TO THE PROBLEM OF ELECTROMAGNETIC WAVES PROPAGATION IN TURBULENT MAGNETIZED PLASMA SLAB

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Abstract

Second order statistical moment of the ordinary and extraordinary waves scattered by turbulent magnetized plasma slab with electron density and magnetic field fluctuations is analytically calculated applying the perturbation method. Numerical calculations are carried out for the anisotropic Gaussian fluctuation spectrum at different anisotropy factor and the angle of inclination of prolate irregularities with respect to the external magnetic field. Phase portraits of correlation function of the phase and amplitude fluctuations of scattered radiation are constructed. It is shown that correlation between ordinary and extraordinary wave decreases in proportion to the anisotropic factor.

1. Introduction

At the present time the features of light propagation in random media have been very well studied. Analysis of the statistical moments of small-amplitude electromagnetic waves scattered by turbulent anisotropic plasma slab is very important in many practical applications associated with both natural and laboratory plasmas. Angular power spectrum, scintillation effects and the angle-of-arrival of scattered electromagnetic waves by turbulent anisotropic magnetized ionospheric plasma slab for both power-law and anisotropic Gaussian correlation functions of electron density fluctuations were investigated analytically (in the geometrical optics approximation) and numerically in [1-3]. Clemmow-Mually-Allias diagrams for two type waves exiting in a gyrotropic medium were given in [4]. Statistical characteristics of the amplitude and phase of scattered radiation by turbulent magnetized plasma slab with electron density and magnetic field fluctuations (vary both in magnitude and direction) via the perturbation method were analyzed in [5]. Phase portraits of the amplitude and phase fluctuations have been constructed. Variance of the Faraday angle and depolarization of metric ordinary and extraordinary waves scattered by magnetized plasma slab at different orientation of the receiving antennas were reported in [6].

Linearly polarized wave in the earth's anisotropic ionosphere generates the ordinary and extraordinary waves with slightly differing phase velocities. Using the perturbation method the second order statistical characteristics - correlation functions of the amplitude and phase fluctuations - are derived for arbitrary correlation function of electron density fluctuations and second-rank tensor of magnetic field fluctuations. Electron density and magnetic field fluctuations are statistically independent since the statistically isotropic scalar field is not correlated with the solenoidal vector field. The obtained expressions are valid for both near and far zones with respect to the turbulent plasma slab. Numerical calculations are carried out for anisotropic Gaussian correlation function for electron density fluctuation and correlation tensor of the second order for the fluctuation of an external magnetic field using the satellite data. Directional fluctuations of an external magnetic field on the statistical characteristics are analyzed by phase portraits of the correlation functions of the amplitude and phase fluctuations.

2. Calculation of second order statistical moment of scattered ordinary and extraordinary waves by magnetized plasma slab

Let's the frequency of an incident wave exceeds the ion gyrofrequency and the conditions are satisfied $\omega >> \Omega_i = eH_0 / Mc$, $\omega >> \nu_{eff}$; M is the mass of an ion, c is the speed of light in vacuum, $H_0 \in yz$ (principal

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plane) is the mean unperturbed strength of the external magnetic field, ν_{eff} is the effective electron collision frequency with the ions and molecules. When the wave passes through a region containing irregularities of refractive index, both amplitude and phase fluctuations arise in the wave front. Using the perturbation method each term could be presented as the sum of the mean value and small fluctuating terms: $\mathbf{E} = <\mathbf{E}> + \mathbf{e}$, $\mathbf{H_0} = <\mathbf{H_0}> + \mathbf{h}$, $N = <N> + n$. Fluctuating values are random functions of the spatial coordinates. Analytical expressions of two-dimensional spectral electric field for scattered ordinary and extraordinary waves propagating along the external magnetic field have the following form [2,5]:

$$e_z^{(O,E)}(k_x,k_y,L) = \frac{2i}{\delta_1 k_0 \varepsilon_{zz}} \tilde{\varepsilon}_{xy} (1-4\zeta_1\zeta_2) \int_0^L dz'\, g_z(k_x,k_y,z')(k_x^2+k_y^2) \cos\left[(L-z')k^{(O,E)} x_1\right] - \frac{4\zeta_3}{\delta_2 \varepsilon_{zz}} \tilde{\varepsilon}_{xy} \cdot$$

$$\cdot \int_0^L dz'\, g_x(k_x,k_y,z')(-i k_x + k_y) \sin\left[(L-z')k^{(O,E)} x_2\right] + \frac{2 i k_0}{\delta_2 \varepsilon_{zz}} \tilde{\varepsilon}_{xy} \int_0^L dz'\, g_z(k_x,k_y,z') \left[2\tilde{\varepsilon}_{xy} + \frac{1}{k_0^2}(1-4\zeta_1\zeta_2)(k_x^2+k_y^2)\right] \cdot$$

$$\cdot \cos\left[(L-z')k^{(O,E)} x_2\right], \tag{1}$$

where: $g_x = <E_x> \Upsilon_0 \left[\Upsilon_1 n_1(\boldsymbol{\kappa},z) + \Upsilon_2 h_z(\boldsymbol{\kappa},z)\right]$, $g_y = i g_x$, $g_z = <E_x> \Upsilon_3 \left[h_x(\boldsymbol{\kappa},z) + i h_y(\boldsymbol{\kappa},z)\right]$, $\Upsilon_1 = 1+\sqrt{u}$, $\Upsilon_0 = -\mathrm{v}/(1-u)$, $\Upsilon_2 = \sqrt{u} + 2u(1+\sqrt{u})/(1-u)$, $\Upsilon_3 = \mathrm{v}(u+\sqrt{u})/(1-u)$, $\mathrm{v} = \omega_p^2/\omega^2$, $\omega_p = (4\pi N_0 e^2/m)^{1/2}$, $u = \Omega_H^2/\omega^2$, $\Omega_H = eH_0/mc$ is the electron gyrofrequency, $x_1 = \zeta_1 - \zeta_2(\gamma_x^2+\gamma_y^2)$, $x_2 = \zeta_3 - \zeta_4(\gamma_x^2+\gamma_y^2)$, $\zeta_1 = (\varepsilon_{xx}+\tilde{\varepsilon}_{xy})^{1/2}$, $\zeta_2 = (\varepsilon_{xx}+\varepsilon_{zz}+\tilde{\varepsilon}_{xy})/4\varepsilon_{zz}\sqrt{\varepsilon_{xx}+\tilde{\varepsilon}_{xy}}$, $\zeta_4 = (\varepsilon_{xx}+\varepsilon_{zz}-\tilde{\varepsilon}_{xy})/4\varepsilon_{zz}\sqrt{\varepsilon_{xx}-\tilde{\varepsilon}_{xy}}$, $\gamma_x = k_x/k_0$, $\gamma_y = k_y/k_0$, $\zeta_3 = (\varepsilon_{xx}-\tilde{\varepsilon}_{xy})^{1/2}$, $\delta_1 = 4\zeta_1\tilde{\varepsilon}_{xy}$, $\delta_2 = 4\zeta_3\tilde{\varepsilon}_{xy}$, $\varepsilon_{xx} = \varepsilon_{yy} = 1-\mathrm{v}/(1-u)$, $\tilde{\varepsilon}_{xy} = \mathrm{v}\sqrt{u}/(1-u)$, $\varepsilon_{zz} = 1-\mathrm{v}$, $k_0 = \omega_0/c$, $\boldsymbol{\kappa} = \{k_x,k_y\}$ is transverse wave vector relative to the external magnetic field, L is a thickness of a turbulent slab, ε_{ij} are components of the second-rank tensor of collisionless magnetized plasma. Plane wave corresponds to the source remote at infinity, which is a good approximation at calculation of the statistical characteristics of fluctuating radio signals radiated from the geostationary satellites.

Correlation function of scattered ordinary and extraordinary waves for arbitrary correlation function of electron density fluctuations could be written as:

$$< e_z^{(O)}(x+\rho_x, y+\rho_y, L)\, e_z^{(E)*}(x,y,L) >_D \equiv -\frac{\Upsilon_0^2 \Upsilon_1^2 L}{2\varepsilon_{zz}^2} E_{x0}^2 \exp(i k_- L + i k_\perp \rho_y) \int_{-\infty}^{\infty} dk_x\, dk_y\, (k_x^2+k_y^2) \exp(i k_x \rho_x + i k_y \rho_y) \cdot$$

$$\cdot \int_{-\infty}^{\infty} d\rho_z\, W_N(k_x,k_y,\rho_z) \left\{ \frac{1}{2}\left[1-\cos(2x_2 k_+ L)\right] \frac{\sin(x_2 k_- \rho_z)}{x_2 k_+ L} + \frac{\sin(2x_2 k_+ L)}{2x_2 k_+ L} \cos(x_2 k_- \rho_z) - \frac{\sin(x_2 k_- L)}{x_2 k_- L} \cos(2x_2 k_- L) \cdot \right.$$

$$\left. \cdot \cos(x_2 k_+ \rho_z) - \frac{\sin(x_2 k_- L)}{x_2 k_- L} \sin(2x_2 k_- L) \sin(x_2 k_+ \rho_z) \right\}, \tag{2}$$

where: $k_\pm = k_0(N_O \pm N_E)/2$, $N_O^2 = 1-\mathrm{v}/(1+\sqrt{u})$, $N_E^2 = 1-\mathrm{v}/(1-\sqrt{u})$. We assume that $E_x^{(O)} = E_x^{(E)} \equiv E_{x0}$, $E_x^{(O)}$ and $E_x^{(E)}$ are the amplitudes of the linearly polarized ordinary and extraordinary waves, E_{x0}^2 is the intensity of an incident wave; angular brackets indicate the statistical average. This expression is valid for near $R << 1$ and far $R >> 1$ zones with respect to plasma slab boundaries, $R = L/k_0 l^2$ is the wavy parameter; $k_\perp$ is connected with the finite width of an incident wave. If $N_o = N_e = 1$ correlation functions of the phase fluctuations $\varphi_1 = \mathrm{Im}(e_i/<E>)$ ($i = x,y,z$) caused only by electron density fluctuations have the form:

$$< \varphi_1(x+\rho_x, y+\rho_y, L)\, \varphi_1^*(x,y,L) >_{zD} \equiv W_{\varphi zD}(\rho_x,\rho_y,L) = \frac{L}{2} \frac{\Upsilon_0^2 \Upsilon_1^2}{\varepsilon_{zz}^2} \int_{-\infty}^{\infty} dk_x dk_y\, k_x^2 \exp(i k_x \rho_x + i k_y \rho_y) \cdot$$

$$\cdot \int_{-\infty}^{\infty} d\rho_z\, W_n(k_x,k_y,\rho_z) \left[\frac{\sin(2x_2 k_0 L)}{2x_2 k_0 L} - \cos(x_2 k_0 \rho_z)\right], \tag{3}$$

In isotropic case ($\chi = 1$), at $\rho_x = \rho_y = 0$, in the absence of an external magnetic field and at $k_0 L << 1$, variance of the phase satisfies the well-known expression: $\sigma_\varphi^2 = \sqrt{\pi}\, \sigma_D^2\, \mathrm{v}^2\, k_0^2 L\, l / 4$. For the external magnetic field fluctuations we suggested correlation tensor of the second rank [2] with characteristic linear scale L_0 :

$$< \mu_i(\mathbf{r_1})\, \mu_j(\mathbf{r_2}) > = \frac{1}{12} < \mu^2 > \left(\frac{\partial^2}{\partial\rho_i \partial\rho_j} - \delta_{ij} \frac{\partial^2}{\partial\rho_s \partial\rho_s} \right) \Phi(\rho) \ , \qquad (4)$$

where $< \mu^2 >$ is the variance of an external magnetic field fluctuation.

3. Numerical calculations

Small-scale irregularities with Gaussian spectrum are responsible for polarization fluctuations at frequencies of 20-50 MHz. The most widely used spectral density function is Gaussian one, which has certain mathematical advantages. In case of forward scattering, $< n_1^2 > k_0 L << 1 << k_0 l_D$, when the single scattering condition is fulfilled $< n_1^2 > k_0^2\, l_D\, L << 1$, the medium is characterized by the Gaussian irregularity spectrum. On the lower boundary of inhomogeneous slab having thickness 100 -200 km and locating at the heights from 300 up to 500 km, the above-mentioned conditions are satisfied for the electromagnetic waves in the range of MHz frequencies and higher frequencies. Therefore, in the analytical calculations we use anisotropic Gaussian correlation function of electron density fluctuation [2]:

$$W_D(k_x, k_y, \rho_z) = \frac{\sigma_N^2}{4\pi} \frac{l_\parallel^2}{\chi\, \Gamma_0} \exp\left(-\frac{m^2}{l_\parallel^2} \rho_z^2 + i\, n\, k_x\, \rho_z \right) \exp\left(-\frac{k_x^2\, l_\parallel^2}{4\Gamma_0^2} - \frac{k_y^2\, l_\parallel^2}{4\,\chi^2} \right) \ , \qquad (5)$$

where: $m^2 = \chi^2 / \Gamma_0^2$, $\Gamma_0^2 = \sin^2\gamma_0 + \chi^2 \cos^2\gamma_0$, $n = (\chi^2 - 1) \sin\gamma_0 \cos\gamma_0 / \Gamma_0^2$, σ_N^2 is the variance of the electron density fluctuations. The average shape of electron density irregularities has the form of elongated ellipsoid. The ellipsoid is characterized with two parameters: the anisotropy factor $\chi = l_\parallel / l_\perp$ (ratio of longitudinal and transverse linear scales of plasma irregularities with respect to the external magnetic field) and the orientation, characterizing by the inclination angle γ_0 of the rotation axis with respect to the magnetic field. Numerical calculations for the electron density fluctuation are carried out in Cartesian coordinate system, and polar coordinate system is applied for the magnetic field fluctuations with so-called Markov assumption. The computation error of the mean-square fluctuations of the log amplitude due to the Markov assumption is proportional to $((\lambda L)^{1/2} / L)^{1/3}$. In the ionosphere L is varying from 50 km to a few hundred kilometers.

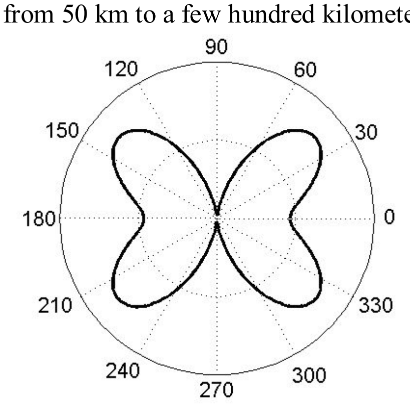

Figure 1. Phase portraits of correlation function of the phase fluctuations of scattered radiation at $k_0 L = 280$, $L = 100$ km, $L_0 = 1.2$ km.

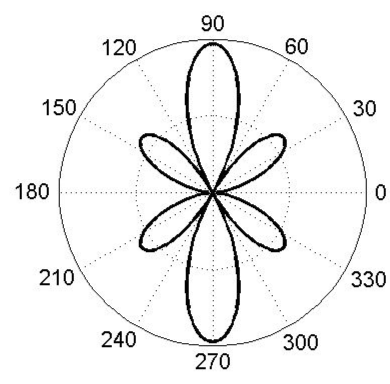

Figure 2. Phase portraits of correlation function of the amplitude fluctuations at $k_0 L = 280$, $L = 100$ km, $L_0 = 400$ m.

Numerical calculations of statistical characteristics of scattered radiation are carried out for correlation function (5) at $k_0 L >> 1$ using stationary phase method. Frequency of a linearly-polarized incident electromagnetic wave is 0.1 MHz. The mean ionospheric height is taken 300 km with plasma parameters $u = 0.22$ and $\mathrm{v} = 0.28$. Figures 1 and 2 illustrate the phase portraits of scattered radiation caused by fluctuations of the direction of an external magnetic field at different relations of the characteristic linear scales of the thickness of plasma layer and external magnetic field fluctuations. From these figures it follows that fluctuations of the direction of an external magnetic field substantially influence the behavior of phase portraits of statistical characteristics of scattered radiation in both principle and perpendicular planes. Therefore, in this case, besides the electron density fluctuations both the value and the direction variations of the external magnetic field should be taken into account.

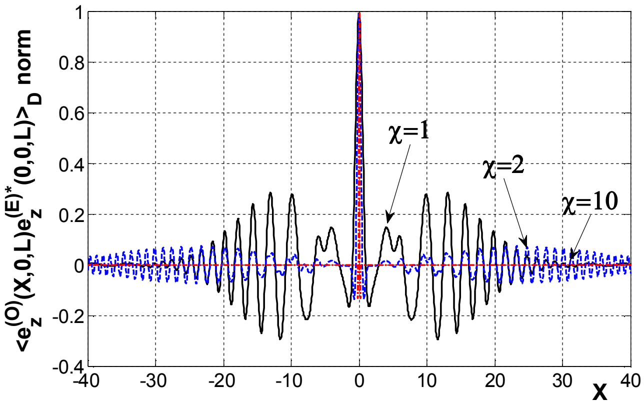

Figure 3. Normalized correlation function of scattered ordinary and extraordinary waves at $Y=0$, $\gamma_0=0$, $k_0L=120$, $L=100$ km, $l_{\parallel}=1$ km, $(k_{\perp}/k_0)<0.8$.

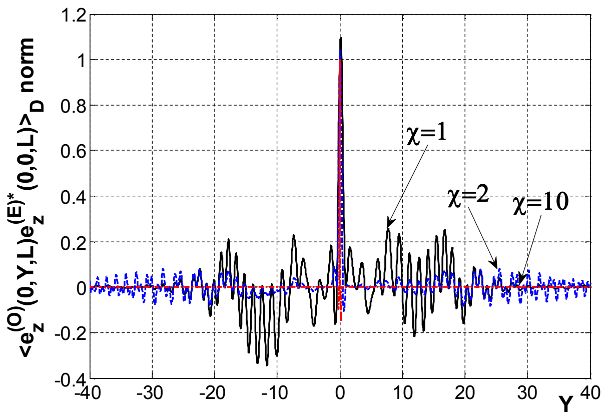

Figure 4. Normalized correlation function of scattered ordinary and extraordinary waves at $X=0$, $\gamma_0=0$, $k_0L=120$, $L=100$ km, $l_{\parallel}=1$ km, $(k_{\perp}/k_0)<0.8$.

Figures 3 and 4 illustrate the dependence of the second order statistical moments versus the distance between observation points. Analysis show that correlation between these waves decreases inversely proportion to the anisotropic factor χ, while the normalized root mean square deviation of the Faraday angle nonlinearly depends on the angle of inclination γ_0 of prolate irregularities and increases in proportion to the anisotropy factor [6].

4. Conclusion

Statistical characteristics of scattered ordinary and extraordinary waves by the magnetized plasma slab with electron density fluctuations have been investigated by the perturbation method taking into account both electron density and magnetic field fluctuations. Mutual correlation functions of these waves for arbitrary correlation function of fluctuating parameters inside magnetized plasma slab have been investigated numerically taking into account anisotropy factor and the angle of inclination of prolate irregularities with respect to the external magnetic field. It is shown that correlation of these waves substantially depends on the anisotropy factor of prolate irregularities.

5. References

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