Proposal of a general method to study wave propagation

Olivier MAURICE[1], Alain REINEIX[2] and Françoise PALADIAN[3]

[1]GERAC, France, URSI commission E vice-president. olivier.maurice@gerac.com
[2]Xlim, Limoges University, France, URSI commission E vice-president. alain.reineix@xlim.fr
[3]Clermont University, Blaise Pascal University, LASMEA, Clermont-Ferrand, France, URSI commission E president. Francoise.PALADIAN@lasmea.univ-bpclermont.fr

Abstract

This paper deals with the techniques to compute the wave propagation and power diffusion in a network. We propose a new approach to obtain the expression of the wave on any port of the network made of junctions. If this approach can appear a little heavy compared to the classical ones that use S parameters, it gives a powerful method to resolve the computation whatever is the complexity of the network.Based on a definition of the topology of the network with junctions and ports, we create a S matrix for the whole network and a propagation matrix G which manage the exchange of waves between the ports.Each time a product with G is applied, the waves are transmitted between the junctions through their input output ports. Once on a input port of a junction, the application of a product with the S matrix implies the transmission of the wave from this input port to an output port of the same junction. The product GS is applied as many time as it is necessary to reproduce the wave propagation in the whole network.

1. Introduction

The commission E of URSI has decided between various actions, to be involved in the dissemination of the knowledges in electromagnetism to the students. In this field, an important part is the understanding of the wave processes. These processes are involved in many jobs of industries, from mechanics to electronics and telecommunications. Starting from a work realized in a more complex objective of modeling of complex networks [1] and from the experience of teaching S parameters to university students, we have developed a simple and systematic approach to compute the transmission through a hyper frequency chain. We present the chain, we recall its resolution using classical S parameters then we construct our topology in order to define the matrices S and G then we show how to use them after having define a wave vector.

2. Hyper frequency chain definition

To illustrate our purpose we define a simple and matched chain. A generator, a perfect line, an amplifier, a divider and an antenna compose it. The S matrix of each element is known. The figure 1 shows the chain that we consider.

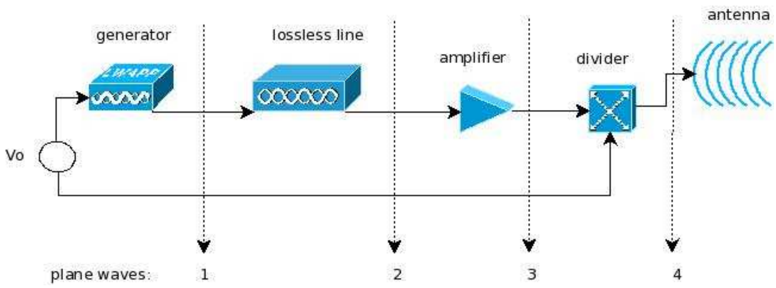

Figure 1: hyper frequency chain

3. Usual computation to obtain the antenna power supply

If a_k are the forward waves and r_k the backward waves, the both at the frontier k, we have to give to each wave plan the matrices S that give the relations between these waves. For the previous chain we obtain:

978-1-4244-6051-9/11/$26.00 ©2011 IEEE

For the waves through the divider: $\begin{bmatrix} a_4 \\ r_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} r_4 \\ a_3 \end{bmatrix}$.

For the waves through the amplifier: $\begin{bmatrix} a_3 \\ r_2 \end{bmatrix} = \begin{bmatrix} 0 & G \\ 0 & \sigma \end{bmatrix} \begin{bmatrix} r_3 \\ a_2 \end{bmatrix}$, G being the gain and σ the reflection coefficient.

For the lossless line of length x and wave vector k: $\begin{bmatrix} a_2 \\ r_1 \end{bmatrix} = \begin{bmatrix} 0 & e^{-jkx} \\ e^{-jkx} & \sigma \end{bmatrix} \begin{bmatrix} r_2 \\ a_1 \end{bmatrix}$

From these three relations, we compute the power going in the antenna of impedance Za:

$$P = \frac{1}{2}\frac{|a_4|^2}{Za} = \frac{1}{8}\frac{(G\alpha V_0)^2}{Za} \qquad (1)$$

But as we see, the process can be quite heavy if the network begins to be wide. In case of multiport networks, it request to identify the ports involved in the energy transmitted to a given output port.In the technique below, whatever are the complexity and the number of ports, once the method understood, the resolution becomes very easy.

3. Topology

The network is divided in junctions and ports. Like in the S parameter method, a port is a line; it means that two poles physically represent it. For our previous case, we give one junction to each element, each junction having one input port and one output port.The junction space in which the whole chain is represented has 6 ports and 3 junctions. Figure 2 represents the chain in a topological graph.

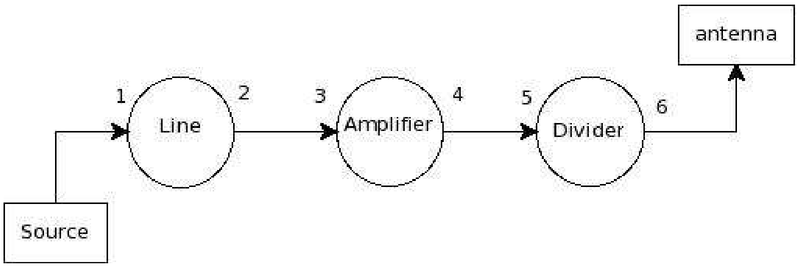

Figure 2: graph of the hyper frequency chain

4.The S matrix

If the network has 6 ports, it means that the S matrix is a 6x6 matrix.This matrix is made of the matrix of each element separated, placed in diagonal in the S matrix of the whole network.But at the opposite of the previous construction, we don't care of the variables order. Implicitly, the S parameters are the links between the ports inside a junction and if a signal is transmitted from port 1 to port 2, it is described by the S21 parameter, as usually. S parameters are null for couple of ports that does not belongs to the same junction. For our graph we obtain:

$$S = \begin{bmatrix} 0 & e^{-jkx} & 0 & 0 & 0 & 0 \\ e^{-jkx} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \end{bmatrix} \qquad (2)$$

Note that the matrix S of the amplifier is this time defined by its gain for S43. S34 is zero. For a perfect amplifier we have no S11 term.

5. The propagation matrix G

This matrix can change with time and presents many characteristics very interesting to translate the properties of the medium. But here, we just have to set to 1 the coefficient of relation between the two ports of a link of two junctions, for establish the matrix G.In our topology, we have only two links: one between ports 2 and 3, and one between ports 4 and 5. These links could have properties described in complex functions. To give the students a simple method to compute the transfer function of networks, we set the coefficients of the G matrix by the same manner as for a connectivity: if two ports *i* and *j* are in relation and if the wave travels from *i* to *j*, the coefficient G_{ji} is set to 1. If it is not the case, the coefficient is zero.Following these rules, we obtain for the matrix G:

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

6. Use of the process to compute the waves

The process uses the natural mechanic of the matrices for transmit the waves from port to port through some coefficients added by the elements that the waves go across.We first define a source wave vector, with its components equal to the generator amplitude when one generator exists on a port.Here we have only one generator and we write the vector p for the sources:

$$p = [g \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{4}$$

It's a vector made of 6 component, one by port, and g is the amplitude of the forward wave of the sourceV_0.In a first step, the wave has to go through the line. We realize a first product: S by p to obtain (all symbols are vectors or matrices):

$$S.p = [0 \quad ge^{-jkx} \quad 0 \quad 0 \quad 0 \quad 0] \tag{5}$$

we see that the wave has traveled from port 1 to port 2 at the output of the line, and so wave amplitude is zero on port 1, and g on port 2.Now, we multiply the previous product by G for realize the wave transfer.The wave vector now is:

$$G.S.p = [0 \quad 0 \quad ge^{-jkx} \quad 0 \quad 0 \quad 0] \tag{6}$$

The wave is located at the input of the amplifier.Let us multiply by S to realize the amplifier process and for obtain:

$$S.G.S.p = [0 \quad 0 \quad 0 \quad Gge^{-jkx} \quad 0 \quad 0] \tag{7}$$

If we have well presented the technique, the reader won't be surprised by the two next operations: we multiply one more time by G:

$$G.S.G.S.p = [0 \quad 0 \quad 0 \quad 0 \quad Gge^{-jkx} \quad 0] \tag{8}$$

we multiply by S:

$$S.G.S.G.S.p = \left[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{Gge^{-jkx}}{2}\right] \tag{9}$$

Which the last term is the solution.

7. Conclusion

The decomposition of a network problem in a space of the junctions can be a very effective way to analyze the propagation of informations like waves inside the network. If we want to make appear a unique time step for all the process that arrive in the network, it is necessary to create identity junctions to delay the wave on some paths for equalize the path time. Another method to do that consists in making the coefficient of G_{ii} equal to 1 (and G_{ij}=0) until the time to pass the wave occurs. At this moment, G_{ii} is set to zero and G_{ij} to 1.Many manipulations can be made, for making that the students to feel good with the matrices mechanisms. The product G.S can be reduced in a single matrix γ. The number of time the product by g is realized is equal to the dimension of a path between two ports. This dimension is the number of links in the path. In our example there was two links between the generator and the antenna. So the propagation operator γ must be set to the power 2: $output = S.\gamma^2.p$.

8. References

1. O.Maurice, A.Reineix, "Use of the Dirac like matrices to compute the wave propagation in various medium," *HaL server archives,*October 2010, http://hal.archives-ouvertes.fr/hal-00528234/fr/.