the raindrop in the tropical region is greater than that in the non tropical.

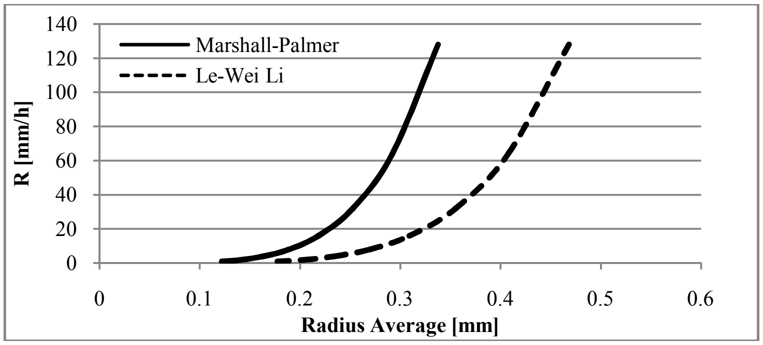


**Figure 1.** Average Comparison of Radius Rain on several DSD

Figure 2 compares specific rain attenuation obtained by Sekine [7] and those computed from the exponential DSD models. It could be seen that the rain attenuation effects starts to increase at frequency of 10 GHz and peaks around 125 GHz. Rain attenuation model using Le-Wei Li exponential DSD is closer to Sekine's result with 1.28638 RMSD.

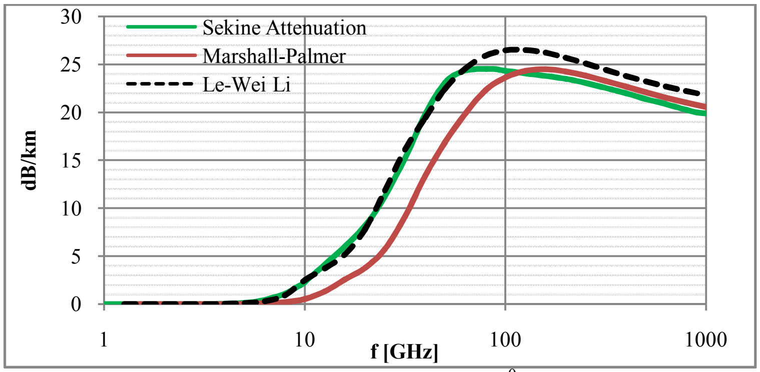


**Figure 2**. Comparison Rain Attenuation with T= $20^0$C, Q= 32, and R= 50 mm/h

A regression analysis between rainfall rate *R* and specific attenuation $\gamma$ was made from measurements on a short 29 GHz link in Surabaya, Indonesia according to the well-known power-law relation $\gamma = kR^\alpha$, with $k$ and $\alpha$ being the power-law coefficients (not to confuse $k$ used here with that in (7)). Consequently, coefficients $k$ = 2.642372 and $\alpha$ = 0.5886 were obtained from the regression [8]. Rain attenuation was subsequently computed [9].

In this study, the calculation of rain attenuation through numerical analysis assumes an average temperature of rain at $T$ = $25^0$C. The results from numerical analysis using exponential DSD and from rain attenuation measurement for rainfall from 1 to 372 mm/h are shown in Figure 3. It can be seen that rain attenuation obtained from the power-law model derived from measurement in Surabaya is closer to, but still higher than, those resulting from calculation using Le-Wei Li exponential DSD with 17.57372 RMSD value. There are two possible reasons that require further study: either that the rain DSD in Surabaya differs from the Singapore or that there are some other mechanisms that have not been accounted for in the numerical computation of attenuation.

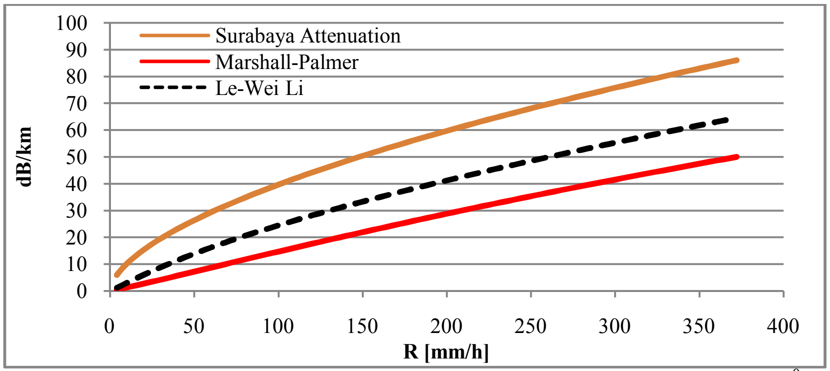
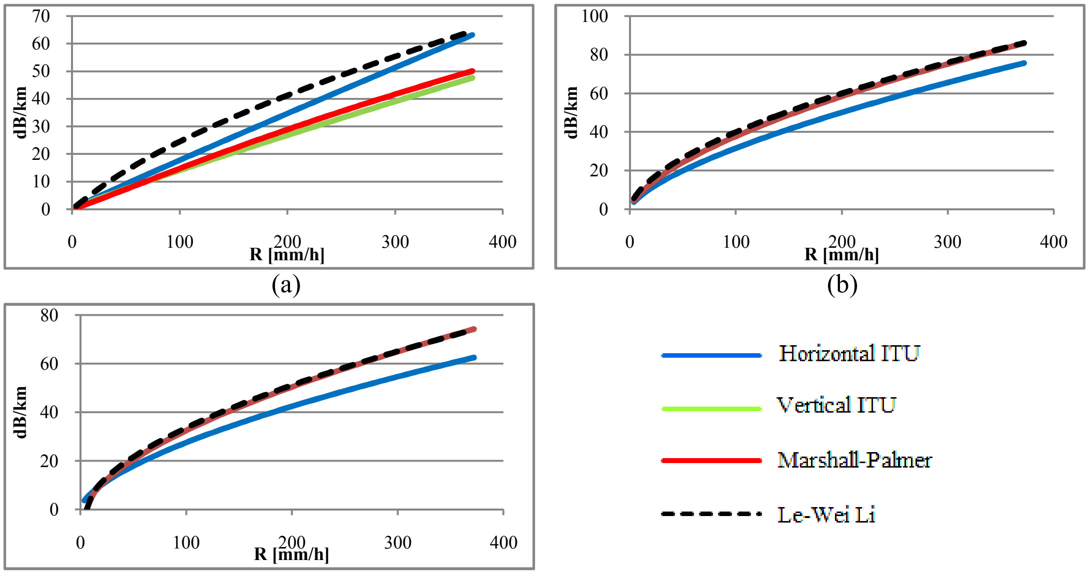


**Figure 3.** Comparison Rain Attenuation with f=28 GHz, Q=32 and T=$25^0$C

In the computation of rain attenuation using numerical analysis we assume $T = 25^0$C and $Q = 32$. Comparison between the results of numerical computation with that obtained using ITU-R P.838-3 [9] can be seen in Figure 4. Since the value of rain attenuation of horizontally-polarized waves approached that of vertical polarization at medium and high frequencies, it is adequate to take only one, which is the horizontal, in the comparison. It can be seen that the higher rainfall rate make the greater rain attenuation value. At 28 GHz rain attenuation model using Marshall-Palmer DSD approaches the ITU-R rain attenuation with vertical polarization, while the rain attenuation obtained using Le-Wei Li exponential DSD is closer to the ITU-R attenuation with horizontal polarization. However, at the higher frequencies, that is 120 GHz and 500 GHz, the numerically computed attenuation based on both exponential DSD models is several dB higher than the ITU-R prediction, with the difference getting larger for higher rain rate.



(a) (b)

(c)

**Figure 4.** Comparison of rain attenuation with calculations at (a) 28 GHz; (b) 120 GHz; and (c) 500 GHz.

## 5. Conclusion

It was found that the average radius of raindrop in the tropical region is greater than that in the non-tropical. Also, rain attenuation computed with Le-Wei Li exponential distribution model derived from measurement in Singapore is larger than that obtained using Marshall-Palmer model. These suggest that tropical regions experience larger rain attenuation. The attenuation effect starts from frequency of 10 GHz and peaks at about 125 GHz. Rain attenuation computed from Le-Wei Li DSD model is closer to, but still differs from, that obtained from a $\gamma$-$R$ power-law model derived from measurement in Surabaya. The difference suggests that a further study is required on the regional variation of DSD or on the numerical computation method. It was also observed that the ITU-R prediction of rain attenuation is lower than those computed from Le-We Lei DSD at 28 GHz. However, at 120 GHz and 500 GHz, both Marshall-Palmer and Wei Li models result in higher prediction of attenuation than the ITU-R method.

## 6. References

1. G. Hendrantoro, A. Matsushima, "Measurement and Modeling of Rain Intensity and Attenuation for the Design and Evaluation of Microwave and Millimeter-wave Communication Systems," in *Microwave and Millimeter-Wave Technology: Modern UWB Antennas and Equipment*, I. Minin (ed.), INTECH, 2010, pp. 271-300.
2. E. Setijadi, A. Matsushima, N. Tanaka, G. Hendrantoro, "Effect of Temperature and Multiple Scattering on Rain Attenuation of Electromagnetic Waves by a Simple Spherical Model." *PIER* 99, 2009, pp. 339-354.
3. J. S. Marshall and W. M. Palmer, "The distribution of raindrops with size," *J. Meteorology*, Vol. 5, 1948, pp. 165-166.
4. Y. T. Soon, P.S. Kooi, M.S Leong, L. W. Li, "Tropical Raindrop Size Distribution for the Prediction of Rain Attenuation of Microwaves in the 10-40 GHz Band," *IEEE Trans. Antennas Propagat.*. Vol. 49. No. 1, Januari 2001.
5. A. I. Tsolakis and W. L. Stutzman, "Multiple Scattering of Electromagnetic Waves by Rain," *Radio Science*, Vol. 17, No. 6, 1982, pp. 1495-1502.
6. H. J. Liebe, G. A. Hufford, and T. Manabe, "A Model for The Complex Permittivity of Water at Frequencies Below 1THz," *Int. J. Infrared Millimeter Waves*, Vol. 12, No. 7, 1991, pp. 659-675.
7. M. Sekine, C.-D. Chen, and T. Musha, "Rain attenuation from Log-normal AND Weibull Raindrop-size distribution," *IEEE Trans. Antennas Propagat.*, Vol. 35, No. 3, 1987, pp. 358-359.
8. R. Ariawan, "Analisis Pola Hubungan Redaman Hujan Radio 28 GHz dengan Curah Hujan Parsivel Disdrometer sebagai Data Pemodelan ARIMA," unpublished, 2010.
9. ITU-R P.838-3, "Specific attenuation model for rain for use in prediction methods," May 2005.