Predicting the Propagation Loss Through a Tree Canopy at Millimeter Frequencies – Forward Scattering Approximation – 3-D Vector Radiative Transport Theory

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Abstract

The vector radiative transport theory is used to compute the attenuation produced by a tree canopy containing random located lossy-dielectric leaves and branches at millimeter wave frequencies. Using this approach, the forward scattering approximation is used to simplify the radiative transport equation. The forward scattering approximation is used since at millimeter frequencies, the leaves and branches are large and thick compared to the wavelength; hence, a leaf or a branch scatter energy strongly in the forward direction and weakly in all other directions. The leaves are modeled as flat-circular lossy-dielectric discs and the branches as lossy-dielectric cylinders with prescribe orientation statistics.

1. Introduction

This work describes the use of the vector radiative transport theory for the specific intensity to understand the attenuation effects of a tree canopy on the propagation of millimeter waves. In this study, the vegetation canopy is modeled by a random collection of leaves and branches; the leaves are replaced by lossy-dielectric flat-circular discs and the branches by lossy dielectric cylinders. At millimeter wave frequencies, the wavelength is small compared to the size of the discs and cylinders and, as a result, a forward scattering approximation is employed to simplify the radiative transport equation. In this analysis, the specific intensity is split into two parts: the coherent and the incoherent intensity. The wave scattering from leaves and branches generates the incoherent waves in addition to the attenuated coherent wave. We show for the forward scattering approximation, how the ratio between the incoherent and coherent intensity varies with penetration depth for different tree canopy parameters and frequencies. Also, by using this approximation, we were able to find the attenuation variation with penetration depths for different tree canopy parameters and frequencies. The organization of the paper is as follows. Section 2 describes how the radiative transport theory is used to find the excess loss above free-space loss produced by random located leaves and branches of a tree canopy. Numerical results are presented in Section 3.

2. Formulation

Consider the canopy of a tree as a layer of thickness d, which is modelled by a slab full of leaves and branches as shown in Fig. 1. The leaves are modelled as randomly positioned flat-circular lossy-dielectric discs of radius a_d, thickness t, and relative dielectric constant ε_r. The branches also are modelled as randomly positioned lossy-dielectric cylinder of radius a_c, length l, and relative dielectric constant ε_r. The orientation of the scatterers is prescribed in terms of the spherical coordinates θ and ϕ [1]. It is assumed that the cylinders and the flat-circular discs are distributed uniformly in azimuthal coordinate ϕ. Here the azimuthal angle ϕ is defined in the plane perpendicular to the slab and parallel to the ground. As shown in Fig. 1, the tree canopy is modelled by a three-layered medium with free space for $z > 0$ and $z < -d$ with free space permeability μ_0 and permittivity ε_0. In the region $-d < z < 0$, the discs are assumed to be identical with a constant volume density ρ_d; similarly, the cylinders are assumed to be identical with a constant volume density ρ_c. A free space background medium is assumed in the slab. The interface between the two mediums is considered to be diffuse, i.e. not introducing any reflections. Now, since the incident field is linearly polarized wave, the scattered wave is generally elliptically polarized wave and its polarization varies randomly due to the randomness of the medium, therefore the scattered wave is partially polarized. In dealing with partially polarized waves where there is coupling between the horizontally and vertically polarized waves, the vector radiative transport equation of the specific intensity must be used and is given by [2]

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$$\hat{o}\cdot\nabla \underline{I}(\boldsymbol{x},\hat{o}) = -\underline{\underline{\kappa}}_e(\hat{o})\underline{I}(\boldsymbol{x},\hat{o}) + \rho(\boldsymbol{x})\int d\hat{o}'\underline{\underline{S}}(\hat{o},\hat{o}')\underline{I}(\boldsymbol{x},\hat{o}') . \quad (1)$$

Where we have assumed that the canopy emission contribution is negligibly and the vector specific intensity $\underline{I}(\boldsymbol{x},\hat{o})$ is a 4 by 1 column matrix whose components are the modified Stokes parameters ($W\, m^{-2}\, sr^{-1}$) as given in [2] and is interpreted as the intensity per solid angle of waves propagating through $\boldsymbol{x}$ in the direction $\hat{o}$, $\rho(\boldsymbol{x})$ represents the number of scatterers per unit volume, $\underline{\underline{\kappa}}_e(\hat{o})$ is the 4 by 4 modified extinction matrix that describes the attenuation of $\underline{I}(\boldsymbol{x},\hat{o})$ due to absorption and scattering in the direction $\hat{o}$, and $\underline{\underline{S}}(\hat{o},\hat{o}')$ is the 4 by 4 modified Stokes matrix that represents the angular distribution of vector intensity incident from $\hat{o}'$ into $\hat{o}$. The bracket represents average over particle angle distribution $\omega = \{\theta, \phi\}$. The modified extinction matrix $\underline{\underline{\kappa}}_e(\hat{o})$ is obtained from the coherent wave propagation of Foldy-Lax's approximation and is equal to [1]

$$\underline{\underline{\kappa}}_e(\hat{o}) = \rho(\boldsymbol{x})\underline{\underline{E}}(\hat{o}) \qquad \text{where} \quad \underline{\underline{E}}(\hat{o}) = \underline{\underline{w}}\left\{-\frac{2\pi i}{k_0}\left[\left\langle \underline{\underline{f}}(\hat{o},\hat{o})\right\rangle \otimes \underline{\underline{I}} - \underline{\underline{I}}\otimes\left\langle \underline{\underline{f}}^*(\hat{o},\hat{o})\right\rangle\right]\right\}\underline{\underline{w}}^{-1} \quad (2)$$

and $\underline{\underline{I}}$ is the 2 by 2 diagonal unit matrix, k_0 is the wave number in free space, $\underline{\underline{f}}(\hat{o},\hat{i})$ is the scattering amplitude matrix for a thick lossy-dielectric scatterer, and $\underline{\underline{w}}$ is a 4 by 4 matrix that makes $\underline{\underline{E}}(\hat{o})$ to be a real matrix. They are given by

$$\underline{\underline{f}}(\hat{o},\hat{i}) = \begin{bmatrix} f_{hh} & f_{hv} \\ f_{vh} & f_{vv} \end{bmatrix} \quad \text{and} \quad \underline{\underline{w}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \end{bmatrix}, \quad (3)$$

note that $\underline{\underline{w}}^{-1}$ is the inverse of matrix $\underline{\underline{w}}$ and $\underline{\underline{f}}^*(\hat{o},\hat{i})$ represents the complex conjugate of $\underline{\underline{f}}(\hat{o},\hat{i})$. Finally in (1), the modified Stokes matrix $\underline{\underline{S}}(\hat{o},\hat{o}')$ is given by

$$\underline{\underline{S}}(\hat{o},\hat{o}') = \underline{\underline{w}}\ \underline{\underline{P}}\ \underline{\underline{w}}^{-1} \qquad \underline{\underline{P}}(\hat{o},\hat{o}') = \left\langle \underline{\underline{f}}(\hat{o},\hat{o}';\hat{n}) \otimes \underline{\underline{f}}^*(\hat{o},o';\hat{n})\right\rangle_{\omega=\{\theta,\varphi\}} \quad (4)$$

and $\hat{n}$ represents the principal axes of the scatterer [1].

2.1 Forward Scattering Approximation – Thick Scatterer

In general, the interaction of the incident wave in direction $\hat{i}$ with the assembly of scatterers in the canopy can be obtained by knowing the modified Stokes matrix $\underline{\underline{S}}(\hat{o},\hat{o}')$ given by (4) and represents the angular distribution of the incident vector intensity into all directions including the direction of interest $\hat{o}$ as shown in Fig. 1. However, for millimeter waves, the leaves and branches are large and thick compared to the wavelength; hence, a leave or a branch scatter energy strongly in the forward direction and weakly in all other directions. Thus, for analytical convenience, the modified Stokes matrix $\underline{\underline{S}}(\hat{o},\hat{o}')$ can be characterized to a reasonable degree of approximation, by a modified Stokes matrix equal to

$$\underline{\underline{S}}(\hat{o},\hat{o}') = \underline{\underline{S}}_{bi}(\hat{o},\hat{o}') + \underline{\underline{E}}_f(\hat{o})\delta(\hat{o}-\hat{o}') . \quad (5)$$

Where $\underline{\underline{S}}_{bi}(\hat{o},\hat{o}')$ represents the bistatic scattering contribution matrix in every direction except in the forward direction, $\underline{\underline{E}}_f(\hat{o})$ represents the forward scattering contribution matrix, and $\delta(.)$ is the delta Dirac function.

Using (5) in (1), assuming $\rho(\boldsymbol{x})$ to be constant ρ in the slab and the canopy to be statistically symmetric about the *z-axis*, and letting $\hat{o}\cdot\nabla$ be equal to $(\hat{o}\cdot\hat{z})d/dz$ or $\cos\theta\ d/dz$; then (1) becomes

$$\mu\frac{d\underline{I}(z,\hat{o})}{dz}=-\rho\underline{\underline{E}}_M(\hat{o})\underline{I}(z,\hat{o})+\rho\int d\hat{o}'\underline{\underline{S}}_{bi}(\hat{o},\hat{o}')\underline{I}(z,\hat{o}') \tag{6}$$

where $\mu=\cos\theta$ and the matrix $\underline{\underline{E}}_M(\hat{o})$ in the forward direction is given by

$$\underline{\underline{E}}_M(\hat{o})=\underline{\underline{E}}(\hat{o})-\underline{\underline{E}}_f(\hat{o}) \tag{7}$$

2.2 Coherent Intensity

It is often useful to split the specific intensity vector $\underline{I}(z,\hat{o})$ into coherent and incoherent parts as follows

$$\underline{I}(z,\hat{o})=\underline{I}_c(z,\hat{o})+\underline{I}_i(z,\hat{o}) \tag{8}$$

where $\underline{I}_c(z,\hat{o})$ is the coherent specific intensity and $\underline{I}_i(z,\hat{o})$ is the incoherent specific intensity yielding a form of the radiative transport equation which offers more physical insight into the problem. Substituting (8) in (6) with the integral term suppressed, the radiative transport equation for the modified coherent specific intensity is given by

$$\mu\frac{d\underline{I}_c(z,\hat{o})}{dz}=-\rho\underline{\underline{E}}_M(\hat{o})\underline{I}_c(z,\hat{o}) \tag{9}$$

which solution is obtained by using the following boundary conditions $\underline{I}_c(z=0,\hat{o})=\underline{I}_{oq}\delta(\hat{o}-\hat{i})$ and $\underline{I}_c(d\to\infty,\hat{o})=0$. Where $\hat{q}=\{\hat{v},\hat{h}\}$ corresponds to vertically or horizontally polarization incident waves respectively and in the free space region for an unit incident plane wave, $\underline{I}_{oq}$ is equal to

$$\underline{I}_{ov}=[1\quad 0\quad 0\quad 0]^T \qquad \text{and} \qquad \underline{I}_{oh}=[0\quad 1\quad 0\quad 0]^T\ . \tag{10}$$

Finally, since we are using the forward scattering approximation which incorporates the strongest energy contributions of the incoherent specific intensity to the coherent specific intensity as shown in (7), then the total intensity is approximated by integrating the modified coherent specific intensity given by the solution of (9), over the entire solid angle $\hat{o}$ as

$$\underline{\Gamma}(z)=\int d\hat{o}\ \underline{I}_c(z,\hat{o})\ . \tag{11}$$

Equation (11) is interpreted as the incoherent power addition of the modified coherent specific intensity over all angles. The modified coherent intensity $\underline{\Gamma}(z)$ is interpreted as having a well defined direction of propagation and may be viewed as the continuation of the incident plane wave into the canopy, where it decreases exponentially due to absorption and scattering. Hence, the modified coherent intensity is the dominant component at intermediate distances into the canopy and this is due to the forward scattering approximation where the forward lobe of the scattering amplitude is narrow. The range of validity of the forward scattering approximation occurs when the actual depth of the tree canopy is less or equal to one or two skin depths [1]. The skin depth calculation includes the forward scattering approximation.

3. Numerical Results

Using the analytical approach presented in Section 2, numerical calculations of the attenuation produced by a canopy of a tree is presented in this section. The model requires specifying the physical and electrical characteristics of a leaf and a branch including their probability density distributions of the inclination angle θ and the azimuthal angle ϕ. In the numerical examples, the semi-empirical formula given in [3] for the complex relative permittivity of leaves is used for a dry matter of m_d = 0.3 and salinity 1% and for the branches a dry matter of m_d = 0.5 and salinity 1%. In addition, the leaves are assumed to have a radius a_d = 5 cm, a thickness t = 0.2 mm, and a density of $\rho_d = 200/m^3$ and the branches are assumed to have a radius a_c = 1.6 cm, a length l = 0.5m, and a density of $\rho_c = 2/m^3$. The probability density of the leaves and branches in the azimuthal coordinate ϕ are assumed to be uniformly distributed from 0^0 to 360^0. The probability density in the θ coordinate is dependent on vegetation type, however, for the leaves and the branches they are considered to be uniformly distributed and given by $p_\theta = 1/(\theta_2 - \theta_1)$. As an example, Fig. 2 shows a plot of tree attenuation versus distance for an incident horizontal polarization plane wave with an angle of incidence of 10^0 at 38 GHz. The inclination angle distribution of the leaves is characterized by $\theta_2 = 180^0$ and $\theta_1 = 0^0$, which implies that they have no preferred inclination. Also, the inclination angle distribution of the branches is characterized by $\theta_2 = 60^0$ and $\theta_1 = 0^0$. The plot shows that at millimeter wave frequencies the incoherent intensity becomes increasingly important of the total intensity as the depth of the canopy increases. This implies that, the attenuation only due to the coherent intensity has a much higher value that the total attenuation due to the coherent and the incoherent intensities. The reason lies that the incoherent intensity in the forward direction is generated due to scattering within the vegetation canopy and coupled into the coherent intensity producing an increase of the total intensity. Finally, the validity of the forward scattering approximation shown in Fig. 2 occurs when the vegetation depth is less than one or two skin depths. The skin depth for this example is about 4.8*m* [1]. Note that good agreement is obtained between the analytical model and measurements given in [4].

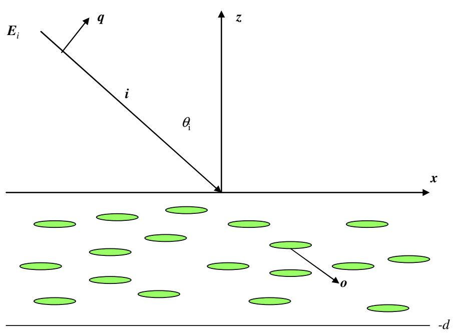

Fig. 1 Incident plane wave on a slab with thick dielectricdiscs and cylinders

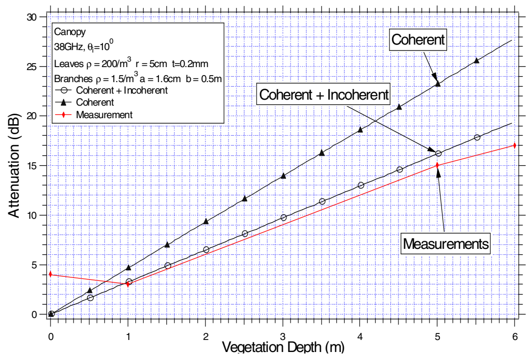

Fig. 2 Tree attenuation vs. distance at 38 GHz

4. Conclusions

A theoretical model has been developed to compute the total attenuation produced by the canopy of a tree. The canopy of the tree is viewed as an ensemble of leaves and branches. The analytical model developed is based on the vector radiative transport theory. The radiative transport equation is simplified and then evaluated by assuming the forward scattering approximation. Results show that by only considering the coherent intensity the excess loss is over estimated due to an ensemble of random located lossy-dielectric discs and cylinders. We could conclude by noting that at intermediate distance into the vegetation, with only few trees on path, the attenuation rate has a higher rate as at larger vegetation depths and which transition occurs when the dominant forward scattering propagation mode changes from the strongly attenuated direct path mode to a multiple scatter mode that is much less attenuated.

7. References

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