CNN computing of the interaction of fluxons

Angela Slavova[1], *Ronald Tetzlaff*[2], *Maya Markova*[3]

[1] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences,Sofia 1113, Bulgaria,slavova@math.bas.bg
[2] Institute of Fundamentals of Electrical Engineering and Electronics, Technical University Dresden, D-01069 Dresden, Germany, Ronald.Tetzlaff@tu-dresden.de
[3] Department of Informatics, University of Russe, Russe 7000, maya.markova@gmail.com

Abstract

In this paper we study Cellular Neural Network (CNN) computing of interaction of fluxons type solutions of the modified sine-Gordon equation. We propose analysis of such solutions by means of classical physical theory. Simulations of the obtained CNN model are given and discussed.

1. Introduction

In this paper we shall study the interaction of the physical object - quantum of magnetic flow, called fluxon. Fluxons are stable in the sense that they can be conserved, their direction can be changed and they can contact electronic devices. Their advantage is that they process information with a very high speed and with a very low energy supply. For this reason, fluxons have applications in information processing of electronic devices. In fact fluxons arise in the well known Josephson Junction (JJ) which is used in many applications in superconductor electronics [5].Recently, JJ have been considered as a source of THz radiation. Fluxons moving coherently in such junctions are a possible source of radiation. The equations which describe JJ are the following:

$$I = I_0 sin\ \varphi, \tag{1}$$

$$\frac{d\varphi}{dt} = \frac{2\pi}{\Phi_0} v, \tag{2}$$

where I is the lossless supercurrent flowing through the JJ, I_0 - its maximum value called critical current of the JJ, $\varphi(t) = \Theta_1 - \Theta_2$ is the difference between the phases of the complex pair wave functions of the both superconductors, v - the voltage drop over the junction, $\Phi_0 = 2.07 mV.ps$ is a fundamental constant called single flux quantum (SFQ).

In the investigation of fluxons, the famous sin-Gordon equation arises [3]:

$$\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial t^2} = sin\ \varphi, \tag{3}$$

In the analysis of JJ more generalized form of equation (1) can be proposed:

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$$I = I_0 sin\ \varphi + [G_0(v) + G_1(v) cos\ \varphi]v, \tag{4}$$

where G_1 and G_0 are rather complex functions of voltage and temperature [3]. If we consider G_1 and G_0 constant, then equation (3) can be written as follows:

$$\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial t^2} - \alpha(1 + \varepsilon cos\ \varphi)\frac{\partial\varphi}{\partial t} = \tag{5}$$
$$= sin\ \varphi - \gamma,$$

γ, $\Gamma \equiv G_0(\Phi_0/2\pi I_0 C)^{1/2} = const > 0$, $0 < \varepsilon \equiv G_1/G_0 \ll 1$, $\alpha \in [10^{-2}, 10^{-4}]$, C is capacitance. If α is very small equation (5) is equivalent to (3). Equation (5) actually describes dissipation in JJ. From the point of view of the analysis it will be easier to consider $G_0 = g_0(v)$ and $G_1 = g_1(v)$, g_0, $g_1 = const$ to be of quadratic form.

In this paper we shall consider the interaction of the solutions of equation (5) called fluxons [2,3]. We shall present Cellular Neural Network (CNN) computing for the interactions of fluxon-antifluxon and a pair of two fluxons. First we shall present the results from the physical point of view. Then we shall construct CNN model of the modified sin-Gordon equation. Finally we shall provide the simulation results and discuss the obtained results.

2. Cellular Neural Network Computing of the Interaction of Fluxons

For solving modified sine-Gordon equation (5) spatial discretization will be applied [4]. The equation is transformed into a system of ordinary differential equations which is identified as the state equations of a CNN with appropriate templates. We map $\varphi(x, t)$ into a CNN layer such that the state voltage of a CNN cell at a grid point is u_j. Let us consider one-dimensional CNN, where the CNN cells consist of a linear capacitor in parallel with a nonlinear inductor described by $i_j = f(\varphi_j) = \alpha(1 + \varepsilon cos\ \varphi_j)u_j - sin\ \varphi_j$ and where these cells are coupled to each other by linear inductors with inductance L. In terms of CNN circuit topology we can identified the following corresponding elements:

1). CNN cell dynamics:

$$\frac{du_j}{dt} = \frac{1}{C}[I_j - f(\varphi_j)], \tag{6}$$

$$\frac{d\varphi_j}{dt} = u_j, 1 \le j \le N; \tag{7}$$

2). CNN synaptic law:

$$I_j = i_{L_j} - i_{L_{j+1}} = \frac{1}{L}(\varphi_{j-1} - 2\varphi_j + \varphi_{j+1}), \tag{8}$$

where $\varphi_j(t) = \int_{-\infty} t u_j(\tau)d\tau$ is the flux-linkage at node j. Observe that the synaptic law (8) is a discrete Laplacian $A = [1, -2, 1]$of the flux linkage φ_j.

Let us write the dynamics of our CNN model (6), (7), (8) in the following form:

$$\begin{aligned} \frac{du_j}{dt} &= (\varphi_{j-1} - 2\varphi_j + \varphi_{j+1}) - \\ &\quad \alpha(1 + \varepsilon cos\ \varphi_j)u_j - sin\ \varphi_j \\ \frac{d\varphi_j}{dt} &= u_j, 1 \le j \le N. \end{aligned} \tag{9}$$

Remark 1. The relation between Cellular Neural Network (CNN) and the Josephson Transmission Line (JTL) array has been studied in many publications see for example [1],[5]. Two-dimensional array of Josephson Junctions is considered in [1]. Authors report the results of a Floquet analysis of such arrays of resistively and capacitively shunted JTLs in an external transverse magnetic field. The Floquet analysis indicates stable phase locking of the active junctions over a finite range of values of the bias current and junction capacitance, even in the absence of an external load.

3. Simulations and Discussions

Simulating our CNN model (9) we obtain the following results.The solutions

$$\varphi_{\pm} = \pm 4arctg\ e^{\pm \frac{\xi - \xi_0}{\sqrt{1-c^2}}} \tag{10}$$

are describing geometrically loops ξ varying from $\xi = -\infty$ to $\xi = +\infty$. If we take in both cases the sign "$+$" we obtain a positive loop, i.e. $\xi \in (-\infty, \infty) \Rightarrow \varphi \in (0, 2\pi)$, while if we take "$-$" in both cases we have again positive loop but $\xi \in (-\infty, +\infty) \Rightarrow \varphi \in (-2\pi, 0)$. The opposite signs give negative loops. Using a physical terminology: positive loops are fluxons and negative loops are antifluxons (Fig.1).

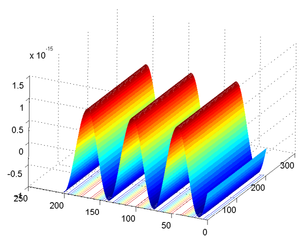

Fig.1.Simulation of the CNN model in the case of antifluxon solution.

When the a pair of fluxons interact they can go through each other as a "double" solution (Fig.2):

$$\varphi_D = 4arctg\{\frac{sh[u(t-t_0)/\sqrt{1-u^2}]}{uch[(x-x_0)/\sqrt{1-u^2}]}\}, \tag{11}$$

or link in "breathon" having the form (see Fig.3):

$$\varphi_B = 4arctg\{\frac{tg\nu sin[(cos\nu)(t-t_0)]}{ch[(sin\nu)(x-x_0)]}\} \tag{12}$$

One can see that the slower fluxon moving to the right with velocity is shifting additionally backward.The faster fluxon moving to the right with velocity is shifting additionally forward.

Remark 2. We consider a CNN programmable realization allowing the calculation of all necessary processing steps in real time. The results are obtained by the CNN simulation system MATCNN.

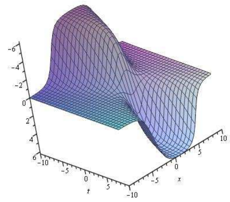

Fig.2. Simulation of the interaction of the pair of fluxons.

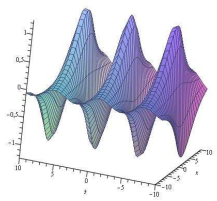

Fig.3.Simulation of "breathon" solution.

4. Conclusion

In this paper we study the interaction of fluxons, which arise in Josephson Junction (JJ). We present theoretical results for the modified sine-Gordon equation from the physical point of view. Using the relation between Cellular Neural Network (CNN) and the JTL we obtain the corresponding CNN model. The obtain simulations (Fig.1,2,3) illustrate the proposed theoretical results.

5. Acknowledgments

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6. References

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